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Research Article

Stability Analysis of Interconnected Fuzzy Systems Using the Fuzzy Lyapunov Method

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The fuzzy Lyapunov method is investigated for use with a class of interconnected fuzzy systems. The interconnected fuzzy systems consist of J interconnected fuzzy subsystems, and the stability analysis is based on Lyapunov functions. Based on traditional Lyapunov stability theory, we further propose a fuzzy Lyapunov method for the stability analysis of interconnected fuzzy systems. The fuzzy Lyapunov function is defined in fuzzy blending quadratic Lyapunov functions. Some stability conditions are derived through the use of fuzzy Lyapunov functions to ensure that the interconnected fuzzy systems are asymptotically stable. Common solutions can be obtained by solving a set of linear matrix inequalities (LMIs) that are numerically feasible. Finally, simulations are performed in order to verify the effectiveness of the proposed stability conditions in this paper.

1. Introduction

In the recent years, a number of research activities have been conducted concerning stability analysis and the stabilization problems of large-scale systems, including electric power systems, nuclear reactors, aerospace systems, large electrical networks, economic systems, process control systems, chemical and petroleum industrial systems, different societal systems, ecological systems, and transportation systems. The interconnected models can be used to represent practical large-scale systems. Moreover, the field of interconnected systems is so broad as to cover the fundamental theory of modeling, optimization and control aspects, and such applications. Therefore, the methodologies used when dealing with interconnected models provide a viable technique whereby the manipulation of system structure can be used to overcome the increasing size and complexity of the relevant mathematical models. Additionally, the fields of analysis, design, and control theory relating to interconnected systems have attained a considerable level of maturity and sophistication. They are currently receiving increasing attention from theorists and practitioners because they are methodologically interesting and have important real-life applications. Such systems comprise numerous interdependent subsystems which serve particular functions, share resources, and are governed by a set of interrelated goals and constraints. Recently, various approaches have been employed to elucidate the stability and stabilization of interconnected systems, as proposed in the literature and the references therein [1–3].

Since Zadeh [4] and Takagi and Sugeno [5] proposed a new concept for a fuzzy inference system which combines the flexibility of fuzzy logic theory and rigorous mathematical analysis tools into a unified framework, the application of fuzzy models has attracted great interest from the engineering and management community (e.g., see [6–20] and the references therein). This kind of fuzzy model suggests an efficient method to represent complex nonlinear systems via fuzzy reasoning. This has enabled the stability issues of fuzzy systems to be extensively applied in system analysis (see [21–26] and the references therein). Similarly, the stability criteria and stabilization problems have been discussed for fuzzy large-scale systems in Wang and Luoh [27], and Hsiao and Hwang [28], where the fuzzy large-scale system consists of J subsystems.

In the aforementioned results for T-S fuzzy models, most of the stability criteria and controller design have usually been derived based on the usage of a single Lyapunov function. However, the main drawback associated with this method is that the single Lyapunov function must work for all linear models of the fuzzy control systems, which in general leads to a conservative controller design [29]. Recently, in order to relax this conservatism, the fuzzy Lyapunov function approach has been proposed in [24, 29–33]. To the best of my knowledge, the stability analyses of interconnected systems based on fuzzy Lyapunov functions have not been discussed yet. Therefore, some novel sufficient conditions are derived from fuzzy Lyapunov functions for stability guarantees by fuzzy for interconnected fuzzy systems in this work.

The organization of the paper is presented as follows. First, the T-S fuzzy modeling is briefly reviewed and the interconnected scheme is used to construct a fuzzy dynamic model. Then, the stability conditions for the fuzzy Lyapunov functions are proposed which guarantee the stability of the interconnected fuzzy systems. In this section, the stability problems can be reformulated into a problem for solving a linear matrix inequality (LMI).

