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Research Article

Exponential Admissibility and Dynamic Output Feedback Control of Switched Singular Systems with Interval Time-Varying Delay

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This paper is concerned with the problems of exponential admissibility and dynamic output feedback (DOF) control for a class of continuous-time switched singular systems with interval time-varying delay. A full-order, dynamic, synchronously switched DOF controller is considered. First, by using the average dwell time approach, a delay-range-dependent exponential admissibility criterion for the unforced switched singular time-delay system is established in terms of linear matrix inequalities (LMIs). Then, based on this criterion, a sufficient condition on the existence of a desired DOF controller, which guarantees that the closed-loop system is regular, impulse free and exponentially stable, is proposed by employing the LMI technique. Finally, some illustrative examples are given to show the effectiveness of the proposed approach.

1. Introduction

The past decades have witnessed an enormous interest in switched systems, due to their powerful ability in modeling of event-driven systems, logic-based systems, parameter- or structure-varying systems, and so forth; for details, see [1–4] and the references therein. Switched systems are a class of hybrid systems, which consist of a collection of continuous- or discrete-time subsystems and a switching rule specifying the switching between them. When focusing on the classification problems in switched systems, it is commonly recognized that there exist three basic problems [1]: (i) finding conditions for stability under arbitrary switching; (ii) identifing the limited but useful class of stabilizing switching signals, and (iii) construct a stabilizing switching signal. Many effective methods have been presented to tackle these three basic problems such as the multiple Lyapunov function approach [5], the piecewise Lyapunov function approach [6], the switched Lyapunov function approach [7], the convex combination technique [8], and the dwell time or average dwell time scheme [9–12]. On the other hand, time-delay is very common in engineering systems and is frequently a source of instability and poor performance [13]. Therefore, control of switched time-delay systems has received more and more attention in the past few years; see [14–23] and the references therein.

As far as we know, singular systems (known also as descriptor, implicit or differential algebraic systems) also provide a natural framework for modeling of dynamic systems and describe a larger class of systems than the regular system models [24]. Switched singular systems have strong engineering background such as electrical networks [25], economic systems [26]. Recently, many results have been obtained in the literature for switched singular systems, such as stability and stabilization [27–30], reachability [31], H_∞ control and filtering problems [32]. For switched singular time-delay (SSTD) systems, due to the coupling between the switching and the time-delay and because of the algebraic constraints in singular model, the behavior of such systems is much more complicated than that of regular switched time-delay systems or switched singular systems, and thus, to date, only a few results have been reported in the literature. In [33], the robust stability and H_∞ control problems for discrete-time uncertain SSTD systems under arbitrary switching were discussed by using switched Lyapunov functions. In [34], a switching signal was constructed to guarantee the asymptotic stability of continuous-time SSTD systems. However, the aforementioned results are focused on the basic problem (i), see [33], and problem (iii), see [34], for SSTD systems. Problem (ii) is to identify stabilizing switching signals on the premise that all the individual subsystems are stable. Basically, we will find that stability is ensured if the switching is sufficiently slow [1], and it is well known that dwell time and average dwell time are two effective tools to define slow switching signals. In [9], it was shown that if all the individual subsystems are exponentially stable and that the dwell time of the switching signal is not smaller than a certain lower bound, then the switched systems is exponentially stable. This result was extended to both continuous-time switched linear time-delay systems [16] and discrete-time cases [17]. Unfortunately, so far, to the best of the authors' knowledge, the problem of solving the basic problem (ii) for SSTD systems via the dwell time or average dwell time scheme remains open and unsolved. On the other hand, the results in [33] are derived based on the state feedback controller. In fact, in many practical systems, state variables are not always available. In this case, the design of a controller that does not require the complete access to the state vector is preferable. An important example of such controller is the dynamic output feedback (DOF) controller. However, little attention has been paid to the DOF controller design problem for SSTD systems. This forms the motivation of this paper.

