Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 176375, 16 pages
doi:10.1155/2012/176375

Research Article

Stability and Stabilization of Impulsive Stochastic Delay Differential Equations

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Received 6 February 2012; Revised 27 March 2012; Accepted 27 March 2012

Academic Editor: Zhijian Ji

We consider the stability and stabilization of impulsive stochastic delay differential equations (ISDDEs). Using the Lyapunov-Razumikhin method, we obtain the sufficient conditions to guarantee the pth moment exponential stability of ISDDEs. Then the almost sure exponential stability is considered and the sufficient conditions of the almost sure exponential stability are obtained. Moreover, the stabilization problem of ISDDEs is studied and the criterion on impulsive stabilization of ISDDEs is established. At last, examples are presented to illustrate the correctness of our results.

1. Introduction

In recent years, the systems with stochastic or impulsive effects were studied by many authors due to their importance in many branches of science and industry, see [1–10] and references therein. In practice, a given system may be with stochastic, delay and impulsive effects simultaneously, so it is necessary to investigate the properties of impulsive stochastic delay differential equations.

There are a lot of papers discussing ISDDEs, for example, see [11–17] and the references therein. In [11, 13], the authors studied the stability of a class of impulsive delay differential equations where the impulsive effects are nonlinear. In [12], the stability of a nonlinear ISDDE was studied and the equivalent relation between the stability of the nonlinear stochastic differential delay system under impulsive control and that of a corresponding nonlinear stochastic differential delay system without impulses was established. In [14], the authors studied the stability of nonlinear impulsive stochastic differential equations in terms of two measures and the concept of perturbing Lyapunov functions is introduced to discuss stability properties. In [15], the pth exponential stability and almost sure exponential stability were studied by the Lyapunov-Krasovskii functional method. In [16], the authors considered the pth moment exponential stability by using an inequality and the propertied of M-cone. In [17], the authors studied the mean square exponential stability of ISDDEs by the formula for the variation of parameters and Cauchy matrix.

From the existing lectures, we can see the stability of ISDDEs is a main research direction. For ISDDEs, there are at least two questions on stability that need be answered, one is that if the stochastic delay differential equation (SDDE) without impulse is stable, what kinds of the impulses can the system tolerate so that it remains stable? The other is if a SDDE without impulse is unstable, what kind of impulsive effects should we take to make the ISDDEs be stable? The first one we call the question of stability, the second one we call the question of stabilization.

As we all know, the Lyapunov-Razumikhin method is a powerful tool to investigate the stability; however, to our best knowledge, there is few work on ISDDEs by using Lyapunov-Razumikhin method.

In this paper, we use the Lyapunov-Razumikhin method to answer the questions of stability and stabilization of ISDDEs, give the sufficient conditions ensuring the pth moment exponential stability of ISDDEs, and present the criteria of almost sure exponential stability of ISDDEs. The Lyapunov-Razumikhin method does not require that the formal derivative of the Lyapunov function falls into a restriction in all time; it just need to satisfy the restriction under some situation; therefore, our results relax the restrictions in some existing lectures. At last, examples are given to illustrate the efficiency of our results.

2. Preliminaries

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all P-null sets). $\mathbb{R}^n$ denotes the n-dimensional Euclidean space with the Euclidean norm $|\cdot|$. Let $PC([-\tau,0],\mathbb{R}^n)$ to denote the set of piecewise right continuous functions $\psi : [-\tau,0] \to \mathbb{R}^n$ with the sup-norm $\|\psi\| = \sup_{-\tau\leqslant\theta\leqslant 0}|\psi(\theta)|$. Take $PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$ to denote the family of all bounded $\mathcal{F}_0$-measurable $PC([-\tau,0],\mathbb{R}^n)$-valued random variables ψ satisfying $\|\psi\| = \sup_{-\tau\leqslant\theta\leqslant 0}\mathbb{E}|\psi(\theta)| < \infty$, and $PC^p_{\mathcal{F}_t}([-\tau,0],\mathbb{R}^n)$ denote the family of all $\mathcal{F}_t$-measurable $PC([-\tau,0],\mathbb{R}^n)$-valued random variables ψ satisfying $\int_{-\tau}^0 \mathbb{E}|\psi(\theta)|^p d\theta < \infty$, where $\mathbb{E}$ denotes the expectation with respect to the given probability measure P.

In this paper, we consider the following impulsive stochastic delay differential system:

$$\begin{gathered} dx(t) = f(x(t), x(t-\tau), t)dt + g(x(t), x(t-\tau), t)dB(t), \quad t \neq t_k, \\ x(t_k) = H_k\left(x\left(t_k^-\right)\right), \end{gathered} \tag{2.1}$$

where $x(t_k^-) = \lim_{h\to 0^+} x(t_k - h)$, $H_k(x(t_k^-)) = (H_{1k}(x(t_k^-)), \dots, H_{nk}(x(t_k^-)))^T$ represents the impulsive perturbation of x at time t_k and satisfies $|H_k(x(t_k^-))| \leqslant \Gamma|x(t_k^-)|, \Gamma \geqslant 0, k = 1,2,\dots$. The impulsive moments t_k satisfy $0 = t_0 < t_1 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = \infty$. The functions $f : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^{n\times m}$ are continuous functions; $B(t) = (B_1(t), B_2(t), \dots, B_m(t))^T$ is an m-dimensional standard Brownian motion defined on $(\Omega, \mathcal{F}, P)$.

The following initial value is imposed on system (2.1)

$$x(s) = \xi(s), \quad s \in [-\tau, 0], \tag{2.2}$$

where $\xi(t) \in PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$.

As a standing hypothesis, we suppose that system (2.1) has a unique solution $x(t,\xi)$ for any given initial value $\xi(t) \in PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$, and there exists an $M(p)$ for any $p > 0$ such that $\mathbb{E}|x(t,\xi)|^p < M(p)$. Suppose $x(t,\xi)$ is left continuous and right limitable. Moreover, we assume that $f(0,0,t) \equiv 0$, $g(0,0,t) \equiv 0$ and $H_k(0) = 0$ for any positive integer k, then system (2.1) admits a trivial solution.

Let v_0 denote the set of nonnegative functions $V(x,t)$ on $\mathbb{R}^n \times ([-\tau,0] \cup \mathbb{R}_+)$, which are twice continuously differential in x and once in t on $\mathbb{R}^n \times [t_k, t_{k+1})$.