2. System Descriptions and Preliminaries

A fuzzy dynamic model was proposed in the pioneering work of Takagi and Sugeno [5] where complex nonlinear systems could be represented using local linear input/output relations. The main feature of this model is that each locally fuzzy implication (rule) is a linear system and the overall system model is achieved through linear fuzzy blending. This fuzzy dynamic model is described by fuzzy IF-THEN rules and are utilized in this study to deal with the stability analysis issue of an interconnected fuzzy system S that is composed of J subsystems Si ($j = 1, \ldots, J$). The ith rule of the interconnected fuzzy model of the jth subsystem is proposed as having the following form.

Plant Rule i:

$$\begin{aligned}&\text{IF } x_{1j}(t) \text{ is } M_{i1j},\dots,x_{gj}(t) \text{ is } M_{igj},\\ &\text{THEN } \dot{x}_j(t) = A_{ij}x_j(t) + \sum_{\substack{n=1\\ n\neq j}}^{J} \widehat{A}_{inj}x_n(t), \quad i = 1,2,\dots,r_j,\end{aligned} \tag{2.1}$$

where r_j is the IF-THEN rule number; A_{ij} and $\widehat{A}_{inj}$ are constant matrices with appropriate dimensions; $x_j(t)$ is the state vector of the jth subsystem; $x_n(t)$ is the interconnection between the nth and jth subsystems; $M_{ipj}(p = 1,2,\dots,g)$ are the fuzzy sets; $x_{1j}(t) \sim x_{g\,j}(t)$ are the premise variables. Through the use of "fuzzy blending," the overall fuzzy model of the jth fuzzy subsystem can be inferred as follows [13]:

$$\dot{x}_j(t) = \frac{\sum_{i=1}^{r_j} w_{ij}(t)\left[A_{ij}x_j(t) + \sum_{n=1,n\neq j}^{J} \widehat{A}_{inj}x_n(t)\right]}{\sum_{i=1}^{r_j} w_{ij}(t)} = \sum_{i=1}^{r_j} h_{ij}(t)\left[A_{ij}x_j(t) + \sum_{\substack{n=1\\ n\neq j}}^{J} \widehat{A}_{inj}x_n(t)\right], \tag{2.2}$$

with

$$w_{ij}(t) \equiv \prod_{p=1}^{g} M_{ipj}\left(x_{pj}(t)\right), \qquad h_{ij}(t) \equiv \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)}, \tag{2.3}$$

in which $M_{ipj}(x_{pj}(t))$ is the grade of membership of $x_{pj}(t)$ in M_{ipj}. In this study, it is assumed that $w_{ij}(t) \geq 0, i = 1,2,\dots,r_j$; $j = 1,2,\dots,J$. Therefore, the normalized membership function $h_{ij}(t)$ satisfies

$$h_{ij}(t) \geq 0, \quad \sum_{i=1}^{r_j} h_{ij}(t) = 1, \quad \forall t. \tag{2.4}$$

In the following, we state the lemmas which are useful to prove the stability of the interconnected fuzzy system $\mathbf{S}$ which consists of J closed-loop subsystems described in (2.1).

Lemma 2.1 (see [13]). *For any A, $B \in R^n$ and for any symmetric positive definite matrix $G \in R^{n\times n}$, one has*

$$2A^T B \leq A^T G A + B^T G^{-1} B. \tag{2.5}$$

Lemma 2.2 (see [34]). *(Schur complements) One has*

$$\textit{The LMI} \begin{bmatrix} Q(x) & S(x) \\ S(x) & R(x) \end{bmatrix} > 0, \tag{2.6}$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$, *and* $S(x)$ *depends on* x *that is equivalent to*

$$R(x) > 0, \quad Q(x) - S(x)R^{-1}(x)S^T(x) > 0. \tag{2.7}$$

In other words, the set of nonlinear inequalities (2.7) *can be represented as the LMI* (2.6).

3. Stability Analysis by a Fuzzy Lyapunov Function

Here we define a fuzzy Lyapunov function and consider the stability conditions for the jth fuzzy subsystem (2.2).

