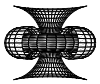
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ON THE SUPERSYMMETRY GROUP OF THE CLASSICAL BOSE-FERMI OSCILLATOR

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Abstract. Applying the concept of a momentum map for supersymplectic supervectorspaces to the one-dimensional Bose-Fermi oscillator, we show that the largest symmetry group that admits a momentum map is the identity component of the intersection of the orthosymplectic group $\mathrm{OSp}(2|2)$ and the group of supersymplectic transformations. This gives a systematic characterization of a certain class of odd supersymmetry transformations that were originally introduced in an ad hoc way.

1. Introduction

Supermechanics is the classical counterpart of quantum field theories involving Bose and Fermi fields. The most prominent use of supermechanics from a mathematical perspective is the role of the classical free particle Lagrangian in the supersymmetric proofs of various index theorems [2]. There has also been some interest in making the geometric description of supermechanics mathematically rigorous, both from a Lagrangian and Hamiltonian point of view [3, 9, 10].

In this note, we are concerned with the classical one-dimensional supersymmetric harmonic oscillator, or Bose-Fermi oscillator. By "classical", we mean that we treat it as a supermechanical system, defined on a supersymplectic flat manifold [10, 13]. It is a simple but a really nontrivial example of a system with supersymmetries, that is, symmetries that mix the fermionic and the bosonic degrees of freedom. It first appeared as one example of a supersymmetric quantum mechanical system in Witten's ground-breaking 1981 paper [15] and was further investigated in the 1980's and 1990's.

The infinitesimal supersymmetry transformations of the harmonic oscillator were initially introduced in an ad hoc way [5]. It was later realized that the stabilizer algebra of the dynamics is the orthosymplectic superalgebra $\mathfrak{osp}(2|2)$ [4]. In this