

## Research Article

# Cauchy-Matern Model of Sea Surface Wind Speed at the Lake Worth, Florida

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We study the Cauchy-Matern (CM) process with long-range dependence (LRD). The closed form of its power spectrum density (PSD) function is given. We apply it to model the autocovariance function (ACF) and the PSD of the sea surface wind speed (wind speed for short) observed in the Lake Worth, Florida, over the 1984–2006 period. The present results exhibit that the wind speed at the Lake Worth over 1984–2006 is of LRD. The present results exhibit that the CM process may yet be a novel model to fit the wind speed there.

## 1. Introduction

Stochastic processes with LRD gain applications in many fields of science and technologies ranging from hydrology to network traffic; see, for example, Mandelbrot [1], Beran [2], and references therein. The fractional Gaussian noise (fGn) introduced by Mandelbrot and van Ness [3] is a widely used model in this field. However, there are other models; see, for example, Lawrance and Kottegoda [4], Kaplan and Kuo [5], Martin and Walker [6], Granger and Ding [7], Beran [8], Yazici and Kashyap [9], Li et al. [10], and Chiles and Delfiner [11], in addition to fGn. In this paper, we focus on the correlation model that was first introduced by Matérn [12]. Later, it was discussed by Yaglom [13] and Chiles and Delfiner applied it to the geo-statistics [11]. Since it is in the Cauchy class, we call a process that obeys that correlation model as the Cauchy-Matern process (CM process for short).

This paper is organized as follows. We will dissertate the CM process and give its closed form of PSD in Section 2. Its application to wind speed is explained in Section 3 and discussions in Section 4, which is followed by conclusions.

## 2. CM Process

Let  $X(t)$  be a random function with zero mean for  $-\infty < t < \infty$ . Denote by  $p_{\text{Cauchy}}(x)$  the probability density function (pdf) of the Cauchy distribution in the form

$$p_{\text{Cauchy}}(x) = \frac{b}{\pi \left[ (x - m)^2 + b^2 \right]}, \quad (2.1)$$

where  $b$  is the half width at half maximum and  $m$  is the statistical median [14, 15]. The term ‘‘Cauchy process’’ conventionally implies that the pdf of  $X(t)$  obeys (2.1); see, for example, Bertoin [16], or its variations; see, for example, Zanzotto [17], Garbaczewski and Olkiewicz [18].

Let  $C(\tau) = E[X(t)X(t + \tau)]$  be the ACF of  $X(t)$ , where  $\tau$  is the time lag. Then, another meaning of the Cauchy process is the ACF of  $X(t)$  being in the form, see Yaglom [13, page 365] or Chilès and Delfiner [11, page 86],

$$C(\tau) = \left( 1 + |\tau|^2 \right)^{-1}, \quad \tau \in \mathbb{R}. \quad (2.2)$$

Obviously, the two are different in meaning. Our research utilizes the term in the sense of (2.2).

Matern generalized the ACF of the ordinary Cauchy model (2.2) to the following:

$$C(\tau) = \left( 1 + \frac{|\tau|^2}{a^2} \right)^{-b_1}, \quad \tau \in \mathbb{R}, \quad a > 0, \quad b_1 > 0. \quad (2.3)$$

Therefore, considering that Matern is a scientist in geosciences, we call a process  $X(t)$  that follows (2.3) the CM process.

Note that Matern did introduce the parameter  $a$  described in (2.3). However, it may be unnecessary because  $\tau$  is a real number. Thus, for the purpose of simplicity, we let  $a = 1$  and  $b_1 = b/2$ . In this way, (2.3) becomes the form

$$C(\tau) = \left( 1 + |\tau|^2 \right)^{-b/2}, \quad \tau \in \mathbb{R}, \quad b > 0. \quad (2.4)$$

The correlation model Matern discussed is of short-range dependence (SRD). In this research, we generalize it such that the LRD condition of  $X(t)$  can be considered when  $0 < b < 1$  in (2.4). Following the tradition in fractal time series, we let  $b/2 = 1 - H$  for  $0.5 < H < 1$ , where  $H$  is the Hurst parameter. Therefore, we may rewrite (2.4) by

$$C(\tau) = \left( 1 + |\tau|^2 \right)^{H-1}. \quad (2.5)$$

The SRD condition of the CM process is described by  $b > 1$ , which implies  $0 < H < 0.5$ .