If $V(x,t) \in v_0$, define an operator $LV : \mathbb{R}^n \times \mathbb{R}^n \times [t_k, t_{k+1}) \to \mathbb{R}$ associated with system (2.1) as follows:

$$LV(x,y,t) = V_t(x,t) + V_x(x,t)f(x,y,t) + \frac{1}{2}\operatorname{trace}\left[g^T(x,y,t)V_{xx}(x,t)g(x,y,t)\right], \tag{2.3}$$

where

$$V_t(x,t) = \frac{\partial V(x,t)}{\partial t}, \qquad V_x(x,t) = \left(\frac{\partial V(x,t)}{\partial x_1},\dots,\frac{\partial V(x,t)}{\partial x_n}\right), \qquad V_{xx} = \left(\frac{\partial^2 V(x,t)}{\partial x_i x_j}\right)_{n\times n}. \tag{2.4}$$

Definition 2.1 (see [15, 16]). The trivial solution of system (2.1) is said to be pth moment exponentially stable ($p > 0$) if there exist positive constants μ and M such that for any initial value $\xi(t) \in PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$,

$$\mathbb{E}|x(t,\xi)|^p \leqslant M\mathbb{E}\|\xi\|^p e^{-\mu t}, \quad t \geqslant 0. \tag{2.5}$$

Definition 2.2 (see [15]). The trivial solution of system (2.1) is said to be almost surely exponentially stable if there exists a positive constant γ such that for any initial value $\xi(t) \in PC^b_{\mathcal{F}_0}([-\tau,0],\mathbb{R}^n)$ and $t \geqslant 0$,

$$\limsup_{t\to\infty}\frac{1}{t}\ln|x(t,\xi)| \leqslant -\gamma \text{ a.s.} \tag{2.6}$$

If the trivial solution of system (2.1) is pth moment exponentially stable or almost surely exponentially stable, we also say the system (2.1) is pth moment exponentially stable or almost surely exponentially stable.

3. Stability and Stabilization of ISDDEs

In this section, we establish the criteria of pth moment exponential stability for system (2.1) by using the Lyapunov-Razumikhin technique, and the almost sure exponential stability is also considered. Moreover, the stabilization theorem is presented for system (2.1). The results show that impulses may change the stability of a given system. Some techniques used in the proof are motivated by the paper [5].

Theorem 3.1. *Assume that there exist a function $V(x,t) \in v_0$ and positive constants $p, c_1, c_2, \lambda, \mu, \gamma$, $q > 1$, $d_k \geqslant 0$, and k is any positive integer, such that the following conditions hold:*

(1) $c_1|x|^p \leqslant V(x,t) \leqslant c_2|x|^p$ *for any* $x \in \mathbb{R}^n$ *and* $t \in [-\tau, 0] \cup \mathbb{R}_+$;

(2) $LV(\varphi(t), \varphi(t-\tau), t) \leqslant -\lambda V(\varphi(t), t)$ *for all* $t \neq t_k$ *in* $\mathbb{R}_+$ *whenever* $V(\varphi(t+s), t+s) \leqslant qV(\varphi(t), t)$ *for any* $s \in [-\tau, 0]$, $q \geqslant e^{\lambda\tau}$;

(3) $V(H_k(x(t_k^-)), t_k) \leqslant (1+d_k)V(x(t_k^-), t_k^-)$;

(4) $\mu \leqslant t_k - t_{k-1}$, $d_k \leqslant \Gamma$, *and* $\ln(1+\Gamma)/\mu < \lambda$.

Then, for any $\xi(t) \in PC^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ *and* $t \geqslant 0$, *the solution* $x(t, \xi)$ *of system* (2.1) *satisfies*

$$\mathbb{E}|x(t,\xi)|^p \leqslant \frac{c_2}{c_1}\mathbb{E}\|\xi\|^p e^{-\gamma t}, \tag{3.1}$$

where $\gamma = \min\{\lambda - \ln(1+\Gamma)/\mu, \ln q/\tau\}$, *that is, system* (2.1) *is pth moment exponentially stable.*

Proof. For a given ξ, let $x(t) = x(t,\xi)$ and write $V(x(t), t) = V(t)$ for the simplicity.

We claim

$$\mathbb{E}V(t) \leqslant c_2 \prod_{i=0}^{n-1}(1+d_i)\mathbb{E}\|\xi\|^p e^{-\lambda t}, \tag{3.2}$$

for any $t \in [t_{n-1}, t_n)$, where $d_0 = 0$.

Let

$$Q(t) = \begin{cases} \mathbb{E}V(t) - c_2\left(\displaystyle\prod_{i=0}^{n-1}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda t}, & t \in [t_{n-1}, t_n), \\ \mathbb{E}V(t) - c_2\mathbb{E}\|\xi\|^p e^{-\lambda t}, & t \in [-\tau, 0]. \end{cases} \tag{3.3}$$

It is easy to check $Q(t)$ is continuous and differentiable in $[t_{n-1}, t_n)$, and

$$Q'(t) = \mathbb{E}LV(t) + \lambda c_2\left(\prod_{i=0}^{n-1}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda t}, \tag{3.4}$$

for $t \in [t_{n-1}, t_n)$.

To verify (3.2), we just need to show that $Q(t) \leqslant 0$ for all $t \geqslant 0$.

We first show that $Q(t) \leqslant 0$ for $t \in (0, t_1)$.

For $t \in [-\tau, 0]$, we have $e^{-\lambda t} \geqslant 1$; using condition 1, we can get $Q(t) \leqslant 0$. Let α be an arbitrary positive constant; we claim

$$Q(t) \leqslant \alpha, \tag{3.5}$$

for $t \in (0, t_1)$.

