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Relaxation Process Modeling in a Turbulent Boundary Layer with Nonzero Free Stream Turbulence

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In order to analyze the relaxation effects in a turbulent boundary layer with zero and nonzero free stream turbulence, the Reynolds-averaged equations of motion and energy are solved. As the closure of the Reynolds-averaged equations, the transport equation for turbulent shear stresses is used. The proposed approach leads to calculation of the relaxation scales in the turbulent boundary layer with zero and nonzero free stream turbulence. Results for friction coefficients, velocity profiles, shear stresses, thickness of the boundary layer and so called "superlayer" in a flat-plate turbulent boundary layer are presented. The results obtained are in agreement with those available from the experimental data.

Keywords: The Reynolds-averaged equations; turbulence modeling; boundary layer; relaxation effects; free stream turbulence

1. INTRODUCTION

The present study is based on the approach proposed by Dyban and Fridman [1] with regard to the numerical solution of the momentum and heat transfer equations in a turbulent boundary layer with nonzero free stream turbulence. This approach is based on the Reynolds-averaged equations of motion and energy:

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$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial \tau_{ef}}{\partial y}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=-\frac{1}{\rho c_p}\frac{\partial q_{ef}}{\partial y}, \qquad (1)$$

with the boundary conditions

$$u=v=0, T=T_w \text{ when } y=0$$

$$u\to U(x), T\to T_\infty \text{ when } y\to\infty. \qquad (2)$$

Unknown effective shear stresses τ_{ef} and effective heat flux q_{ef} are defined as

$$\tau_{ef}=\tau+\tau_T=\tau-\rho\overline{u'v'},$$

$$q_{ef}=q+q_T=q-\rho c_p\overline{v'T'}. \qquad (3)$$

Then, as a closure for the system (1), instead of the usually employed Boussinesq hypothesis $-\overline{u'v'}=\nu_T\,\partial u/\partial y=(\nu_{ef}-\nu)\,\partial u/\partial y$, Dyban and Fridman [1] used the relaxation equations for turbulent shear stresses and for turbulent heat flux

$$-\overline{u'v'}=(\nu_{ef}-\nu)\frac{\partial u}{\partial y}+L_x\frac{\partial\overline{u'v'}}{\partial x}+L_y\frac{\partial\overline{u'v'}}{\partial y} \qquad (4)$$

$$-\overline{v'T'}=(a_{ef}-a)\frac{\partial T}{\partial y}+L_{x_\theta}\frac{\partial\overline{v'T'}}{\partial x}+L_{y_\theta}\frac{\partial\overline{v'T'}}{\partial y}, \qquad (5)$$

where L_x, L_y are relaxation scales in longitudinal and transverse directions, respectively, and L_{x_θ}, L_{y_θ} are thermal relaxation scales in longitudinal and transverse directions, respectively.

Equation (4) was proposed by Hinze [2] in order to account for memory effects which, as he has shown, could be significant in a turbulent boundary layer flow. To account for memory effects, Hinze [2] represented turbulent stresses as consisting of two parts: the first one was defined by the local gradients of averaged motion (Boussinesq hypothesis), and the second by memory effects.

There are two reasons for using the relaxation equation (4) in the case of the boundary layer with nonzero free stream turbulence. The first reason is that the Boussinesq hypothesis does not work for flow patterns where the time averaged product of the velocity fluctuations $|\overline{u'v'}|$ and the gradient $\partial u/\partial y$ do not simultaneously approach zero. As Fig. 1 shows, such a situation exists in a turbulent boundary layer with nonzero free stream turbulence. Indeed, when $y = \delta$, the value of $|\overline{u'v'}|$ differs considerably from zero, and this difference increases with the increasing of the parameter ν_{T_δ}/ν_{T_0}. The latter parameter is a ratio of the turbulent viscosity in the free stream, ν_{T_δ}, and the value of turbulent viscosity in a "reference" turbulent boundary layer with zero free stream turbulence, ν_{T_0}; it reflects the level of turbulence in a free stream [3]. While the value of $|\overline{u'v'}|$ does not equal zero, the velocity gradient is practically equal to zero in the region close to the outer edge of the boundary layer. The second reason for using the relaxation equation (4) is that the larger the scale of turbulent