In this paper, we are concerned with the problems of exponential admissibility and DOF control for a class of continuous-time switched singular systems with interval time-varying delay. A full-order, dynamic, synchronously switched DOF controller is designed. First, by using the average dwell time approach and the piecewise Lyapunov function technique, a delay-range-dependent exponential admissibility criterion is derived in terms of LMIs, which guarantees the regularity, nonimpulsiveness, and exponential stability of the unforced system. A estimation of the convergence of the system is also explicitly given. Then, the corresponding solvability condition for the desired DOF controller is established by employing the LMI technique. Finally, some illustrative examples are given to show the effectiveness of the proposed approach.

Notation. Throughout this paper, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space and $\mathbf{R}^{n\times n}$ is the set of all $n \times n$ real matrices. $P > 0$ ($P \geq 0$) means that matrix P is positive definite (semipositive definite). $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) denotes the minimum (maximum) eigenvalue of symmetric matrix P. $\mathcal{C}_{n,d} := \mathcal{C}([-d,0],\mathbf{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d,0]$ to $\mathbf{R}^n$. Let $x_t \in \mathcal{C}_{n,d}$ be defined by $x_t := x(t+\theta)$, $\theta \in [-d,0]$. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix and $\|x_t\|_d = \sup_{-d\le\theta\le 0}\|x(t+\theta)\|$. The superscript "$T$" represents matrix transposition, the symmetric terms in a matrix are denoted by "$*$". diag$\{\cdots\}$ stands for a block-diagonal matrix and Sym$\{A\}$ is the shorthand notation for $A + A^T$.

2. Preliminaries and Problem Formulation

Consider a class of SSTD systems of the form

$$\begin{aligned} E\dot{x}(t) &= A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + B_{\sigma(t)}u(t), \\ y(t) &= C_{\sigma(t)}x(t) + C_{d\sigma(t)}x(t-d(t)), \\ x(t) &= \phi(t), \quad t \in [-d_1 - d_2, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in \mathbf{R}^n$ is the system state, $u(t) \in \mathbf{R}^m$ is the control input, and $y(t) \in \mathbf{R}^p$ is the measured output; $\sigma(t) : [0,+\infty) \to \mathcal{I} = \{1,2,\ldots,N\}$ with integer $N > 1$ is the switching signal; $E \in \mathbf{R}^{n\times n}$ is a singular matrix with rank $E = r \le n$; for each possible value $\sigma(t) = i, i \in \mathcal{I}$, A_i, A_{di}, B_i, C_i and C_{di} are constant real matrices with appropriate dimensions; $\phi(t) \in \mathcal{C}_{n,d_1+d_2}$ is a compatible vector valued initial function; $d(t)$ is an interval time-varying delay satisfying

$$d_1 \le d(t) \le d_1 + d_2, \quad \dot{d}(t) \le \mu, \tag{2.2}$$

where $d_1 \ge 0$, $d_2 > 0$ and $0 \le \mu < 1$ are constants.

Remark 2.1. Model (2.1) can describe many practical time-delay systems (e.g., chemical engineering systems, lossless transmission lines, partial element equivalent circuit, etc.) with time-varying parameters or structures, which may be caused by random failures and repairs of the components, sudden environment changes, and varying of the operating point of a system [13, 35]. In real application, the importance of the study of controller design problem for model (2.1) also arises from the extensive applications in networked control [36].

Since rank $E = r \le n$, there exist nonsingular transformation matrices $P, Q \in \mathbf{R}^{n\times n}$ such that $PEQ = \mathrm{diag}\{I_r, 0\}$. In this paper, without loss of generality, let

$$E = \mathrm{diag}\{I_r, 0\}. \tag{2.3}$$

Corresponding to the switching signal $\sigma(t)$, we denote the switching sequence by $\mathcal{S} := \{(i_0,t_0),\ldots,(i_k,t_k) \mid i_k \in \mathcal{I},\ k = 0,1,\ldots\}$ with $t_0 = 0$, which means that the i_k subsystem is activated when $t \in [t_k, t_{k+1})$. To present the objective of this paper more precisely, the following definitions are introduced.