Definition 3.1. Equation (3.1) is said to be a fuzzy Lyapunov function for the T-S fuzzy system (2.2) if the time derivative of $V(t)$ is always negative

$$V(t) = \sum_{j=1}^{J} v_j(t) = \sum_{j=1}^{J} \sum_{l=1}^{r_j} h_{lj}(t) x_j^T(t) P_{lj} x_j(t), \tag{3.1}$$

where P_{lj} is a positive definite matrix.

Because the fuzzy Lyapunov function shares the same membership functions with the T-S fuzzy model of a system, the time derivative of the fuzzy Lyapunov function contains the time derivative of the premise membership functions. Therefore, how to deal with the time derivative of the time derivative of the premise membership functions is an important consideration.

By taking the time derivative of (3.1), the following stability condition of open-loop system (2.7) will be obtained.

Theorem 3.2. *The fuzzy system* (2.7) *is stable in the large if there exist common positive definite matrices* $P_1, P_2, \ldots, P_r$ *such that the following inequality is satisfied:*

$$\sum_{\rho=1}^{r_j} \dot{h}_{\rho j}(t) P_{\rho j} + \sum_{l=1}^{r_j} \sum_{i=1}^{r_j} h_{lj}(t) h_{ij}(t) \left[A_{ij}^T P_{li} + P_{li} A_{ij} + \alpha (J-1) I + \sum_{n=1}^{J} \alpha^{-1} P_j \widehat{A}_{inj} \widehat{A}_{inj}^T P_j \right] < 0. \tag{3.2}$$

Proof. Consider the Lyapunov function candidate for the fuzzy system (2.2)

$$V(t) = \sum_{j=1}^{J} v_j(t) = \sum_{j=1}^{J} \sum_{l=1}^{r_j} h_{lj}(t) x_j^T(t) P_{lj} x_j(t). \tag{A1}$$

The time derivative of V is

$$\dot{V}(t) = \sum_{j=1}^{J}\sum_{\rho=1}^{r_j}\dot{h}_{\rho j}(t)x_j^T(t)P_{\rho j}x_j(t) + \sum_{j=1}^{J}\sum_{l=1}^{r_j}h_{lj}(t)\left\{\dot{x}_j^T(t)P_{lj}x_j(t) + x_j^T(t)P_{lj}\dot{x}_j(t)\right\} \tag{A2}$$

$$\begin{aligned}
&= \sum_{j=1}^{J}\sum_{\rho=1}^{r_j}\dot{h}_{\rho j}(t)x_j^T(t)P_{\rho j}x_j(t) \\
&\quad + \sum_{j=1}^{J}\sum_{l=1}^{r_j}h_{lj}(t)\left\{\left[\sum_{i=1}^{r_j}h_{ij}(t)\left(A_{ij}x_j(t) + \sum_{\substack{n=1\\ n\neq j}}^{J}\widehat{A}_{inj}x_n(t)\right)\right]^T P_{li}x_j(t)\right. \\
&\qquad\qquad \left. + x_j^T(t)P_{lj}\left[\sum_{i=1}^{r_j}h_{ij}(t)\left(A_{ij}x_j(t) + \sum_{\substack{n=1\\ n\neq j}}^{J}\widehat{A}_{inj}x_n(t)\right)\right]\right\}
\end{aligned} \tag{A3}$$

$$\begin{aligned}
&= \sum_{j=1}^{J}\sum_{\rho=1}^{r_j}\dot{h}_{\rho j}(t)x_j^T(t)P_{\rho j}x_j(t) + \sum_{j=1}^{J}\sum_{l=1}^{r_j}\sum_{i=1}^{r_j}h_{lj}(t)h_{ij}(t)x_j^T(t)\left[A_{ij}^T P_{li} + P_{li}A_{ij}\right]x_j(t) \\
&\quad + \sum_{j=1}^{J}\sum_{l=1}^{r_j}\sum_{i=1}^{r_j}\sum_{\substack{n=1\\ n\neq j}}^{J}h_{lj}(t)h_{ij}(t)\left[x_n^T(t)\widehat{A}_{inj}^T P_j x_j(t) + x_j^T(t)P_j\widehat{A}_{inj}x_n(t)\right].
\end{aligned} \tag{A4}$$