If (3.5) is not true, then there must exist a $\bar{t} \in (0, t_1)$, such that $Q(t) > \alpha$, which implies that there exists a $t^* \in (0, \bar{t})$ such that $Q(t^*) = \alpha$ and $Q(t) \leqslant \alpha$ for $t \in [-\tau, t^*]$ being the continuity of $Q(t)$ in $[-\tau, t^*]$. Noting

$$\mathbb{E}V(t^*) = Q(t^*) + c_2\mathbb{E}\|\xi\|^p e^{-\lambda t^*} = \alpha + c_2\mathbb{E}\|\xi\|^p e^{-\lambda t^*}, \tag{3.6}$$

using the fact $q \geqslant e^{\lambda\tau}$, $Q(t^* + s) = \mathbb{E}V(t^* + s) - c_2\mathbb{E}\|\xi\|^p e^{-\lambda(t^*+s)}$, $Q(t^* + s) \leqslant \alpha$ for $s \in [-\tau, 0]$, we have, for any $s \in [-\tau, 0]$,

$$\begin{aligned} \mathbb{E}V(t^* + s) &= Q(t^* + s) + c_2\mathbb{E}\|\xi\|^p e^{-\lambda(t^*+s)} \\ &\leqslant \alpha + c_2\mathbb{E}\|\xi\|^p e^{-\lambda(t^*-\tau)} \\ &\leqslant \left(\alpha + c_2\mathbb{E}\|\xi\|^p e^{-\lambda t^*}\right)e^{\lambda\tau} \\ &= \mathbb{E}V(t^*)e^{\lambda\tau} \leqslant q\mathbb{E}V(t^*). \end{aligned} \tag{3.7}$$

By virtue of condition 2, we can obtain $\mathbb{E}LV(t^*) \leqslant -\lambda\mathbb{E}V(t^*)$; then

$$\begin{aligned} Q'(t^*) &= \mathbb{E}LV(t^*) + \lambda c_2\mathbb{E}\|\xi\|^p e^{-\lambda t^*} \\ &\leqslant -\lambda\left(\mathbb{E}V(t^*) - c_2\mathbb{E}\|\xi\|^p e^{-\lambda t^*}\right) \\ &= -\lambda Q(t^*) = -\lambda\alpha < 0, \end{aligned} \tag{3.8}$$

which contradicts the definition of t^*. So we get $Q(t) \leqslant \alpha$ for all $t \in [0, t_1)$. Let $\alpha \to 0^+$; we obtain $Q(t) \leqslant 0$ for $t \in [0, t_1)$.

Now, assume $Q(t) \leqslant 0$ for $t \in [0, t_m)$. In view of condition 3, we have

$$\begin{aligned} Q(t_m) &= \mathbb{E}V(t_m) + c_2\left(\prod_{i=0}^{m}(1 + d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda t_m} \\ &\leqslant (1 + d_m)\left(\mathbb{E}V(t_m^-) + c_2\left(\prod_{i=0}^{m-1}(1 + d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda t_m}\right) \\ &= (1 + d_m)Q(t_m^-) \leqslant 0. \end{aligned} \tag{3.9}$$

Next, we will show, for arbitrary $\alpha > 0$,

$$Q(t) \leqslant \alpha \quad \text{for } t \in [t_m, t_{m+1}). \tag{3.10}$$

For the sake of contradiction, suppose (3.10) is not true. Define

$$\tilde{t} = \inf\{t \in [t_m, t_{m+1}) \mid Q(t) > \alpha\}. \tag{3.11}$$

From (3.9), we have $\tilde{t} \neq t_m$. The continuity of $Q(t)$ in $[t_m, t_{m+1})$ yields $Q(\tilde{t}) = \alpha$ and $Q(t) \leqslant \alpha$ for $t \in [t_m, \tilde{t}]$.

Since $\mathbb{E}V(\tilde{t}) = Q(\tilde{t}) + c_2\prod_{i=0}^{m}(1+d_i)\mathbb{E}\|\xi\|^p e^{-\lambda\tilde{t}}$, then, for $s \in [-\tau, 0]$,

$$\begin{aligned}
\mathbb{E}V\left(\tilde{t}+s\right) &= Q\left(\tilde{t}+s\right) + c_2\left(\prod_{i=0}^{m}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda(\tilde{t}+s)} \\
&\leqslant \alpha + c_2\left(\prod_{i=0}^{m}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda(\tilde{t}-\tau)} \\
&\leqslant \left[\alpha + c_2\left(\prod_{i=0}^{m}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda\tilde{t}}\right]e^{\lambda\tau} \\
&= \mathbb{E}V\left(\tilde{t}\right)e^{\lambda\tau} \leqslant q\mathbb{E}V\left(\tilde{t}\right).
\end{aligned} \tag{3.12}$$

In view of condition 2, we obtain $\mathbb{E}LV(\tilde{t}) \leqslant -\lambda\mathbb{E}V(\tilde{t})$, then

$$\begin{aligned}
Q'\left(\tilde{t}\right) &= \mathbb{E}LV\left(\tilde{t}\right) + \lambda c_2\left(\prod_{i=0}^{m}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda\tilde{t}} \\
&\leqslant -\lambda\left[\mathbb{E}V\left(\tilde{t}\right) - c_2\left(\prod_{i=0}^{m}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda\tilde{t}}\right] \\
&= -\lambda Q\left(\tilde{t}\right) = -\lambda\alpha < 0,
\end{aligned} \tag{3.13}$$

which contradicts the definition of $\tilde{t}$. Therefore, $Q(t) \leqslant \alpha$ for all $t \in [t_m, t_{m+1})$. Let $\alpha \to 0^+$; we have $Q(t) \leqslant 0$ for $t \in [t_m, t_{m+1})$. Thus, by induction, we obtain $Q(t) \leqslant 0$ holds for $t \geqslant 0$; hence

$$\mathbb{E}V(t) \leqslant c_2\left(\prod_{i=0}^{n-1}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda t} \quad \text{for } t \in [t_{n-1}, t_n). \tag{3.14}$$

Then by condition 1, we have

$$\begin{aligned}
\mathbb{E}|x|^p &\leqslant \frac{c_2}{c_1}\left(\prod_{i=0}^{\mathrm{n}-1}(1+d_i)\right)\mathbb{E}\|\xi\|^p e^{-\lambda t} \\
&\leqslant \frac{c_2}{c_1}\mathbb{E}\|\xi\|^p \exp\{n\,\ln(1+\Gamma) - \lambda t\} \\
&\leqslant \frac{c_2}{c_1}\mathbb{E}\|\xi\|^p \exp\left\{\frac{\ln(1+\Gamma)}{\mu}t - \lambda t\right\} \\
&\leqslant \frac{c_2}{c_1}\mathbb{E}\|\xi\|^p e^{-\gamma t}.
\end{aligned} \tag{3.15}$$

This completes the proof. □

The following theorem states the almost sure exponential stability of system (2.1). In the proof, the classical method used in [4] is borrowed and this method was also adopted in paper [15].