Definition 2.2 (see [16, 37]). For the switching signal $\sigma(t)$ and any delay $d(t)$ satisfying (2.2), the unforced part of system (2.1)

$$\begin{aligned} E\dot{x}(t) &= A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)), \\ x_{t_0}(\theta) &= x(t_0+\theta), \quad \theta \in [-d_1-d_2, 0] \end{aligned} \tag{2.4}$$

is said to be

(1) regular if $\det(sE - A_i)$ is not identically zero for each $\sigma(t) = i$, $i \in \mathcal{I}$,

(2) impulse free if $\deg(\det(sE - A_i)) = \operatorname{rank} E$ for each $\sigma(t) = i$, $i \in \mathcal{I}$,

(3) exponentially stable under the switching signal $\sigma(t)$ if the solution $x(t)$ of system (2.4) satisfies $\|x(t)\| \leq \iota e^{-\lambda(t-t_0)}\|x_{t_0}\|_{d_1+d_2}$, for all $t \geq t_0$, where $\lambda > 0$ and $\iota > 0$ are called the decay rate and decay coefficient, respectively;

(4) exponentially admissible if it is regular, impulse free and exponentially stable under the switching signal $\sigma(t)$.

Definition 2.3 (see [9]). For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the number of switching of $\sigma(t)$ over (T_1, T_2). If $N_\sigma(T_1, T_2) \leq N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \geq 0$, then T_a is called average dwell time. As commonly used in the literature, we choose $N_0 = 0$.

This paper considers the full-order DOF controller of the following form:

$$\begin{aligned} E_c\dot{x}_c(t) &= A_{c\sigma(t)}x_c(t) + B_{c\sigma(t)}y(t), \\ u(t) &= C_{c\sigma(t)}x_c(t) + D_{c\sigma(t)}y(t), \end{aligned} \tag{2.5}$$

where $x_c(t) \in \mathbf{R}^n$ is the controller state vector, and E_c, A_{ci}, B_{ci}, C_{ci} and D_{ci}, $\sigma(t) = i$, $i \in \mathcal{I}$, are appropriately dimensioned constant matrices to be determined.

Then, the problem to be addressed in this paper can be formulated as follows: given the SSTD system (2.1), identify a class of switching signal $\sigma(t)$ and design a DOF controller of the form (2.5) such that the resultant closed-loop system is exponentially admissible under the switching signal $\sigma(t)$.

Before ending this section, we introduce the following lemma, which is essential for the development of our main results.

Lemma 2.4. *For any constant matrix $Z \in R^{n\times n}$, $Z = Z^T > 0$, positive scalar α, and vector function $\dot{x} : [-\tau, \infty) \to R^n$ such that the following integration is well defined, then*

$$\frac{e^{\alpha\tau}-1}{\alpha}\int_{t-d(t)}^{t} e^{\alpha(s-t)}\dot{x}^T(s)E^TZE\dot{x}(s)ds \geq \left(\int_{t-d(t)}^{t} E\dot{x}(s)ds\right)^T Z\left(\int_{t-d(t)}^{t} E\dot{x}(s)ds\right), \quad t \geq 0, \tag{2.6}$$

where $0 \leq d(t) \leq \tau$.

Proof. The proof is almost the same as Lemma 1 in [20]. Using Schur complement, we have

$$\begin{bmatrix} e^{\alpha(s-t)}\dot{x}^T(s)E^TZE\dot{x}(s) & \dot{x}^T(s)E^T \\ * & e^{\alpha(t-s)}Z^{-1} \end{bmatrix} \geq 0. \tag{2.7}$$

Integrating it from $t-d(t)$ to t, we get

$$\begin{bmatrix} \int_{t-d(t)}^{t} e^{\alpha(s-t)}\dot{x}^T(s)E^TZE\dot{x}(s)ds & \int_{t-d(t)}^{t} \dot{x}^T(s)E^Tds \\ * & \dfrac{(e^{\alpha\tau}-1)Z^{-1}}{\alpha} \end{bmatrix} \geq 0. \tag{2.8}$$

Using Schur complement again, we find that Lemma 2.4 holds. □

3. Main Results

In this section, we first apply the average dwell time approach to investigate the exponential admissibility for SSTD system (2.4), and give the following result.

