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A LORENTZIAN SURFACE IN A FOUR-DIMENSIONAL MANIFOLD OF NEUTRAL SIGNATURE AND ITS REFLECTOR LIFT

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Abstract. A Lorentzian surface in a four-dimensional manifold of neutral signature is called super-extremal if its reflector lift is horizontal. We give an elementary proof of a rigidity theorem for super-extremal surfaces in the space of constant curvature and neutral signature. As corollary, a characterization of the immersion of the Veronese type is given.

1. Introduction

The twistor lifts play an important role for oriented surfaces in oriented four-dimensional Riemannian manifolds and have been studied by many researchers (see [1, 3–5, 7–10] for example). In geometry of pseudo-Riemannian manifold of neutral signature, the reflector bundle is the corresponding object to the twistor space. For Lorentzian surfaces in four-dimensional manifolds of neutral signature, the reflector lifts are defined in [12], which are corresponding to the twistor lifts in Riemannian case. In this paper, we study Lorentzian surfaces in four-dimensional manifolds of neutral signature with horizontal reflector lifts, which are corresponding to superminimal surfaces in Riemannian geometry. In pseudo-Riemannian geometry, because of the failure of definiteness for metrics, different situations often occur from Riemannian cases. For example, in Riemannian case, a connected minimal surface of constant Gaussian curvature in the Euclidean space must be flat and an open part of a two-plane (see [2]). But one can find many non-totally geodesic extremal flat surfaces in the pseudo-Euclidean space of neutral signature (see Section 4), where extremal means vanishing of the mean curvature vector field. We say that a Lorentzian surface is super-extremal if its reflector lift is horizontal. Note that the notions of the reflector lift and super-extremal surface can be defined for higher even-dimentional cases. In [11], a rigidity theorem for super-extremal surfaces is obtained.

The purpose of this paper is to give several examples of surfaces such that their reflector lifts are horizontal or para-holomorphic and an elementary proof of a