Theorem 3.2. *Suppose all of the conditions of Theorem 3.1 are satisfied and in addition $p \geqslant 2$. If there exist positive constants T and K such that $0 < t_k - t_{k-1} \leqslant T$ and for all $t \geqslant 0$*

$$\begin{aligned} &\mathbb{E}\left|f(\phi(t),\phi(t-\tau),t)\right|^p + \mathbb{E}\Big(\operatorname{trace}\Big[g^T(\phi(t),\phi(t-\tau),t)g(\phi(t),\phi(t-\tau),t)\Big]\Big)^{p/2} \\ &\quad \leqslant K \sup_{-\tau\leqslant\theta\leqslant 0} \mathbb{E}\left|\phi(t+\theta)\right|^p, \end{aligned} \tag{3.16}$$

then for all $\xi \in PC^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$,

$$\limsup_{t\to\infty} \frac{1}{t} \ln|x(t,\xi)| \leqslant -\frac{\gamma}{p} \text{ a.s.} \tag{3.17}$$

Proof. From system (2.1), we can get, for $t \in (t_{n-1}, t_n)$,

$$\begin{aligned} \mathbb{E}\left(\sup_{t_{n-1}<t<t_n} |x(t)|^p\right) \leqslant 3^{p-1}\Bigg(&\mathbb{E}|x(t_{n-1})|^p + \mathbb{E}\left[\int_{t_{n-1}}^{t_n} \left|f(x(s),x(s-\tau),s)\right| ds\right]^p \\ &+\mathbb{E}\left[\sup_{t_{n-1}<t<t_n}\left|\int_{t_{n-1}}^{t} g(x(s),x(s-\tau),s)dB(s)\right|^p\right]\Bigg). \end{aligned} \tag{3.18}$$

Using Hölder's inequality, condition (3.16), and Theorem 3.1, we derive that

$$\begin{aligned} \mathbb{E}\left[\int_{t_{n-1}}^{t_n} \left|f(x(s),x(s-\tau),s)\right| ds\right]^p &\leqslant (t_n - t_{n-1})^{p-1} \int_{t_{n-1}}^{t_n} \mathbb{E}\left|f(x(s),x(s-\tau),s)\right|^p ds \\ &\leqslant T^{p-1} K \int_{t_{n-1}}^{t_n} \sup_{-\tau\leqslant\theta\leqslant 0} \mathbb{E}|x(t+\theta)|^p ds \\ &\leqslant T^{p-1} K \int_{t_{n-1}}^{t_n} \left(\frac{c_2}{c_1}\right) \mathbb{E}\|\xi\|^p e^{-\gamma s} ds \\ &\leqslant \frac{c_2}{c_1} T^p K \mathbb{E}\|\xi\|^p e^{-\gamma t_{n-1}} \\ &\leqslant \frac{c_2}{c_1} T^p e^T K \mathbb{E}\|\xi\|^p e^{-\gamma t_n}. \end{aligned} \tag{3.19}$$

In virtue of Burkholder-Davis-Gundy inequality, we have

$$\mathbb{E}\left[\sup_{t_{n-1}<t<t_n} \int_{t_{n-1}}^{t} \left|g(x(s),x(s-\tau),s)dB(s)\right|^p\right] \leqslant C_p \mathbb{E}\left(\int_{t_{n-1}}^{t_n} \operatorname{trace}\left[g^T g\right] ds\right)^{p/2}, \tag{3.20}$$

where C_p is a positive constant dependent on p only. One can then show in the same way as in (3.19) that

$$\mathbb{E}\left[\sup_{t_{n-1}<t<t_n} \int_{t_{n-1}}^{t} \left|g(x(s),x(s-\tau),s)dB(s)\right|^p\right] \leqslant \frac{c_2}{c_1} C_p T^{p/2} e^T K \mathbb{E}\|\xi\|^p e^{-\gamma t_n}. \tag{3.21}$$

Substituting (3.15), (3.19), and (3.21) into (3.18) yields

$$\mathbb{E}\left(\sup_{t_{n-1}<t<t_n}|x(t)|^p\right) \leqslant K_1 e^{-\gamma t_n}, \tag{3.22}$$

where

$$K_1 = \frac{3^{p-1}c_2}{c_1}\mathbb{E}\|\xi\|^p\left[1 + Ke^T\left(T^p + C_p T^{p/2}\right)\right]. \tag{3.23}$$

When $t = t_{n-1}$, keeping $|H_{n-1}(x(t_{n-1}^-))| \leqslant \Gamma|x(t_{n-1}^-)|$ and (3.15) in mind, we can get

$$\mathbb{E}|x(t_{n-1})| = \mathbb{E}\left|H_{n-1}\left(x\left(t_{n-1}^-\right)\right)\right| \leqslant \Gamma\mathbb{E}\left|x\left(t_{n-1}^-\right)\right| \leqslant \frac{c_2}{c_1}\Gamma e^T\mathbb{E}\|\xi\|^p e^{-\gamma t_n}. \tag{3.24}$$

Taking $K_2 = \max\{K_1, c_2/c_1\Gamma e^{T+t_0}\mathbb{E}\|\xi\|^p\}$, we have

$$\mathbb{E}\left(\sup_{t_{n-1}\leqslant t<t_n}|x(t)|^p\right) \leqslant K_2 e^{-\gamma t_n}. \tag{3.25}$$

We now show that (3.25) implies the required (3.17).

Let ϵ be an arbitrary constant satisfying $0 < \epsilon < \gamma$. By virtue of (3.25) and Markovian inequality, we have

$$P\left(\omega : \sup_{t_{n-1}\leqslant t<t_n}|x(t)|^p > e^{-(\gamma-\epsilon)t_n}\right) \leqslant e^{(\gamma-\epsilon)t_n}\mathbb{E}\left(\sup_{t_{n-1}\leqslant t<t_n}|x(t)|^p\right) \leqslant K_2 e^{-\epsilon t_n} \leqslant K_2 e^{t_0}e^{-n\mu\epsilon}. \tag{3.26}$$

In view of Borel-Cantelli lemma, we can obtain that for almost all $\omega \in \Omega$,

$$\sup_{t_{n-1}\leqslant t<t_n}|x(t)|^p \leqslant e^{-(\gamma-\epsilon)t_n} \tag{3.27}$$

holds for all but finitely many n. Hence there exists an $N(\omega)$, for all $\omega \in \Omega$ but a P-null set, such that (3.27) holds when $n > N(\omega)$. Then we have, for almost all $\omega \in \Omega$, if $t_{n-1} \leqslant t < t_n$, $n > N(\omega)$,

$$|x(t)|^p \leqslant e^{-(\gamma-\epsilon)t_n} \leqslant e^{-(\gamma-\epsilon)t}, \tag{3.28}$$

that is,

$$\frac{1}{t}\ln|x(t)| \leqslant -\frac{\gamma-\epsilon}{p} \tag{3.29}$$

and (3.17) follows by letting $\epsilon \to 0$. $\square$

In the following, we give two corollaries.