Theorem 3.1. *For prescribed scalars $\alpha > 0$, $d_1 \geq 0$, $d_2 > 0$ and $0 \leq \mu < 1$, if for each $i \in \mathcal{I}$, there exist matrices $Q_{il} > 0$, $Z_{il} > 0$, $l = 1,2$, and P_i of the following form*

$$P_i = \begin{bmatrix} P_{i11} & 0 \\ P_{i21} & P_{i22} \end{bmatrix} \tag{3.1}$$

with $P_{i11} \in \mathbf{R}^r$, $P_{i11} > 0$, and P_{i22} being invertible, such that

$$\Phi_i = \begin{bmatrix} \Phi_{i11} & P_i^TA_{di} + A_i^TU_iA_{di} & c_1E^TZ_{i1}E \\ * & \Phi_{i22} & c_2E^TZ_{i2}E \\ * & * & \Phi_{i33} \end{bmatrix} < 0, \tag{3.2}$$

where $\Phi_{i11} = \mathrm{Sym}\{P_i^TA_i\} + \sum_{l=1}^{2} Q_{il} + \alpha E^TP_i - c_1E^TZ_{i1}E + A_i^TU_iA_i$, $\Phi_{i22} = -(1-\mu)e^{-\alpha(d_1+d_2)}Q_{i2} - c_2E^TZ_{i2}E + A_{di}^TU_iA_{di}$, $\Phi_{i33} = -e^{-\alpha d_1}Q_{i1} - c_1E^TZ_{i1}E - c_2E^TZ_{i2}E$, $c_1 = (\alpha d_1)/(e^{\alpha d_1}-1)$, $c_2 = (\alpha d_2)/(e^{\alpha d_2}-1)$ and $U_i = d_1^2Z_{i1} + d_2^2e^{\alpha d_1}Z_{i2}$. Then, system (2.4) with $d(t)$ satisfying (2.2) is exponentially admissible for any switching sequence $\mathcal{S}$ with average dwell time $T_a \geq T_a^* = (\ln\beta)/\alpha$, where $\beta \geq 1$ satisfies

$$P_{i11} \leq \beta P_{j11}, \quad Q_{il} \leq \beta Q_{jl}, \quad Z_{il} \leq \beta Z_{jl}, \quad l = 1,2, \quad \forall i,j \in \mathcal{I}. \tag{3.3}$$

Moreover, an estimate on the exponential decay rate is $\lambda = (1/2)(\alpha - (\ln\beta)/T_a)$.

Proof. The proof is divided into three parts: (i) to show the regularity and nonimpulsiveness; (ii) to show the exponential stability of the differential subsystem; and (iii) to show the exponential stability of the algebraic subsystem.

Part (i) regularity and nonimpulsiveness. According to (2.3), for each $i \in \mathcal{I}$, denote

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \qquad Z_{i1} = \begin{bmatrix} Z_{i111} & Z_{i112} \\ Z_{i112}^T & Z_{i122} \end{bmatrix}, \tag{3.4}$$

where $A_{i11} \in \mathbf{R}^r$ and $Z_{i111} \in \mathbf{R}^r$. From (3.2), it is easy to see that $\Phi_{i11} < 0$, $i \in \mathcal{I}$. Noting $Q_{il} > 0$ and $Z_{il} > 0$, $l = 1,2$, we get $\mathrm{Sym}\{P_i^T A_i\} + \alpha E^T P_i - c_1 E^T Z_{i1} E < 0$. Substituting P_i, A_i, Z_{i1} and E given as (3.1), (3.4) and (2.3) into this inequality and using Schur complement, we have $\mathrm{Sym}\{A_{i22}^T P_{i22}\} < 0$, which implies that A_{i22}, $i \in \mathcal{I}$, is nonsingular. Then by [24] and Definition 2.2, system (2.4) is regular and impulse free.