Because  $C(\tau)$  is nonintegrable for  $0 < b < 1$ , the Fourier transform of  $C(\tau)$  denoted by  $S(\omega)$  does not exist in the domain of ordinary functions for  $0 < b < 1$ . This reminds

us that PSD of the CM process with LRD should be treated as a generalized function over the Schwartz space of test functions. In the domain of generalized functions (see [19, §2.5, Chapter 2]), the PSD of the CM process is given by

$$S(\omega) = \int_{-\infty}^{\infty} (1 + |\tau|^2)^{-b/2} e^{-j\omega\tau} d\tau = \frac{2^{(1-b)/2}}{\sqrt{\pi}\Gamma(b/2)} |\omega|^{1/2(b-1)} K_{1/2(b-1)}(|\omega|), \quad (2.6)$$

where  $K_\nu(\cdot)$  is the modified Bessel function of the second kind (Gradshteyn and Ryzhik [20]), which is expressed by

$$K_\nu(z) = \frac{\Gamma(\nu + 1/2)(2z)^\nu}{\sqrt{\pi}} \int_0^\infty \frac{\cos t dt}{(t^2 + z^2)^{\nu+1/2}}. \quad (2.7)$$

In the case of LRD, that is,  $0 < b < 1$ , one can infer that  $S(\omega) \sim 1/\omega$  for  $\omega \rightarrow 0$ ; see the details in Li and Zhao [15] for this inference. Thus, in order to plot the PSD of the CM process, we need regularizing  $S(\omega)$  such that the regularized PSD is finite for  $\omega \rightarrow 0$ . The regularization can be done in the following way. Denote by  $S_0(\omega)$  the regularized PSD. Then,

$$S_0(\omega) = \frac{S(\omega)}{\lim_{\omega \rightarrow 0} S(\omega)}. \quad (2.8)$$

The above implies that  $S_0(0) = 1$ . The plots below for PSD are in the sense of regularized PSD. Figures 1(a) and 1(b) indicate  $C(\tau)$  and  $S_0(\omega)$  for three values of  $H$ , respectively.

It is noted that the CM process is non-Markovian since its correlation  $C(t_1, t_2)$  does not satisfy the triangular relation given by

$$C(t_1, t_3) = \frac{C(t_1, t_2)C(t_2, t_3)}{C(t_2, t_2)}, \quad t_1 < t_2 < t_3, \quad (2.9)$$

which is a necessary condition for a Gaussian process to be Markovian; see Todorovic [21] for details. In fact, up to a multiplicative constant, the Ornstein-Uhlenbeck process is the only stationary Gaussian Markov process (Lim and Muniandy [22], Wolpert and Taqqu [23]).

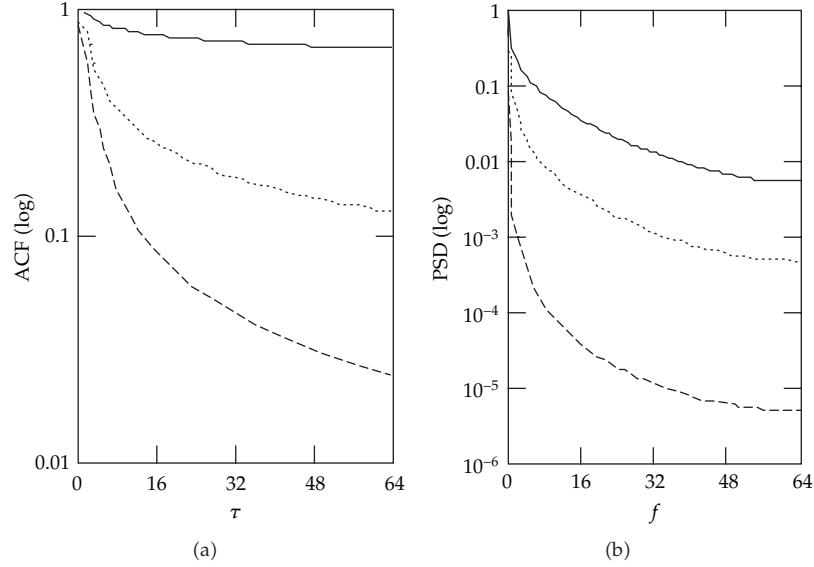
The CM process is not self-similar but its Lamperti transformation is self-similar. For a stationary process  $X(t)$ , if  $\lambda > 0$ , the Lamperti transform of  $X(t)$  is given by

$$Y(t) = t^\lambda X(\ln t), \quad \text{for } t > 0, \quad Y(0) = 0. \quad (2.10)$$

