

# GEOMETRY OF THE SHILOV BOUNDARY OF A BOUNDED SYMMETRIC DOMAIN

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**Abstract.** In the first part, the theory of bounded symmetric domains is presented along two main approaches: as special cases of Riemannian symmetric spaces of the noncompact type on one hand, as unit balls in positive Hermitian Jordan triple systems on the other hand. In the second part, an invariant for triples in the Shilov boundary of such a domain is constructed. It generalizes an invariant constructed by E. Cartan for the unit sphere in  $\mathbb{C}^2$  and also the triple Maslov index on the Lagrangian manifold.

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