Based on Lemma 2.1, we have

$$\begin{aligned}
&\sum_{j=1}^{J}\sum_{l=1}^{r_j}\sum_{i=1}^{r_j}\sum_{\substack{n=1\\ n\neq j}}^{J}h_{lj}(t)h_{ij}(t)\left\{x_n^T(t)\widehat{A}_{inj}^T P_j x_j(t) + x_j^T(t)P_j\widehat{A}_{inj}x_n(t)\right\} \\
&\quad \leq \sum_{j=1}^{J}\sum_{l=1}^{r_j}\sum_{i=1}^{r_j}\sum_{\substack{n=1\\ n\neq j}}^{J}h_{lj}(t)h_{ij}(t)\left\{\alpha\left[x_n^T(t)x_n(t)\right] + \alpha^{-1}\left[x_j^T(t)P_j\widehat{A}_{inj}\widehat{A}_{inj}^T P_j x_j(t)\right]\right\}
\end{aligned} \tag{A5}$$

$$\begin{aligned}
&= \sum_{j=1}^{J}\sum_{l=1}^{r_j}\sum_{i=1}^{r_j}\sum_{n=1}^{J}h_{lj}(t)h_{ij}(t)\left\{\alpha\left[\left(1-\frac{1}{J}\right)x_j^T(t)x_j(t)\right]\right. \\
&\qquad \left. + \alpha^{-1}\left[x_j^T(t)P_j\widehat{A}_{inj}\widehat{A}_{inj}^T P_j x_j(t) - \frac{1}{J}x_j^T(t)P_j\widehat{A}_{ijj}\widehat{A}_{ijj}^T P_j x_j(t)\right]\right\}.
\end{aligned} \tag{A6}$$

(Based on the concept of interconnection, the matrix $\widehat{A}_{ijj}$ is equal to zero.) From (A4) and (A6), we obtain

$$\dot{V}(t) \le \sum_{j=1}^{J} \sum_{\rho=1}^{r_j} \dot{h}_{\rho j}(t) x_j^T(t) P_{\rho j} x_j(t) + \sum_{j=1}^{J} \sum_{l=1}^{r_j} \sum_{i=1}^{r_j} h_{lj}(t) h_{ij}(t) x_j^T(t) \times \left[A_{ij}^T P_{li} + P_{li} A_{ij} + \alpha(J-1)I + \sum_{n=1}^{J} \alpha^{-1} P_j \widehat{A}_{inj} \widehat{A}_{inj}^T P_j \right] x_j(t). \tag{A7}$$

Therefore, $\dot{V}(t) < 0$ if (3.2) holds.However, condition (3.2) cannot be easily solved numerically because we need to consider the term of the time derivative $\dot{h}_{\rho j}(t)$. Eq. (3.2) is thus transformed into numerically feasible conditions described in Theorem 3.3 and upper bounds of the time derivative are used in place of the $h_\rho(t)$. □

Theorem 3.3. *The fuzzy system* (2.7) *is stable in the large if there exist common positive definite matrices* $P_{1j}, P_{2j}, \ldots, P_{rj}$ *such that inequality* $|\dot{h}_{\rho j}(t)| \le \phi_{\rho j}$ *is satisfied and*

$$\sum_{\rho=1}^{r_j} \phi_{\rho j} P_{\rho j} + A_{ij}^T P_{li} + P_{li} A_{ij} + \alpha(J-1)I + \sum_{n=1}^{J} \alpha^{-1} P_j \widehat{A}_{inj} \widehat{A}_{inj}^T P_j. \tag{3.3}$$

Proof.