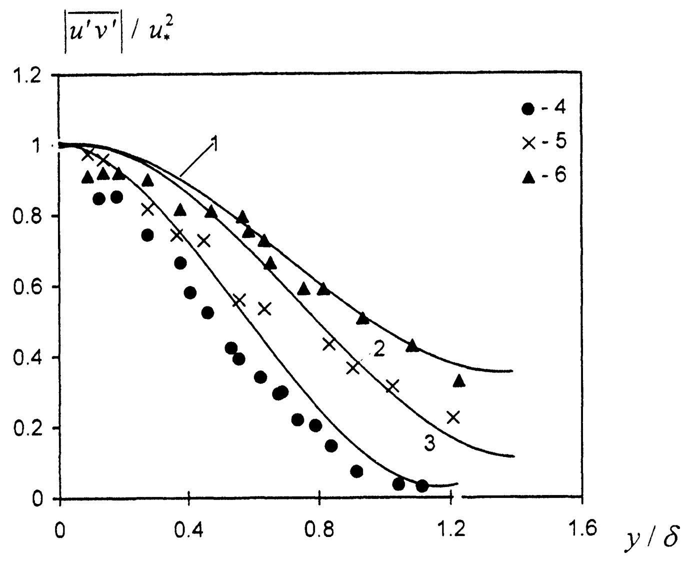

FIGURE 1 Turbulent shear stresses in the presence of free stream turbulence. Calculations: 1- $\nu_{T_\delta}/\nu_{T_0} = 10$, 2- $\nu_{T_\delta}/\nu_{T_0} = 5$, 3- $\nu_{T_\delta}/\nu_{T_0} = 1$. Experiments [3]: 4- $\nu_{T_\delta}/\nu_{T_0} = 1$, 5- $\nu_{T_\delta}/\nu_{T_0} = 3.6$, 6- $\nu_{T_\delta}/\nu_{T_0} = 13.6$.

motion and the more homogeneous is the velocity field ($\partial u/\partial y \rightarrow 0$), the more pronounced the memory effects are for the flow pattern [2]. Such a situation takes place at the outer edge of the boundary layer when the level of turbulence in the free stream is significant.

The main difficulty in using Eq. (4) is in defining the longitudinal L_x and transverse L_y relaxation scales. Loytsyanskiy [4] proposed to interpret the relaxation scales L_x and L_y as the Prandtl mixing lengths in longitudinal and transverse directions respectively. In the present study another method of defining the relaxation scales L_x and L_y is proposed.

2. ANALYSIS

The transport equation for the Reynolds shear stresses can be written as [5]

$$u\frac{\partial \overline{u'v'}}{\partial x} + v\frac{\partial \overline{u'v'}}{\partial y} = C_s\frac{\partial}{\partial y}\left(\frac{E^2}{\epsilon}\frac{\partial \overline{u'v'}}{\partial y}\right) - C_{\phi_1}\left(\frac{\epsilon}{E}\overline{u'v'} + C_\mu E\frac{\partial u}{\partial y}\right), \quad (6)$$

where

$$C_\mu = 0.09;\ C_s = 0.1;\ C_{\phi_1} = 2.8. \quad (6a)$$

Usually, to define turbulent kinetic energy, E, and kinetic energy dissipation rate, ϵ, two more equations need to be solved together with Eq. (6). However, if one takes into account an expression for eddy viscosity $\nu_T = C_\mu E^2 / \epsilon$ and an estimation $|\overline{u'v'}|/E \sim 0.3$ [6], that is valid for the most part of the turbulent boundary layer, then the Eq. (6) becomes:

$$u\frac{\partial \overline{u'v'}}{\partial x} + v\frac{\partial \overline{u'v'}}{\partial y} = C_s\frac{\partial}{\partial y}\left(\frac{\nu_T}{C_\mu}\frac{\partial \overline{u'v'}}{\partial y}\right) + \frac{C_{\phi_1}C_\mu}{0.3}\overline{u'v'}\left(\frac{\overline{u'v'}}{\nu_T} + \frac{\partial u}{\partial y}\right) \quad (7)$$