Part (ii) exponential stability of the differential subsystem. Define the piecewise Lyapunov functional candidate for system (2.4) as follows

$$\begin{aligned} V(x_t) &= V_{\sigma(t)}(x_t) \\ &= x^T(t)E^T P_{\sigma(t)} x(t) + \int_{t-d_1}^{t} e^{\alpha(s-t)} x^T(s) Q_{\sigma(t)1} x(s) ds \\ &\quad + \int_{t-d(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_{\sigma(t)2} x(s) ds \\ &\quad + d_1 \int_{-d_1}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} (E\dot{x}(s))^T Z_{\sigma(t)1} (E\dot{x}(s)) ds\, d\theta \\ &\quad + d_2 \int_{-d_1-d_2}^{-d_1} \int_{t+\theta}^{t} e^{\alpha(s-t+d_1)} (E\dot{x}(s))^T Z_{\sigma(t)2} (E\dot{x}(s)) ds\, d\theta. \end{aligned} \tag{3.5}$$

As mentioned earlier, the i_kth subsystem is activated when $t \in [t_k, t_{k+1})$. Then, along the solution of system (2.4) under the switching sequence $\mathcal{S}$, for $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} \dot{V}_{i_k}(x_t) + \alpha V_{i_k}(x_t) \le{}& 2x^T(t) P_{i_k}^T E\dot{x}(t) + x^T(t) Q_{i_k 1} x(t) - e^{-\alpha d_1} x^T(t-d_1) Q_{i_k 1} x(t-d_1) \\ &+ x^T(t) Q_{i_k 2} x(t) - (1-\mu) e^{-\alpha(d_1+d_2)} x^T(t-d(t)) Q_{i_k 2} x(t-d(t)) \\ &+ (E\dot{x}(t))^T \left(d_1^2 Z_{i_k 1} + d_2^2 e^{\alpha d_1} Z_{i_k 2} \right) (E\dot{x}(t)) + \alpha x^T(t) E^T P_{i_k} x(t) \\ &- d_1 \int_{t-d_1}^{t} e^{\alpha(s-t)} (E\dot{x}(s))^T Z_{i_k 1} (E\dot{x}(s)) ds \\ &- d_2 \int_{t-d(t)}^{t-d_1} e^{\alpha(s-t+d_1)} (E\dot{x}(s))^T Z_{i_k 2} (E\dot{x}(s)) ds. \end{aligned} \tag{3.6}$$

By replacing $E\dot{x}(t)$ with $A_{i_k}x(t)+A_{di_k}x(t-d(t))$ and using Lemma 2.4 and Schur complement, LMI (3.2) yields

$$\dot{V}_{i_k}(x_t) + \alpha V_{i_k}(x_t) < 0. \tag{3.7}$$

Integrating (3.7) from t_k to t_{k+1} gives

$$V_{\sigma(t)}(x_t) = V_{i_k}(x_t) \le e^{-\alpha(t-t_k)}V_{i_k}(x_{t_k}), \quad t \in [t_k, t_{k+1}). \tag{3.8}$$

Let $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, where $x_1(t) \in \mathbf{R}^r$ and $x_2(t) \in \mathbf{R}^{n-r}$. From (2.3) and (3.1), it can be seen that for each i, $i \in \mathcal{I}$, $x^T(t)E^TP_ix(t) = x_1^T(t)P_{i11}x_1(t)$. Noting this, and using (3.3) and (3.5), at switching instant t_k, we have

$$V_{i_k}(x_{t_k}) \le \beta V_{\sigma(t_k^-)}\left(x_{t_k^-}\right) = \beta V_{i_{k-1}}\left(x_{t_k^-}\right), \quad k = 1, 2, \ldots, \tag{3.9}$$

where t_k^- denotes the left limitation of t_k. Therefore, it follows from (3.8), (3.9) and the relation $k = N_\sigma(t_0, t) \le (t - t_0)/T_a$ that

$$\begin{aligned} V_{\sigma(t)}(x_t) &\le e^{-\alpha(t-t_k)}\beta V_{\sigma(t_k^-)}\left(x_{t_k^-}\right) \le \cdots \le e^{-\alpha(t-t_0)}\beta^k V_{\sigma(t_0)}(t_0) \\ &\le e^{-(\alpha-(\ln\beta)/T_a)(t-t_0)}V_{\sigma(t_0)}(x_{t_0}). \end{aligned} \tag{3.10}$$