Then,  $Y(t)$  is a  $\lambda$  self-similar process (Lamperti [24], Flandrin et al. [25]). Applying the Lamperti transformation to the CM process  $X(t)$  results in the covariance given by

$$E[Y(t)Y(s)] = (ts)^\lambda \left[ 1 + \left| \frac{\ln t}{s} \right|^2 \right]^{-b/2}, \quad t, s > 0. \quad (2.11)$$

The above exhibits that  $Y(t)$  is a Gaussian nonstationary process with the self-similarity index  $\lambda$ .



**Figure 1:** (a) ACF of CM process. Solid line for  $H = 0.95$ , dot line for  $H = 0.75$ , and dash line for  $H = 0.55$ . (b) Regularized PSD of CM process. Solid line for  $H = 0.55$ , dot line for  $H = 0.75$ , and dash line for  $H = 0.95$ .

### 3. Application of the CM Model to a Set of Wind Speed Data

Wind speed plays a role in several areas of science and engineering, such as ocean physics, ocean engineering, wind engineering, and meteorology; see, for example, Massel [26], Li [27]. In this section, we shall apply the CM process to the ACF and the PSD of the wind speed (m/s) observed at the Station LKWF1 (Lake Worth, Florida) [28]. The data are available from the category of Standard Meteorological [29]. They were averaged over an eight-minute period for buoys and a two-minute period for land stations (Gilhousen [30, 31]).

Denote the data series by  $x_{yyyy}(t)$ , where yyyy stands for the index of year. Denote  $C_{yyyy}(k)$  ( $k = 0, 1, \dots$ ) and  $S_{yyyy}(f)$  as the measured ACF and the measured PSD in the year of yyyy, respectively. For instance,  $x_{2003}(t)$  and  $S_{2003}(f)$  represent the measured time series and the measured PSD at the station LKMF1 in 2003, respectively.

Note that the measured ACF and PSD are estimates of the true ACF and PSD. Therefore, a PSD or ACF estimation of wind speed should be reliable and traceable. For that reason, we estimated the ACFs and the PSDs with the recognized instrument Solartron 1200 Real Time Signal Processor [32]. Practically, an ACF or PSD is estimated on a block-by-block basis by averaging PSD (or ACF) estimates of blocks of data with a certain window weight function for the sake of variance reduction (Mitra and Kaiser [33]). Let  $B$  be the block size and  $M$  be the average count, respectively. We sectioned the data in the nonoverlapping case. On Solartron 1200 Real Time Signal Processor, a Hanning widow was set.  $M$  is selected such that  $0 < [L - (B \times M)] < B$ , where  $L$  is the total length of  $x_{yyyy}(t)$ . Table 1 lists the measured data and the settings for the PSD (or ACF) estimation, where  $B = 128$ .

The key parameter in the CM model is  $H$ . The literature regarding  $H$  estimation is affluent. Commonly used estimators of  $H$  are  $R/S$  analysis, maximum likelihood method, variogram-based methods, box-counting, detrended fluctuation analysis, spectrum regression, and correlation regression; see, for example, [1, 2, 34–36]. In this paper, we use the