Following Loytsyanskiy [4], assume that the turbulent relaxation time can be expressed as $\lambda_T = C_\mu E / \epsilon$. Then, multiplying Eq. 7 by λ_T and rearranging the expression for the turbulent relaxation time as

$$\lambda_T = C_\mu \frac{E}{\epsilon} = \frac{\nu_T}{\overline{u'v'}} \left|\frac{\overline{u'v'}}{E}\right| = -0.3 \frac{\nu_T}{\overline{u'v'}}, \tag{8}$$

one can get

$$\lambda_T u \frac{\partial \overline{u'v'}}{\partial x} + \lambda_T v \frac{\partial \overline{u'v'}}{\partial y} = -0.3 C_s \frac{\nu_T}{\overline{u'v'}} \frac{\partial}{\partial y}\left(\frac{\nu_T}{C_\mu}\frac{\partial \overline{u'v'}}{\partial y}\right) - C_{\phi_1} C_\mu \left(\overline{u'v'} + \nu_T \frac{\partial u}{\partial y}\right). \tag{9}$$

The longitudinal and transverse relaxation lengths can, then, be defined as [4]

$$L_x = \lambda_T u, L_y = \lambda_T v \tag{10}$$

Thus, Eq. (9) becomes:

$$L_x \frac{\partial \overline{u'v'}}{\partial x} + L_y \frac{\partial \overline{u'v'}}{\partial y} = -0.3 C_s \frac{\nu_T}{\overline{u'v'}} \frac{\partial}{\partial y}\left(\frac{\nu_T}{C_\mu}\frac{\partial \overline{u'v'}}{\partial y}\right) - C_{\phi_1} C_\mu \left(\overline{u'v'} + \nu_T \frac{\partial u}{\partial y}\right). \tag{11}$$

Hence, to account for the relaxation effects in the longitudinal direction as well as in the transverse direction, Eqs. (1)–(2) should be solved simultaneously with the modified transport equation for the Reynolds shear stresses (Eq. 11). In order to do so, the distributions of the longitudinal L_x and transverse L_y relaxation scales need to be known.

Another way to account for the relaxation effects in a turbulent boundary layer is to solve the system (1)–(2) using Eqs. (4) and (5) as the closure model. The relaxation scales functions in the turbulent boundary layer can be defined by integrating Eqs. (1)–(2) together with Eq. (7) and then substituting the results into Eqs. (8) and (10). The latter approach is employed in the present study.

Calculations have been carried out with the usual values for constants (6a) in Eq. (7), except for constant C_s which has been changed to $C_s = 0.1 Re_x^n$, where $n = 0.16$ and $n = 0.2$ for zero and nonzero free stream turbulence, respectively. Such a choice is made in order to provide better agreement between the calculation results and the experimental data. The functions for turbulent viscosity ν_T are defined according to the recommendations given by Dyban and Fridman [1] (Fig. 2).

3. NUMERICAL PROCEDURE

The numerical solutions have been carried out by finite difference method with variables x and $\ln(1 + y / a_1\sqrt{x})$. The Crank-Nicolson implicit scheme [7] has been employed for finite-difference approximation of the corresponding differential equations. This scheme is considered unconditionally stable and has second-order accuracy $O(\Delta x)^2 + O(\Delta y)^2$. The solution of the discretization equations is obtained by iterations using TriDiagonal-Matrix Algorithm [7]. Iterations were repeated until the magnitude of errors for $|\overline{u'v'}|/u_*^2$ approached $\pm 1\%$ from its maximum value, which corresponds to the absolute value of approximately $\pm 0.1\ cm^2/s^2$.