According to (3.5) and (3.10), we obtain

$$\lambda_1\|x_1(t)\|^2 \le V_{\sigma(t)}(x_t), \qquad V_{\sigma(t_0)}(x_{t_0}) \le \lambda_2\|x_{t_0}\|^2_{d_1+d_2} \tag{3.11}$$

where $\lambda_1 = \min_{\forall i\in\mathcal{I}}\lambda_{\min}(P_{i11})$, and $\lambda_2 = \max_{\forall i\in\mathcal{I}}\lambda_{\max}(P_{i11}) + (1/\alpha)(1-e^{-\alpha d_1})\max_{\forall i\in\mathcal{I}}\lambda_{\max}(Q_{i1}) + (1/\alpha)(1 - e^{-\alpha(d_1+d_2)})\max_{\forall i\in\mathcal{I}}\lambda_{\max}(Q_{i2}) + (d_1/\alpha^2)(\alpha d_1 - 1 + e^{-\alpha d_1})\max_{\forall i\in\mathcal{I}}(2\lambda_{\max}(Z_{i1})(\|A_i\| + \|A_{di}\|)) + (1/\alpha^2)(-d_2 + \alpha d_2^2 e^{\alpha d_1} + d_2 e^{-\alpha d_2})\max_{\forall i\in\mathcal{I}}(2\lambda_{\max}(Z_{i2})(\|A_i\| + \|A_{di}\|))$. Then, combining (3.10) with (3.11) yields

$$\|x_1(t)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}}e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}. \tag{3.12}$$

Part (iii) exponential stability of the algebraic subsystem. Since A_{i22}, $i \in \mathcal{I}$, is nonsingular, set $G_i = \begin{bmatrix} I_r & -A_{i12}A_{i22}^{-1} \\ 0 & A_{i22}^{-1} \end{bmatrix}$ and $H = \begin{bmatrix} I_r & 0 \\ 0 & I_{n-r} \end{bmatrix}$. Then, it is easy to get

$$\widehat{E} := G_iEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad \widehat{A}_i := G_iA_iH = \begin{bmatrix} \widehat{A}_{i11} & 0 \\ \widehat{A}_{i21} & I_{n-r} \end{bmatrix}, \qquad \widehat{P}_i := G_i^{-T}P_iH = \begin{bmatrix} \widehat{P}_{i11} & 0 \\ \widehat{P}_{i21} & \widehat{P}_{i22} \end{bmatrix}, \tag{3.13}$$

where $\hat{A}_{i11} = A_{i11} - A_{i12}A_{i22}^{-1}A_{i21}$, $\hat{A}_{i21} = A_{i22}^{-1}A_{i21}$, $\hat{P}_{i11} = P_{i11}$, $\hat{P}_{i21} = A_{i12}^{T}P_{i11} + A_{i22}^{T}P_{i21}$, and $\hat{P}_{i22} = A_{i22}^{T}P_{i22}$. According to (3.13), denote

$$\hat{A}_{di} := G_i A_{di} H = \begin{bmatrix} \hat{A}_{di11} & \hat{A}_{di12} \\ \hat{A}_{di21} & \hat{A}_{di22} \end{bmatrix}, \qquad \hat{Q}_{il} := H^T Q_{il} H = \begin{bmatrix} \hat{Q}_{il11} & \hat{Q}_{il12} \\ \hat{Q}_{il21} & \hat{Q}_{il22} \end{bmatrix},$$
$$\hat{Z}_{il} := G_i^{-T} Z_{il} G_i^{-1} = \begin{bmatrix} \hat{Z}_{il11} & \hat{Z}_{il12} \\ \hat{Z}_{il21} & \hat{Z}_{il22} \end{bmatrix}, \quad l = 1,2. \tag{3.14}$$

