NATURAL COORDINATE SYSTEM IN CURVED SPACE-TIME

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Abstract. In this paper we establish a generally and globally valid coordinate system in curved space-time with the simultaneous hypersurface orthogonal to the time coordinate. The time coordinate can be presented according to practical evolving process and keep synchronous with the evolution of the realistic world. In this coordinate system, it is convenient to express the physical laws and to calculate physical variables with clear geometrical meaning. We call it "natural coordinate system". The constructing method for the natural coordinate system is concretely provided, and its physical and geometrical meanings are discussed in detail. In natural coordinate system, we make classical approximation of spinor equation to get Newtonian mechanics, and then make weak field approximation of Einstein's equation and low speed approximation of particles moving in the space-time. From the analysis and examples we find it is helpful to understand the nature of space-time.

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1. Introduction

The selection of coordinate system in general relativity is important to conveniently express and solve physical equations. In a suitable coordinate system, the physical equations have simple and concise forms and definite physical meanings. In general relativity, the special coordinate systems we usually used for theoretical discussion are Gaussian normal coordinate system and harmonic coordinate system [24, Ch7.4, Ch10.4, Ch11.8]. However, the Gaussian normal coordinate system only exists locally. Other coordinate systems, such as Weyl-Lewis-Papapetrou one [17], all depend on some special structure and symmetry of space-time.

It was proved in [3–6] that, under some normal conditions, we can set up a coordinate system with global space-like Cauchy hypersurface. A more important problem is that, the realistic space-time is an evolving Lorentz manifold which has one and only one simultaneous hypersurface $f(x^\mu) = C$. This can be proved by contradiction of causal relation between events in neighbourhood similarly to the case of special relativity [9, 10]. Because the realistic multi-simultaneity enables us