Corollary 3.3. *Assume there exist positive constants* $\lambda, \alpha, \mu, p, c_1, c_2$ *and a function* $V(x,t) \in \nu_0$ *such that*

(1) $c_1|x|^p \leqslant V(x,t) \leqslant c_2|x|^p$ *for all* $(x,t) \in \mathbb{R}^n \times [-\tau, \infty)$;

(2) $V_t(x,t) + V_x(x,t) f(x, x(t-\tau), t) + (1/2)trace[g^T(x, x(t-\tau), t) V_{xx}(x(t), t) g(x, x(t-\tau), t)] \leqslant -\lambda V(x,t) + \alpha V(x(t-\tau), t-\tau)$ *for all* $t \in (t_{k-1}, t_k)$, *where* $\lambda > e^{\lambda\tau}\alpha$;

(3) $V(H_k(x(t_k^-)), t_k) \leqslant (1+d_k) V(x(t_k^-), t_k^-)$;

(4) $\mu \leqslant t_k - t_{k-1}$, $d_k \leqslant \Gamma$, *and* $\ln(1+\Gamma)/\mu < \lambda - e^{\lambda\tau}\alpha$.

Then, the trivial solution of system (2.1) *is pth moment exponentially stable.*

Furthermore, if there exists a positive constant T such that $t_k - t_{k-1} \leqslant T$ *and the following inequality holds for any* $x, y \in \mathbb{R}^n$:

$$\left|f(x,y,t)\right| \vee \left|g(x,y,t)\right| \leqslant K\left(|x| + |y|\right), \tag{3.30}$$

then the trivial solution of system (2.1) *is almost surely exponentially stable.*

Proof. Take $q = e^{\lambda\tau}$. Obviously, we just need to verify the condition 2 of Theorem 3.1.

$$\begin{aligned} LV(x(t), x(t-\tau), t) &= V_t(x(t), t) + V_x(x(t), t) f(x(t), x(t-\tau), t) \\ &\quad + \frac{1}{2}\text{trace}\Big(g^T(x(t), x(t-\tau), t) V_{xx}(x(t), t) g(x(t), x(t-\tau), t)\Big) \\ &\leqslant -\lambda V(x(t), t) + \alpha V(x(t-\tau), t-\tau). \end{aligned} \tag{3.31}$$

If $t \geqslant 0$ and $t \in (t_k, t_{k+1})$, $s \in [-\tau, 0]$, the following inequality holds

$$V(x(t+s), t+s) \leqslant qV(x(t), t), \tag{3.32}$$

then

$$\begin{aligned} LV(x(t), x(t-\tau), t) &\leqslant -\lambda V(x(t), t) + \alpha q V(x(t), t) \\ &= -(\lambda - \alpha q) V(x(t), t) = -\left(\lambda - \alpha e^{\lambda\tau}\right) V(x(t), t). \end{aligned} \tag{3.33}$$

Condition 2 of Theorem 3.1 is verified, then the pth moment exponential stability for the trivial solution of system (2.1) is obtained. The almost sure exponential stability is followed directly by virtue of Theorem 3.2. □

The 2th moment exponential stability; is also called mean square exponential stability, the following corollary presents the criteria of mean square exponential stability of system (2.1).

Corollary 3.4. *For system* (2.1), *assume there exist positive constants* a, b, c, d, μ, β_k *such that*

(1) $x^T f(x,y,t) \leqslant -a|x|^2 + b|y|^2$, *and* $|g(x,y,t)|^2 \leqslant c|x|^2 + d|y|^2$, $2a - c > 0$ *and* $2a - c - e^{(2a-c)\tau}(2b+d) > 0$;

(2) $|H_k(x)| \leqslant \beta_k|x|$;

(3) $\mu \leqslant t_{k+1} - t_k$, $\beta_k^2 \leqslant \Gamma + 1$, $\ln(\Gamma+2)/\mu < 2a - c - e^{(2a-c)\tau}(2b+d)$.

Then the trivial solution of system (2.1) *is mean square exponentially stable.*

Proof. Let $V(x,t) = |x|^2$; then

$$\begin{aligned} LV(x(t), x(t-\tau), t) &= 2x^T f(x(t), x(t-\tau), t) + \left|g(x(t), x(t-\tau), t)\right|^2 \\ &\leqslant -2a|x|^2 + 2b|x(t-\tau)|^2 + c|x|^2 + d|x(t-\tau)|^2 \\ &= (-2a+c)|x|^2 + (2b+d)|x(t-\tau)|^2. \end{aligned} \tag{3.34}$$

The conditions of Corollary 3.3 are easily to be verified, and the required result can be obtained. □

Now we are on the position to state the stabilization theorem.

Theorem 3.5. *Assume there exist a function $V(x,t) \in v_0$ and positive constants p, c_1, c_2, c, λ, α such that*

(1) $c_1|x|^p \leqslant V(x,t) \leqslant c_2|x|^p$, *for any* $x \in \mathbb{R}^n$ *and* $t \in \mathbb{R}_+$;

(2) $LV(x(t), x(t-\tau), t) \leqslant cV(x(t), t)$, *for all* $t \in [t_k, t_{k+1})$, *whenever* $qV(x(t), t) \geqslant V(t+s, x(t+s))$, *for* $s \in [-\tau, 0]$, *where* $q \geqslant \max\{e^{2\lambda\alpha}, e^{c\alpha}\}$;

(3) $V(H_k(x(t_k^-)), t_k) \leqslant d_k V(t_k^-, x(t_k^-))$, *where* $d_k > 0$;

(4) $\tau \leqslant t_{k+1} - t_k \leqslant \alpha$ *and* $\ln d_k + \lambda\alpha < -\lambda(t_{k+1} - t_k)$.

