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Some New Cyclotomic Strongly Regular Graphs

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Given a finite field F and a subset D of F^* such that $D = -D$, we can define a graph Γ with vertex set F by letting $x \sim y$ whenever $y - x \in D$. (Here $\sim$ denotes adjacency.) The spectrum of Γ consists of the numbers $\sum_{d \in D} \chi(d)$, where χ runs through the (additive) characters of F. In particular, the trivial character χ_0 yields the eigenvalue $|D|$, the valency of Γ.

One might wonder in what cases the graph Γ is strongly regular, and there has been done a lot of work on this question, see, e.g., Delsarte [4], van Lint and Schrijver [6], Calderbank and Kantor [3], Brouwer [1], de Resmini [7] and de Resmini and Migliori [8]. (As Delsarte showed, there is a one-to-one correspondence between (i) sets D closed under multiplication by elements of the prime field of F (and yielding a strongly regular Γ), and (ii) projective two-weight codes, and (iii) subsets of projective spaces such that the cardinality of the intersection with a hyperplane takes only two values. Work on this problem occurs in each of these three terminologies.)

In [5], we constructed four new examples, which will be described below. Our sets D will be unions of a number of cosets of a subgroup K of F^*, i.e., $D = ZK$ for some set $Z \subseteq F^*$. The field F is described by its characteristic p and a primitive polynomial defining it over its prime field. For the resulting strongly regular graphs we give the standard parameters $v, k, \lambda, \mu, r, s, f, g$ (cf. Brouwer and van Lint [2]).

Examples

	p	F	K	Z
a	3	$\alpha^8 = \alpha^3 + 1$	$\langle\alpha^{20}\rangle$	$\{1, \alpha, \alpha^4, \alpha^8, \alpha^{11}, \alpha^{12}, \alpha^{16}\}$
b	3	$\alpha^8 = \alpha^3 + 1$	$\langle\alpha^{16}\rangle$	$\{1, \alpha, \alpha^2, \alpha^8, \alpha^{10}, \alpha^{11}, \alpha^{13}\}$
c	2	$\alpha^{12} = \alpha^6 + \alpha^4 + 1$	$\langle\alpha^{45}\rangle$	$\{1, \alpha^5, \alpha^{10}\}$

	v	k	λ	μ	r	s	f	g
a	6561	2296	787	812	28	−53	4264	2296
b	6561	2870	1249	1260	35	−46	3690	2870
c	4096	273	20	18	17	−15	1911	2184
d	4096	1911	950	840	119	−9	273	3822

Here Example d is the dual of Example c. (Examples a and b are formally self-dual.)

Example c is interesting: it can be viewed as a graph with vertex set $\mathbb{F}_q^3$ for $q = 16$, such that each vertex has a unique neighbour in each of the $q^2 + q + 1 = 273$ directions. Probably some generalization is possible.

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