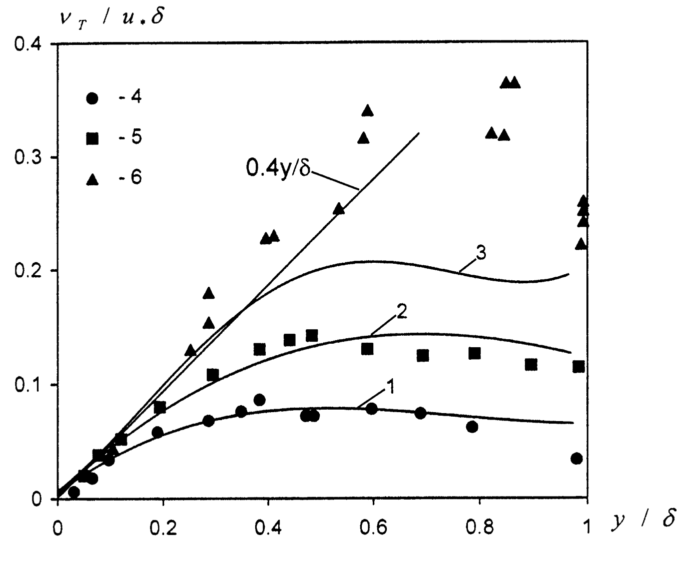

FIGURE 2 Effective coefficient of turbulent viscosity in the turbulent boundary layer with nonzero stream turbulence. Calculations: 1- $\nu_{T_\delta}/\nu_{T_0} = 1$, 2- $\nu_{T_\delta}/\nu_{T_0} = 5$, 3- $\nu_{T_\delta}/\nu_{T_0} = 10$. Experiment: 4- $\nu_{T_\delta}/\nu_{T_0} = 1$, 5- $\nu_{T_\delta}/\nu_{T_0} = 3$, 6- $\nu_{T_\delta}/\nu_{T_0} = 8.8$.

4. RESULTS AND DISCUSSION

The calculation results of the characteristics of turbulent boundary layer with zero and nonzero free stream turbulence are described below.

Figure 3 presents the relaxation scales in longitudinal L_x and transverse L_y directions for a flat-plate turbulent boundary layer with zero and nonzero free stream turbulence. The L_x and L_y functions are consistent with the notion [2] that the relaxation effects are pronounced in the outer part of the turbulent boundary layer and decreased with the decreasing distance from the wall. The obtained relaxation scales are employed to simulate the relaxation effects in the turbulent boundary layer with nonzero free stream turbulence.

In order to test the described method, the calculations of turbulent boundary layer with zero free stream turbulence have been performed and a comparison of the results with well known experimental data [8] has been carried out (Fig 4). The friction coefficient in the turbulent boundary layer with nonzero free stream turbulence (Fig. 4, broken line) has also been calculated. These results show an increase in friction coefficient under the influence of the free stream turbulence. This is in agreement with our previous results approximated by the expression [9]

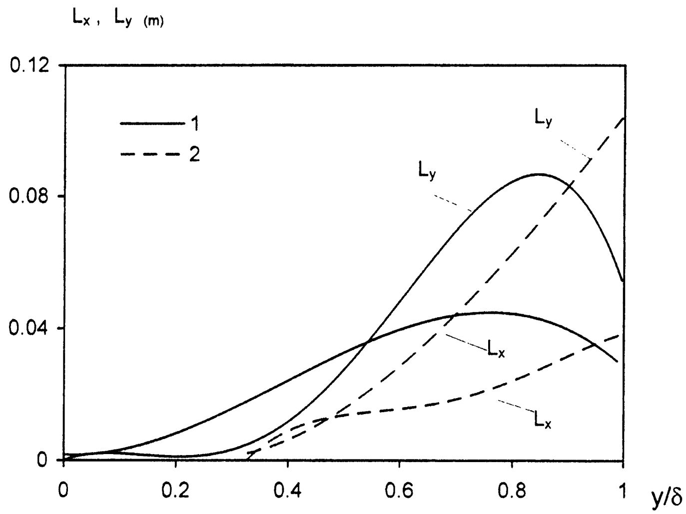

FIGURE 3 The relaxation scales in a flat-plate turbulent boundary layer with nonzero (1) and zero (2) free stream turbulence.

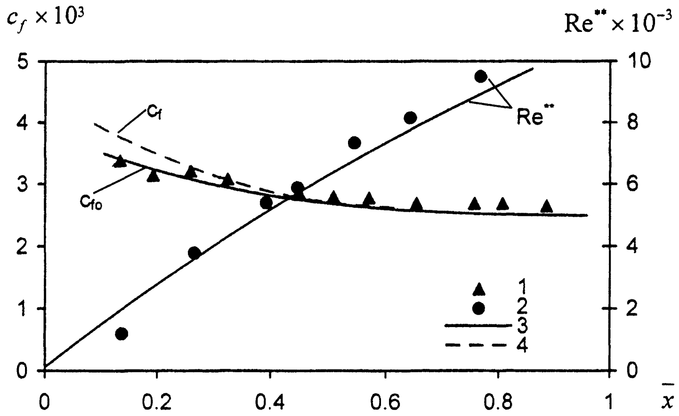

FIGURE 4 The comparison of the calculation results of characteristics of the flat plate turbulent boundary layer with experimental data. Experiment: 1,2- $Tu_\infty = 0$. Calculations: 3- $Tu_\infty = 0$, 4- $Tu_\infty > 0$.