**Table 1:** Measured data at LKWF1, the settings for signal processing, and the modeling results.

Series	Record date and time	$L$	$M$	$b$	$H$	MSE
$x_{1984}(t)$	19:00, 20 Jul–23:00, 31 Dec. 1984	3175	24	0.262	0.869	$9.861 \times 10^{-4}$
$x_{1985}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1985	7848	61	0.342	0.829	$8.353 \times 10^{-4}$
$x_{1986}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1986	8376	65	0.320	0.840	$7.509 \times 10^{-4}$
$x_{1987}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1987	8682	67	0.361	0.819	$1.170 \times 10^{-4}$
$x_{1988}(t)$	1:00, 1 Jan.–23:00, 31 Dec. 1988	8598	67	0.338	0.831	$9.666 \times 10^{-4}$
$x_{1989}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1989	8694	67	0.330	0.850	$8.300 \times 10^{-4}$
$x_{1990}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1990	8604	67	0.300	0.850	$7.778 \times 10^{-4}$
$x_{1991}(t)$	0:00, 1 Jan.–18:00, 31 Oct. 1990	7273	56	0.304	0.848	$6.964 \times 10^{-4}$
$x_{1993}(t)$	11:00, 20 Dec.–23:00, 31 Dec. 1993	277	2	0.276	0.862	$1.952 \times 10^{-3}$
$x_{1994}(t)$	0:00, 1 Jan.–18:00, 31 Oct. 1994	8571	66	0.332	0.834	$7.433 \times 10^{-4}$
$x_{1995}(t)$	0:00, 1 Jan.–18:00, 31 Oct. 1995	8503	66	0.302	0.849	$6.950 \times 10^{-4}$
$x_{1996}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1996	8112	63	0.299	0.850	$6.459 \times 10^{-4}$
$x_{1997}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1997	8184	63	0.318	0.841	$6.242 \times 10^{-4}$
$x_{1998}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1998	8784	68	0.326	0.837	$8.667 \times 10^{-4}$
$x_{1999}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 1999	8760	68	0.310	0.845	$6.162 \times 10^{-4}$
$x_{2000}(t)$	0:00, 1 Jan.–17:00, 26 Feb. 2000	8072	63	0.311	0.844	$6.643 \times 10^{-4}$
$x_{2001}(t)$	17:00, 8 Aug.–23:00, 31 Dec. 2001	8760	68	0.310	0.845	$8.636 \times 10^{-4}$
$x_{2002}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 2002	8760	68	0.293	0.853	$7.555 \times 10^{-4}$
$x_{2003}(t)$	0:00, 1 Jan.–23:00, 31 Dec. 2003	8650	67	0.300	0.850	$9.175 \times 10^{-4}$
$x_{2004}(t)$	0:00, 1 Jan.–14:00, 5 Oct. 2004	6650	52	0.330	0.835	$8.911 \times 10^{-4}$
$x_{2006}(t)$	0:00, 31 May–23:00, 31 Dec. 2006	5149	40	0.353	0.824	$1.169 \times 10^{-4}$

regression method to estimate  $H$ . The following uses the PSD regression to estimate  $H$  that is equivalent to the ACF regression owing to the Wiener-Khinchin theorem.

After obtaining a measured PSD  $S_{yyyy}(f)$ , we input it into a PC to do the data fitting with the theoretic PSD  $S_0(f)$  by using the least square fitting. Denote the cost function by

$$J(b) = \frac{2}{B} \sum_k [S_0(f) - S_{yyyy}(f)]^2, \quad (3.1)$$

where  $S_{yyyy}(f)$  is in the normalized case. The derivative of  $J$  with respect to  $b$ , which will be zero when  $J$  is minimum, yields the estimate  $b$  or equivalently the  $H$  estimate, which is the solution of  $dJ/db = 0$ .

Figures 2(a) and 2(b) indicate 2 series  $x_{1990}(t)$  and  $x_{2004}(t)$  at the station LKWF1, respectively. Each starts from the first data point to the 256th one, that is, about the first 10 days of data. The data fitting between the measured PSD and the theoretical one for each series is demonstrated in Figures 3(a) and 3(b). By the least square fitting, we have the estimated  $H$  values 0.850, 0.835 for  $x_{1990}(t)$ ,  $x_{2004}(t)$ , respectively (Table 1). The MSEs for the data fitting of the most series are in the order of magnitude of  $10^{-4}$  except that  $x_{1993}(t)$  has the MSE in the order of magnitude of  $10^{-3}$ , likely due to the too short series (Table 1). Figures 3(c) and 3(d) illustrate the data fitting for  $C_{1990}(k)$  and  $C_{2004}(k)$ , respectively. Hence, from the modeling results, we experimentally infer that the CM model well fits the wind speed observed. The  $H$  estimates for all series are summarized in Table 1, exhibiting the LRD property of wind speed due to  $0.5 < H < 1$ .