Then the trivial solution of system (2.1) *is pth moment exponentially stable.*

Remark 3.6. From condition 2, we can see that the formal derivative of $V(x,t)$ can be positive since c is a positive constant; this means that the original system without impulses may be unstable. Therefore, this theorem is called the stabilization theorem.

Proof. Let $x(t) = x(t,\xi)$ be a solution of (2.1) with $x(t) = \xi(t)$, $t \in [-\tau, 0]$ and write $V(x(t), t) = V(t)$ for simplicity. Choose $M \geqslant 1$ such that

$$c_2\mathbb{E}\|\xi\|^p < M\mathbb{E}\|\xi\|^p e^{-\lambda t_1} e^{-\alpha c} < M\mathbb{E}\|\xi\|^p e^{-\lambda t_1} \leqslant qc_2\mathbb{E}\|\xi\|^p. \tag{3.35}$$

We will show, for any positive integer k,

$$\mathbb{E}V(t) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_k}, \quad t \in [t_{k-1}, t_k). \tag{3.36}$$

We first show that

$$\mathbb{E}V(t) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_1}, \quad t \in [0, t_1). \tag{3.37}$$

From condition 1 and (3.35), we have, for $t \in [-\tau, 0]$,

$$\mathbb{E}V(t) \leqslant c_2\mathbb{E}|x(t)|^p \leqslant c_2\mathbb{E}\|\xi\|^p < M\mathbb{E}\|\xi\|^p e^{-\lambda t_1} e^{-\alpha c}. \tag{3.38}$$

If (3.37) is not true, then there must be a $\bar{t} \in (0, t_1)$ such that

$$\mathbb{E}V\left(\bar{t}\right) > M\mathbb{E}\|\xi\|^p e^{-\lambda t_1} > M\mathbb{E}\|\xi\|^p e^{-\lambda t_1} e^{-\alpha c} > c_2\mathbb{E}\|\xi\|^p \geqslant \mathbb{E}V(s), \tag{3.39}$$

where $s \in [-\tau, 0]$.

Then there exists a $t^* \in (0, \bar{t})$ such that

$$\mathbb{E}V(t^*) = M\mathbb{E}\|\xi\|^p e^{-\lambda t_1}, \quad \mathbb{E}V(t) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_1}, \quad t \in [-\tau, t^*], \tag{3.40}$$

and there is a $t^{**} \in [0, t^*)$ such that

$$\mathbb{E}V(t^{**}) = c_2\mathbb{E}\|\xi\|^p, \quad \mathbb{E}V(t) \geqslant c_2\mathbb{E}\|\xi\|^P, \quad t^{**} \leqslant t \leqslant t^*. \tag{3.41}$$

Then we have, for any $t \in [t^{**}, t^*]$,

$$\mathbb{E}V(t+s) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_1} \leqslant qc_2\mathbb{E}\|\xi\|^P \leqslant q\mathbb{E}V(t), \quad s \in [-\tau, 0]. \tag{3.42}$$

From condition 2, we have

$$\mathbb{E}LV(t) \leqslant c\mathbb{E}V(t) \tag{3.43}$$

for $t \in [t^{**}, t^*]$. Since $t^* - t^{**} < \alpha$, we get

$$\begin{aligned} \mathbb{E}V(t^*)e^{-\alpha c} - \mathbb{E}V(t^{**}) &\leqslant \mathbb{E}V(t^*)e^{-c(t^*-t^{**})} - \mathbb{E}V(t^{**}) \\ &= \int_{t^{**}}^{t^*} e^{-c(s-t^{**})}(-c\mathbb{E}V(s) + \mathbb{E}LV(s))ds \leqslant 0, \end{aligned} \tag{3.44}$$

that is,

$$\mathbb{E}V(t^*)e^{-\alpha c} \leqslant \mathbb{E}V(t^{**}), \tag{3.45}$$

then

$$M\mathbb{E}\|\xi\|^p e^{-\lambda t_1} e^{-\alpha c} \leqslant c_2\mathbb{E}\|\xi\|^P, \tag{3.46}$$

which is in conflict with (3.35). We obtain that (3.37) holds, that is, (3.36) holds when $k = 1$.

Now we assume that (3.36) holds when $k = 1, 2, 3, \ldots, m$, m is a positive integer and $m \geqslant 1$, then

$$\mathbb{E}V(t) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_k}, \quad t \in [t_{k-1}, t_k), \tag{3.47}$$

especially,

$$\mathbb{E}V\left(t_m^-\right) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_m}. \tag{3.48}$$

From conditions 3, 4 and (3.48), we have

$$\begin{aligned}\mathbb{E}V(t_m) &\leqslant d_m \mathbb{E}V\left(t_m^-\right)\\ &< e^{-\lambda\alpha-\lambda(t_{m+1}-t_m)} M\mathbb{E}\|\xi\|^p e^{-\lambda t_m}\\ &= e^{-\lambda\alpha} M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}}\\ &< M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}}.\end{aligned} \tag{3.49}$$

Now we will show that (3.36) holds when $k = m+1$, that is,

$$\mathbb{E}V(t) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}}, \quad t \in [t_m, t_{m+1}). \tag{3.50}$$

If (3.50) is not true, we define

$$\bar{t} = \inf\left\{t \in [t_m, t_{m+1}) \mid \mathbb{E}V(t) > M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}}\right\}. \tag{3.51}$$

From (3.49), we know $\bar{t} \neq t_m$; by the continuity of $\mathbb{E}V(t)$ in $[t_m, t_{m+1})$, we get

$$\begin{gathered}\mathbb{E}V\left(\bar{t}\right) = M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}},\\ \mathbb{E}V(t) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}}, \quad t \in \left[t_m, \bar{t}\right].\end{gathered} \tag{3.52}$$

From (3.49), we have

$$\mathbb{E}V(t_m) < e^{-\lambda\alpha} M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}} < EV\left(\bar{t}\right); \tag{3.53}$$

there must be a $t^* \in (t_m, \bar{t})$ such that

$$\begin{gathered}\mathbb{E}V(t^*) = e^{-\lambda\alpha} M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}},\\ \mathbb{E}V(t^*) \leqslant \mathbb{E}V(t) \leqslant \mathbb{E}V\left(\bar{t}\right), \quad t \in \left[t^*, \bar{t}\right].\end{gathered} \tag{3.54}$$