$$c_f = \left[1 + 0.045\left(\frac{\nu_{T_\delta}}{\nu_{T_0}}\right)^{0.5}\right]c_{f_0}, \tag{12}$$

where

$$c_{f_0} = 0.027[1 + 0.051g(Re^{**} - 3.3) + 0.1(Re^{**} - 3.3)^2]Re^{**^{-0.268}}$$

and ν_{T_δ}/ν_{T_0} is a parameter that reflects the level of turbulence in a free stream. When, for example, $\nu_{T_\delta}/\nu_{T_0} = 4$, $Re^{**} = 2500$ and $Re_x = 1.4 \times 10^6$, the increase in the friction coefficient is about 11% while the formula (12) gives the quantity about 10%.

Fig. 5 represents velocity profiles in the range of the Reynolds number $Re^{**} = 2000$–4000. A commonly employed expression that describes a velocity profile in a turbulent boundary layer is

$$u^+ = 2.5\ln y+ + 5.1 + 2.5\Pi\omega(y/\delta),$$

where the profile parameter Π grows with the increase in the Reynolds number and becomes $\Pi \sim 0.5$ in the fully developed turbulent boundary layer. The calculated velocity profiles (Fig. 5) are in agreement with available experimental data in the near wall region. They also demonstrate that

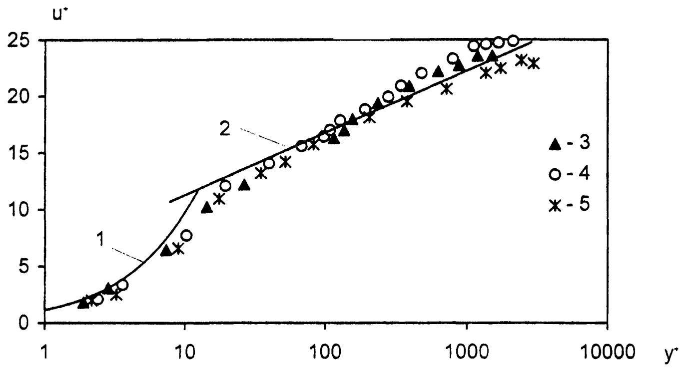

FIGURE 5 The velocity profiles in turbulent boundary layer with zero (3, 4) and nonzero (5) free stream turbulence. 1- $u^+ = y^+$, 2- $u^+ = 2.5\ln y^+ + 5.1$.

the profile parameter Π in the wake region increases from value $\Pi = 0.325$ at Re** = 2400 (profile 3) to $\Pi = 0.54$ at Re** = 4000 (profile 4) in the fully developed turbulent boundary layer. These results are in agreement with experimental data [3] as well. Under the influence of free stream turbulence, the velocity profile changes in such a way that profile parameter Π decreases and in some cases becomes negative. As one can see, this is the case presented in Fig. 5 (profile 5): the profile parameter is negative $\Pi = -0.2$ while the level of free stream turbulence is nonzero ($\nu_{T_\delta}/\nu_{T_0} = 4$).

Distributions of shear stresses for turbulent boundary layer with zero and nonzero free stream turbulence are presented in Fig. 1. The results of calculations correspond to the notion that values of shear stresses at the outer edge of the turbulent boundary layer with nonzero free stream turbulence differed from zero. Thus, when the parameter $\nu_{T_\delta}/\nu_{T_0} = 5$, the value of $|\overline{u'v'}|/u_*^2$ at the outer edge of the boundary layer ($y = \delta$) approaches quantity ~0.3. The thickness of the so-called "superlayer" (the layer above the boundary layer where the gradient $\partial|\overline{u'v'}|/\partial y$ asymptotically approaches zero [3]) is 1.8δ in this particular case. With the decrease of the parameter ν_{T_δ}/ν_{T_0}, which corresponds to the decrease of the free stream turbulence, the values of $|\overline{u'v'}|/u_*^2$ and the "superlayer's" thickness decrease as well. So, the thickness of the boundary layer where gradient of the mean velocity $\partial u/\partial y$ approaches zero does not equal the thickness of the layer where the gradient of the shear stresses $\partial|\overline{u'v'}|/\partial y$ approaches zero in the boundary layer with nonzero free stream turbulence. The higher the level of the free stream turbulence the larger the difference between these two.