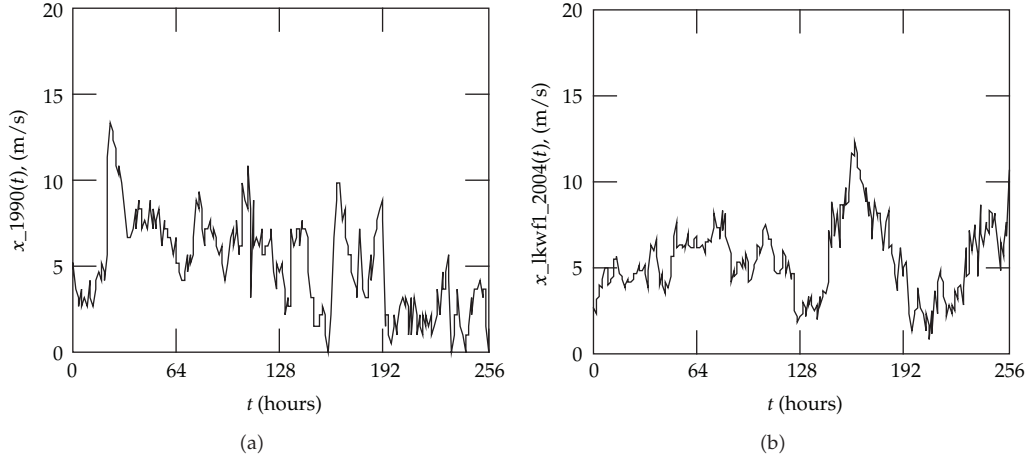


Figure 2: Measured data at LKWF1. (a)  $x_{1990}(t)$ . (b)  $x_{2004}(t)$ .

#### 4. Discussions

In order to exhibit that the present CM process is a novel model of wind speed, we brief some results with respect to several models of wind speed, which are used in wind engineering.

In the aspect of PSD of wind speed, Davenport [37] proposed a well-known form of the normalized PSD given by

$$\frac{f S_{\text{Dav}}(f)}{u_f^2} = 4 \frac{u^2}{(1 + u^2)^{4/3}}, \quad u = \frac{1200n}{z}, \quad (4.1)$$

where  $f$  is the frequency (Hz),  $u_f$  friction velocity ( $\text{ms}^{-1}$ ), and  $n$  is the normalized frequency ( $fz/U$  (10 m)), where  $U$  (10 m) is the mean wind speed ( $\text{ms}^{-1}$ ) measured at height 10 m,  $U(z)$  mean wind speed ( $\text{ms}^{-1}$ ) measured at height  $z$ . Kaimal et al. [38] introduced the following PSD:

$$\frac{f S_{\text{Kai}}(f)}{u_f^2} = \frac{100n}{(0.44 + 33n)^{5/3}}. \quad (4.2)$$

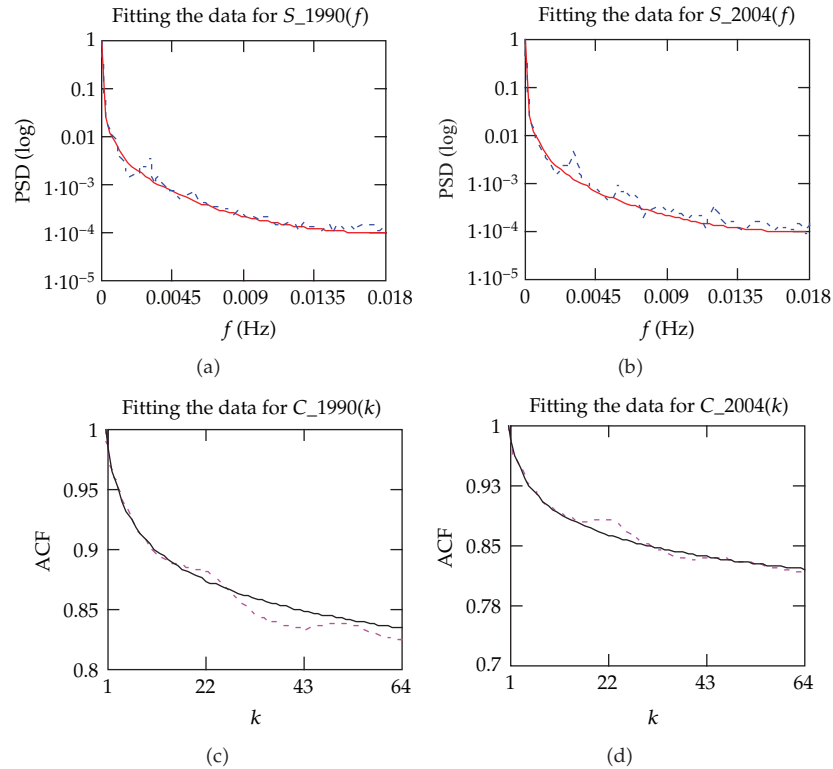
Antoniou et al. [39] discussed the one expressed by