Since $\tau \leqslant t_{k+1} - t_k \leqslant \alpha$, and $s \in [-\tau, 0]$, when $t \in [\mathrm{t}^*, \bar{t}]$, we get $t + s \in [t_{m+1}, \bar{t}]$. From (3.48) and (3.53), when $t \in [t^*, \bar{t}]$, $s \in [-\tau, 0]$, we have

$$\begin{aligned}\mathbb{E}V(t+s) &\leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_m}\\ &= M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}} e^{\lambda(t_{m+1}-t_m)}\\ &\leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}} e^{\lambda\alpha}\\ &= e^{2\lambda\alpha}\mathbb{E}V(t^*) \leqslant q\mathbb{E}V(t).\end{aligned} \tag{3.55}$$

From condition 2, we get

$$\mathbb{E}LV(t) \leqslant c\mathbb{E}V(t). \tag{3.56}$$

But

$$\begin{aligned}\mathbb{E}V\left(\bar{t}\right)e^{-\alpha c} - \mathbb{E}V(t^*) &\leqslant \mathbb{E}V\left(\bar{t}\right)e^{-(\bar{t}-t^*)c} - \mathbb{E}V(t^*) \\ &= \int_{t^*}^{\bar{t}} e^{-c(s-t^*)}(-c\mathbb{E}V(s) - \mathbb{E}LV(s))ds < 0,\end{aligned} \tag{3.57}$$

we get

$$\begin{aligned}\mathbb{E}V\left(\bar{t}\right) &\leqslant \mathbb{E}V(t^*)e^{\alpha c} = M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}} e^{-\lambda\alpha} e^{\alpha c} \\ &= M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}} e^{-(\lambda-c)\alpha} \\ &< M\mathbb{E}\|\xi\|^p e^{-\lambda t_{m+1}} = \mathbb{E}V\left(\bar{t}\right).\end{aligned} \tag{3.58}$$

Then (3.36) holds when $k = m+1$. By induction, we have that (3.36) holds, and

$$\mathbb{E}V(t) \leqslant M\mathbb{E}\|\xi\|^p e^{-\lambda t}, \quad t \in [t_{k-1}, t_k). \tag{3.59}$$

From condition 1, we have

$$\mathbb{E}|x(t)|^p \leqslant M^*\mathbb{E}\|\xi\|^p e^{-\lambda t}, \quad t \in [t_k, t_{k+1}),\ k \in \mathbb{N}, \tag{3.60}$$

where $M^* \geqslant \max\{1, (M/c_1)^{1/p}\}$. This completes the proof. □

4. Applications and Examples

In this section, we consider a nonlinear impulsive stochastic delay differential system. We present the stability criterion and stabilization criterion for this system, then we illustrate the correctness of our results using the numerical experiments.

The following nonlinear impulsive stochastic delay differential system is considered:

$$\begin{gathered}dx(t) = \left(ax(t) + bx(t-\tau)\exp\left[-x^2(t-\tau)\right]\right)dt + cx(t-\tau)dW(t), \quad t \neq t_k, \\ x(t_k) = \beta_k x(t_k^-).\end{gathered} \tag{4.1}$$

By virtue of Corollary 3.4, we can get the following corollary directly.

Corollary 4.1. *Assume there exist positive constants μ and T such that the impulsive moments t_k satisfy $\mu \leqslant t_k - t_{k-1} \leqslant T$, and the following inequalities hold:*

$$\begin{gathered}2a + |b| + e^{(-2a-|b|)\tau}\left(2b + c^2\right) < 0, \\ \beta_k^2 < \Gamma + 1, \frac{\ln(\Gamma+2)}{\mu} < -2a - |b| - e^{(-2a-|b|)\tau}\left(2b + c^2\right),\end{gathered} \tag{4.2}$$

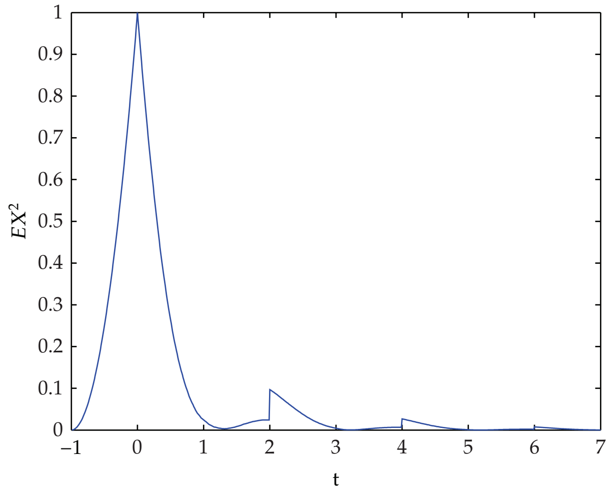

Figure 1: Mean square exponential stability of system (4.1).

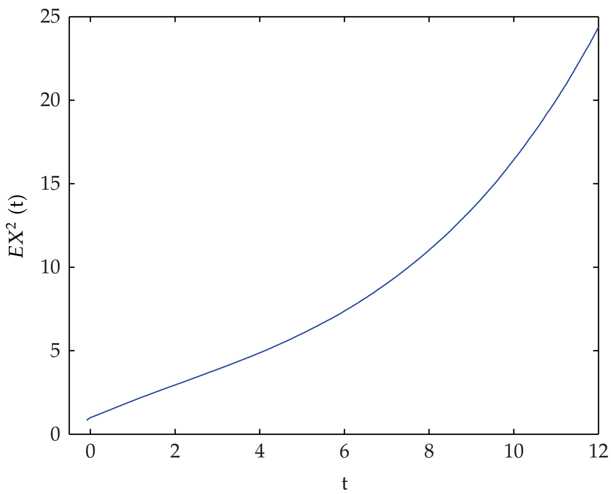

Figure 2: Nonstability of system (4.1) without impulse.

then the trivial solution of system (4.1) *is mean square exponentially stable and almost surely exponentially stable.*

Using Theorem 3.5 and taking $V(x,t) = |x|^2$*, we can easily get the following stabilization corollary for system* (4.1).

Corollary 4.2. *Assume there exist constants* $\alpha, \lambda > 0$ *satisfing*

(1) *there exists a constant* $q > 0$ *such that* $q > \max\{e^{2\lambda\alpha}, e^{\gamma\alpha}\}$*, where* $\gamma = 2a + |b| + |b|q + c^2 q$*;*

(2) $\tau \leqslant t_{k+1} - t_k \leqslant \alpha$ *and* $\ln \beta_k + \lambda\alpha/2 < -(\lambda/2)(t_{k+1} - t_k)$.