5. CONCLUSION

The proposed method allows the modeling of the relaxation effects in a turbulent boundary layer. The calculation of relaxation scales makes it possible to use the relaxation turbulence model [1] instead of the usually employed Boussinesq hypothesis for the prediction of the velocity and temperature profiles for turbulent boundary layers with nonzero free stream turbulence. The calculation results for heat transfer and friction coefficients, for velocity and temperature profiles, for Reynolds stresses and turbulent heat fluxes could be used for predicting the heat transfer rates in the highly turbulent flow such as flow that associated with elements of turbomachinery.

NOMENCLATURE

α	=	$k/\rho c_p$-thermal diffusivity
c_f	=	skin friction coefficient
c_p	=	fluid specific heat
E	=	$(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$, $k = 2 * E$ turbulent kinetic energy
L_x, L_y	=	relaxation scales in longitudinal and transverse directions
L_{x_θ}, L_{y_θ}	=	thermal relaxation scales in longitudinal and transverse directions
q	=	heat flux
Re**	=	Reynolds number based on momentum thickness
R_T	=	ν_T/ν turbulent Reynolds number
Re_θ^{**}	=	Reynolds number based on enthalpy thickness
Re_x	=	Reynolds number based on plate location
T	=	mean temperature
T'	=	temperature fluctuation
Tu	=	ratio of rms velocity fluctuation to mean value
u^+	=	u/u_*—mean velocity scaled on inner variables
u_*	=	friction velocity

U	= mean convective velocity
u, v	= longitudinal and transverse components of mean velocity
u', v$'$	= longitudinal and transverse components of turbulent velocity
x, y	= Cartesian coordinates
y^+	= yu_*/ν—dimensionless distance from the wall
δ	= momentum boundary layer thickness
ϵ	= kinetic energy dissipation rate
ν	= viscosity
ρ	= fluid density
τ	= shear stresses
ω_0	= wake function for velocity profile

Subscripts

ef, T, w δ θ, 0, ∞	= effective, turbulent, wall, outer edge of the boundary layer, thermal boundary layer, zero free stream turbulence and free stream.

References

[1] Dyban, Ye. P. and Fridman, E. (1987). Relaksatsionnaya Model' Perenosa Teploty i Impulsa v Pristennykh Pogranichnykh Sloyakh Turbulizirovannykh Potokov (The Relaxation Model of Heat and Momentum Transfer in Boundary Layers with Nonzero Free Stream Turbulence), *Prom. Teplotechnika*, Vol. 9, 4, 16–27 (In Russian).

[2] Hinze, J. O. (1976). Gedachtniseffekte in der Turbulenz, *ZAMM*, **B 56**, 403–415.

[3] Dyban, Ye. P. and Epik, E. Ya. (1985). *Teplomassoobmen i Gidrodinamika Turbulizirovannykh Potokov* (Heat and Mass Transfer and Hydrodynamics in Turbulent Flow), Naukova Dumka Press, Kiev, p. 350 (In Russian).

[4] Loytsyanskiy, L. G. (1982). Nasledstvennye Yavleniya v Turbylentnykh Potokakh (Memory Effects in Turbulent Flows), Isv. AN SSSR, *Mechanika Zidkosty i Gasa*, **2**, 5–19 (In Russian).

[5] Frost, W. and Moulden, T. H. (1977). *Handbook of Turbulence. Fundamentals and Applications*, Plenum Press, New York, pp. 187–231.

[6] Bradshaw, P., Ferriss, D. H. and Atwell, N. P. (1967). Calculation of boundary-layer development using the turbulent energy equation, *J. Fluid Mech.*, **28**, 593–616.

[7] Patankar, S. V. (1980). *Numerical Heat Transfer and Fluid Flow*, Hemisphere Publishing Corporation, New York, pp. 41–73.

[8] Wieghardt, K. (1944). Uber die turbulente Stromung imrohr und langs der Platte. *ZAMM*, **24**, 294–296.

[9] Fridman, E. Simulation of Heat Transfer from Flow with High Free Stream Turbulence to Turbine Blades, Presented at the *ASME Cogen Turbo Power '95 Conference*, Vienna, Austria, August 1995.