$$\frac{f S_{\text{Ant}}(f)}{u_f^2} = \frac{18n}{(0.44 + 5n)^{5/3}}. \quad (4.3)$$

For this class of spectra, Hiriart et al. [40] gave a general form written by

$$\frac{f S_{\text{Hir}}(f)}{u_f^2} = \frac{4n^2}{(1 + n^2)^{2\gamma}}, \quad \gamma > 0, \quad (4.4)$$

which generalized the Davenport PSD form by using the spectral index  $\gamma$ .



**Figure 3:** Modeling demonstrations. Solid line: theoretic PSD/ACF. Dot line: measured PSD/ACF. (a) Fitting the data for  $S_{1990}(f)$ . (b) Fitting the data for  $S_{2004}(f)$ . (c) Fitting the data for  $C_{1990}(k)$ . (d) Fitting the data for  $C_{2004}(k)$ .

The spectra mentioned above, including the Hojstrup-type PSD [41], are finite near the origin. Therefore, the inverse Fourier transforms of those PSDs are summable [42]. Hence, they are PSDs of processes with SRD. Though the models discussed in [37–41] are of SRD, the slow-decayed ACF of wind speed (slower than an exponential-type function) was noticed as can be seen from Brett and Tuller [43, abstract section] that is actually the  $1/f$  noise behavior of a time series [42]. Recently, fractal descriptions of wind speed were reported (Kavasseri and Nagarajan [44, 45], Santhanam and Kantz [46]). Nevertheless, the closed form of either ACF or PSD of wind speed in the LRD case is rarely seen, to the best of our knowledge. Consequently, different from the models commonly used in the field, the present CM model provides a closed form of either ACF or PSD of wind speed with LRD.

The CM model (2.4) differs from those, that is, (4.1)–(4.4), conventionally used in wind engineering. However, it must be noting that our research used the data measured above sea surface while others, we mean those discussed in [37–40], utilized data recorded above ground. For instance, the data Davenport utilized were measured using cup anemometers mounted at 12.2 m, 64 m, and 153 m on a radio mast [37, page 195]. Kaimal et al. studied the data measured on a 32 m tower [38, page 563], and Hiriart et al. investigated the data for the purpose of selecting the site of the new Mexican Optical-Infrared Telescope (TIM) installed at the Sierra of San Pedro Martir [40, page 213]. One thing worth keeping in mind is that the present model in this paper never implies that one model may be superior to another. More precisely, we consider that a model may be site dependent.



Note that the CM process is LRD. Thus, its PSD follows power law; see (2.6). In fact, any random functions that are LRD have power-law-type PSDs [15]. From that point of view, consequently, one thing in common for different models described by say (2.6), (4.1)–(4.4) is that their PSDs all follow power laws, which implies that all may be explained from the point of view of fractal time series [42, 47].

The main point in this paper is to exhibit the possible LRD property of sea surface wind speed in addition to its CM model. The research is a beginning in this regard. The other properties of sea surface wind speed, such as fractal structure, periodicity, probability distributions, and complex dynamics [48–60], remain to be studied from that point of view in the future.

## 5. Conclusions

We have studied the Cauchy-Matern process with LRD. The closed form of its PSD has been obtained. We also consider its application to wind speed modeling in the Lake Worth, Florida. The modeling results are satisfactory, suggesting a new model of wind speed. Though the climatologic study of wind speed is in general site dependent, one thing in common for different models appears that PSDs of the different models all are of power-law type.

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