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INFINITE DIMENSIONAL LIE GROUPS WITH APPLICATIONS TO MATHEMATICAL PHYSICS

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Abstract. We give a survey of infinite dimensional Lie groups and show some applications and examples in mathematical physics. This includes diffeomorphism groups and their natural subgroups like volume preserving, symplectic and contact transformations, as well as gauge groups, quantomorphisms and loop groups. Various applications include fluid dynamics, Maxwell's equations, plasma physics and BRST symmetries in quantum field theory. We discuss the Lie group structures of pseudodifferential and Fourier integral operators, both on compact and noncompact manifolds and give applications to the KdV equation and quantization.

1. Introduction

Lie Groups play an important role in physical systems both as phase spaces and as symmetry groups. Infinite dimensional Lie groups occur in the study of dynamical systems with an infinite number of degrees of freedom such as PDEs and in field theories. For such infinite dimensional dynamical systems diffeomorphism groups and various extensions and variations thereof, such as gauge groups, loop groups and groups of Fourier integral operators occur as symmetry groups and phase spaces. Symmetries are fundamental for Hamiltonian systems. They provide conservation laws (Noether currents) and reduce the number of degrees of freedom, i.e. the dimension of the phase space. Cohomological aspects of Lie groups come into the picture when studying anomalies and BRST symmetries in quantum field theory.

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The topics selected for these lectures aim to illustrate some of the ways infinite