$$\begin{aligned}
\dot{V}(t) \le{}& \sum_{j=1}^{J} \sum_{\rho=1}^{r_j} \dot{h}_{\rho j}(t) x_j^T(t) P_{\rho j} x_j(t) \\
&+ \sum_{j=1}^{J} \sum_{l=1}^{r_j} \sum_{i=1}^{r_j} h_{lj}(t) h_{ij}(t) x_j^T(t) \left[A_{ij}^T P_{li} + P_{li} A_{ij} + \alpha(J-1)I + \sum_{n=1}^{J} \alpha^{-1} P_j \widehat{A}_{inj} \widehat{A}_{inj}^T P_j \right] x_j(t) \\
\le{}& \sum_{j=1}^{J} \sum_{l=1}^{r_j} \sum_{i=1}^{r_j} h_{lj}(t) h_{ij}(t) \left\{ \sum_{\rho=1}^{r_j} x_j^T(t) \left[\phi_{\rho j} P_{\rho j} \right] x_j(t) \right\} \\
&+ \sum_{j=1}^{J} \sum_{l=1}^{r_j} \sum_{i=1}^{r_j} h_{lj}(t) h_{ij}(t) x_j^T(t) \left[A_{ij}^T P_{li} + P_{li} A_{ij} + \alpha(J-1)I + \sum_{n=1}^{J} \alpha^{-1} P_j \widehat{A}_{inj} \widehat{A}_{inj}^T P_j \right] x_j(t) \\
={}& \sum_{j=1}^{J} \sum_{l=1}^{r_j} \sum_{i=1}^{r_j} h_{lj}(t) h_{ij}(t) x_j^T(t) \left[\sum_{\rho=1}^{r_j} \phi_{\rho j} P_{\rho j} + A_{ij}^T P_{li} + P_{li} A_{ij} + \alpha(J-1)I \right. \\
&\left. + \sum_{n=1}^{J} \alpha^{-1} P_j \widehat{A}_{inj} \widehat{A}_{inj}^T P_j \right] x_j(t),
\end{aligned} \tag{A8}$$

$\dot{V}(t) < 0$ if (3.3) holds. □

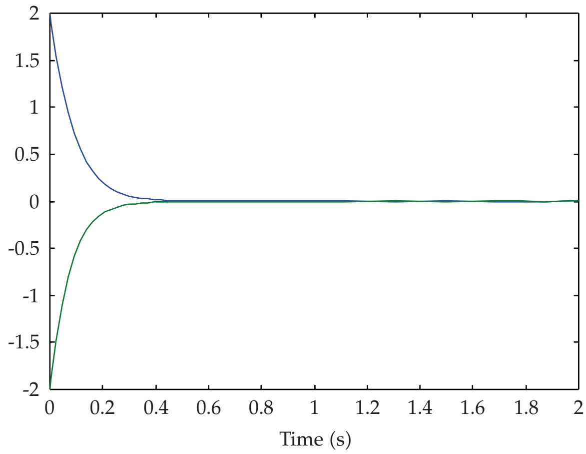

Figure 1: The state response of subsystem 1.

Remark 3.4. A special case of sufficient conditions without $\phi_{\rho j}$ is proposed to guarantee the asymptotically stability of fuzzy large-scale system **S**. If there exist symmetric positive definite matrices P_j which satisfy that each isolated subsystem is asymptotically stable as described in (3.4), the trajectories of the interconnected system are stable:

$$A_{ij}{}^{T}P_j + P_jA_{ij} + \sum_{n=1}^{J}\alpha^{-1}P_j\widehat{A}_{inj}\widehat{A}_{inj}^{T}P_j + \alpha(J-1)I < 0 \tag{3.4}$$

for $i = 1, 2, \ldots, r_j$; $j = 1, 2, \ldots, J$.