Then the trivial solution of system (4.1) *is pth moment exponentially stable.*

Now let us illustrate the correctness of Corollaries 4.1 and 4.2.

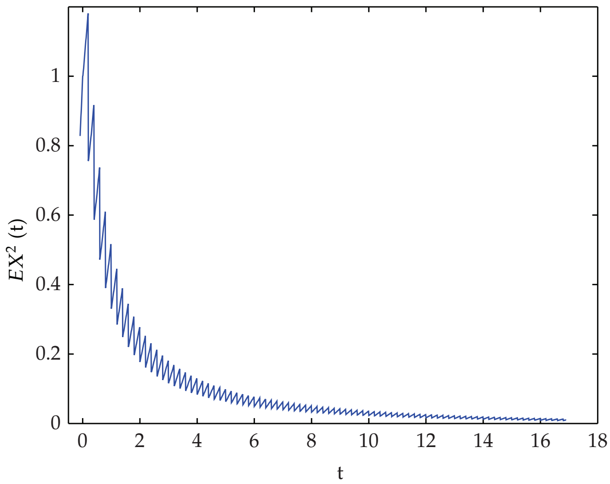

Figure 3: Mean square exponential stability of system (4.1).

Let $a = -1$, $b = -1$, $c = 1$, $\tau = 1$, and $\beta_k = 2$. Take $x(s) = s + 1$ when $s \in [-1, 0]$ and $t_{k+1} - t_k = 2$. Let $\mu = 1$ and $\Gamma = 7$; it is easy to verify the conditions of Corollary 4.1 are satisfied. Then the trivial solution of system (4.1) is mean square exponentially stable. The mean square exponential stability is showed in Figure 1. This illustrates the correctness of Corollary 4.1.

Now we consider the stabilization of system (4.1). Take $a = 0.1$, $b = 1$, $c = 0.1$, and $\tau = 0.1$. Let $x(s) = s + 1$ when $s \in [-0.1, 0]$. It is easy to see that the trivial solution of system (4.1) without impulsive effects is not mean square stable, see Figure 2. Then we take $t_{k+1} - t_k = 0.2$, $\beta_k = 0.8$ and $\alpha = 0.2$, $\lambda = 1$, $q = 2$; it can be verified that the conditions of Corollary 4.2 are satisfied; the the trivial solution of system (4.1) is mean square exponentially stable, see Figure 3.

Acknowledgments

This paper was supported by the National Natural Science Foundation of China under Grant 11026189 and by the Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (HIT.NSRIF.201014).

References

[1] H. Wu and J. Sun, "p-moment stability of stochastic differential equations with impulsive jump and Markovian switching," *Automatica*, vol. 42, no. 10, pp. 1753–1759, 2006.

[2] W. Zhang, G. Feng, and Q. Li, "Robust H_∞ filtering for general nonlinear stochastic state-delayed systems," *Mathematical Problems in Engineering*, vol. 2012, Article ID 231352, 15 pages, 2012.

[3] J. Sun, Y. Zhang, and Q. Wu, "Less conservative conditions for asymptotic stability of implusive control systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 5, pp. 829–831, 2003.

[4] X. Mao, "Razumikhin-type theorems on exponential stability of stochastic functional-differential equations," *Stochastic Processes and their Applications*, vol. 65, no. 2, pp. 233–250, 1996.

[5] Q. Wang and X. Liu, "Impulsive stabilization of delay differential systems via the Lyapunov-Razumikhin method," *Applied Mathematics Letters*, vol. 20, no. 8, pp. 839–845, 2007.

[6] M. De la Sen, "Stability of impulsive time-varying systems and compactness of the operators mapping the input space into the state and output spaces," *Journal of Mathematical Analysis and Applications*, vol. 321, no. 2, pp. 621–650, 2006.

[7] M. De la Sen and N. Luo, "A note on the stability of linear time-delay systems with impulsive inputs," *IEEE Transactions on Circuits and Systems I*, vol. 50, no. 1, pp. 149–152, 2003.
[8] H. Bereketoglu, G. Seyhan, and A. Ogun, "Advanced impulsive differential equations with piecewise constant arguments," *Mathematical Modelling and Analysis*, vol. 15, no. 2, pp. 175–187, 2010.
[9] M. De la Sen, P. Agarwal Ravi, and A. Ibeas, "On a generalized time-varying SEIR epidemic model with mixed point and distributed time-varying delays and combined regular and impulsive vaccination controls," *Advances in Difference Equations*, vol. 2010, Article ID 281612, 2010.
[10] M. de la Sen, "Global stability of polytopic linear time-varying dynamic systems under time-varying point delays and impulsive controls," *Mathematical Problems in Engineering*, vol. 2010, Article ID 693958, 33 pages, 2010.
[11] C. Li, J. Sun, and R. Sun, "Stability analysis of a class of stochastic differential delay equations with nonlinear impulsive effects," *Journal of the Franklin Institute*, vol. 347, no. 7, pp. 1186–1198, 2010.
[12] C. Li and J. Sun, "Stability analysis of nonlinear stochastic differential delay systems under impulsive control," *Physics Letters A*, vol. 374, no. 9, pp. 1154–1158, 2010.
[13] C. Li, J. Shi, and J. Sun, "Stability of impulsive stochastic differential delay systems and its application to impulsive stochastic neural networks," *Nonlinear Analysis: Theory, Methods & Applications A*, vol. 74, no. 10, pp. 3099–3111, 2011.
[14] S. Zhang, J. Sun, and Y. Zhang, "Stability of impulsive stochastic differential equations in terms of two measures via perturbing Lyapunov functions," *Applied Mathematics and Computation*, vol. 218, pp. 5181–5186, 2012.
[15] L. Pan and J. Cao, "Exponential stability of impulsive stochastic functional differential equations," *Journal of Mathematical Analysis and Applications*, vol. 382, no. 2, pp. 672–685, 2011.
[16] Z. Yang, D. Xu, and L. Xiang, "Exponential p-stability of impulsive stochastic differential equations with delays," *Physics Letters A*, vol. 359, no. 2, pp. 129–137, 2006.
[17] L. Xu and D. Xu, "Mean square exponential stability of impulsive control stochastic systems with time-varying delay," *Physics Letters A*, vol. 373, pp. 328–333, 2009.