Remark 3.5. Equation (3.4) can be recast as an LMI problem based on Lemma 2.2. Therefore, new variables $W_j = P_j^{-1}$ and $\overline{\alpha} = \alpha^{-1}$ are introduced and (3.4) is rewritten as

$$\begin{bmatrix} W_jA_{ij}^{T} + A_{ij}W_j + \sum_{n=1}^{J}\overline{\alpha}\widehat{A}_{inj}\widehat{A}_{inj}^{T} & W_j \\ W_j & -\overline{\alpha}\left(\dfrac{1}{J-1}\right)I \end{bmatrix} < 0 \quad \text{for } i = 1, 2, \ldots, r_j. \tag{3.5}$$

4. A Numerical Example

Consider a interconnected fuzzy system **S** which consists of two fuzzy subsystems described by (2.2) with one rule.

Subsystem 1:

$\dot{x}_1(t) = h_{11}(t)\lfloor A_{11}x_1(t) + \widehat{A}_{121}x_2(t)\rfloor,$

Subsystem 2:

$\dot{x}_2(t) = h_{12}(t)\lfloor A_{12}x_2(t) + \widehat{A}_{112}x_1(t)\rfloor,$

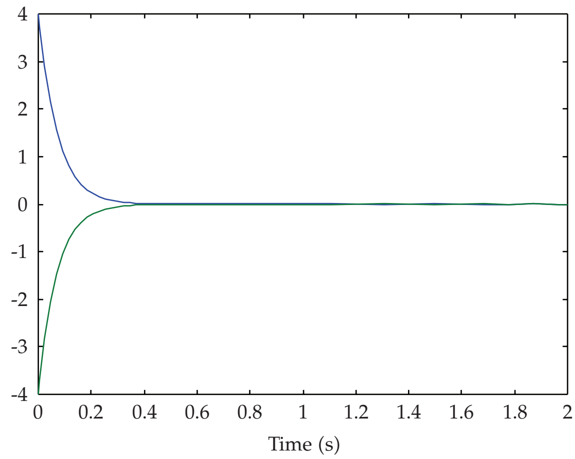

Figure 2: The state response of subsystem 2.

in which

$$x_1^T(t) = [x_{11}(t)x_{21}(t)], \quad x_2^T(t) = [x_{12}(t)x_{22}(t)], \quad A_{11} = \begin{bmatrix} -16.252 & -6.222 \\ -6.084 & -24.674 \end{bmatrix},$$
$$\widehat{A}_{121} = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.4 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -21.800 & -7.710 \\ -7.150 & -23.210 \end{bmatrix}, \quad \widehat{A}_{112} = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}. \tag{4.1}$$

At first, based on (3.5), we can get the common solutions W_j and $\overline{\alpha}$ via the Matlab LMI optimization toolbox

$$W_1 = \begin{bmatrix} 0.811 & -0.2411 \\ -0.2411 & 0.342 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 2.9482 & 1.1952 \\ 1.1952 & 3.1873 \end{bmatrix}, \qquad \overline{\alpha} = 1. \tag{4.2}$$

Then, the following positive definite matrices $P_j(= W_j^{-1})$ and α can be obtained such that (3.4) is satisfied

$$P_1 = \begin{bmatrix} 1.5601 & 1.1002 \\ 1.1002 & 3.7001 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 0.4002 & -0.1511 \\ -0.1512 & 0.3703 \end{bmatrix}, \qquad \alpha = 1. \tag{4.3}$$

Therefore, based on the theorem, the interconnected fuzzy system **S** described in (4.1) is guaranteed to be asymptotically stable. From the simulation of Figures 1 and 2 and given the initial conditions, $x_{11}(0) = 2$, $x_{21}(0) = -2$, $x_{12}(0) = 4$, $x_{22}(0) = -4$, we observe that the interconnected fuzzy system **S** is asymptotically stable, because the trajectories of two subsystems starting from non-zero initial states both approach close to the origin.

5. Conclusions

For the class of continuous interconnected fuzzy system **S**, LMI-based stability conditions have been derived based on the new fuzzy Lyapunov function. Sufficient stability conditions were derived based on the asymptotically stability of the existence of a common positive definite matrix Pi which is able to satisfy the Lyapunov equation or the LMI for each subsystem Si. Finally, a numerical example was given to illustrate the effectiveness and ease of implementation of this approach.

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