Let

$$\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} := H^{-1}x(t) = x(t), \tag{3.15}$$

where $\xi_1(t) \in \mathbf{R}^r$ and $\xi_2(t) \in \mathbf{R}^{n-r}$. Then, for any fixed $\sigma(t) = i$, $i \in \mathcal{I}$, system (2.4) is restricted system equivalent (r.s.e.) to

$$\dot{\xi}_1(t) = \hat{A}_{i11}\xi_1(t) + \hat{A}_{di11}\xi_1(t - d(t)) + \hat{A}_{di12}\xi_2(t - d(t)), \tag{3.16}$$

$$-\xi_2(t) = \hat{A}_{i21}\xi_1(t) + \hat{A}_{di21}\xi_1(t - d(t)) + \hat{A}_{di22}\xi_2(t - d(t)). \tag{3.17}$$

By (3.2) and Schur complement, we have $\left[\begin{smallmatrix} \Phi'_{i11} & P_i^T A_{di} \\ * & \Phi'_{i22} \end{smallmatrix}\right] < 0$, where $\Phi'_{i11} = \mathrm{Sym}\{P_i^T A_i\} + \sum_{l=1}^{2} Q_{il} + \alpha E^T P_i - c_1 E^T Z_{i1} E$ and $\Phi'_{i22} = -(1-\mu)e^{-\alpha(d_1+d_2)}Q_{i2} - c_2 E^T Z_{i2} E$. Pre- and postmultiplying this inequality by $\mathrm{diag}\{H^T, H^T\}$ and $\mathrm{diag}\{H, H\}$, respectively, noting the expressions in (3.13) and (3.14), and using Schur complement, we have

$$\begin{bmatrix} \mathrm{Sym}\left\{\hat{P}_{i22}^{T}\right\} + \sum_{l=1}^{2}\hat{Q}_{il22} & \hat{P}_{i22}^{T}\hat{A}_{di22} \\ * & -(1-\mu)e^{-\alpha(d_1+d_2)}\hat{Q}_{i222} \end{bmatrix} < 0. \tag{3.18}$$

Pre- and postmultiplying this inequality by $[-\hat{A}_{di22}^{T} I]$ and its transpose, respectively, and noting $\hat{Q}_{i122} > 0$ and $0 \le \mu < 1$, we obtain $\hat{A}_{di22}^{T}\hat{Q}_{i222}\hat{A}_{di22} - e^{-\alpha(d_1+d_2)}\hat{Q}_{i222} < 0$. Then, according to Lemma 7 in [38], we can deduce that there exist constants $\hbar_i > 1$ and $\eta_i > 0$ such that

$$\left\| \left(e^{(1/2)\alpha(d_1+d_2)}\hat{A}_{di22} \right)^l \right\| \le \hbar_i e^{-\eta_i l}, \quad l = 0, 1, \ldots, \ \forall i \in \mathcal{I}. \tag{3.19}$$

Define

$$t^0 = t, \quad t^j = t^{j-1} - d\left(t^{j-1}\right), \quad j = 1, 2, \ldots, \tag{3.20}$$

$$\left\|\hat{A}_{21}\right\| = \max_{\forall i \in \mathcal{I}}\left\|\hat{A}_{i21}\right\|, \quad \left\|\hat{A}_{d21}\right\| = \max_{\forall i \in \mathcal{I}}\left\|\hat{A}_{di21}\right\|, \quad \left\|\hat{A}_{d22}\right\| = \max_{\forall i \in \mathcal{I}}\left\|\hat{A}_{di22}\right\|, \quad \forall i \in \mathcal{I}. \tag{3.21}$$

As mentioned earlier, under the switching sequence $\mathcal{S}$, for $t \in [t_k, t_{k+1})$, the i_kth subsystem is activated. Then, from (3.17) and (3.20), we have

$$\xi_2(t) - \widehat{A}_{i_k 21}\xi_1\left(t^0\right) - \widehat{A}_{di_k 21}\xi_1\left(t^1\right) - \widehat{A}_{di_k 22}\xi_2\left(t^1\right). \tag{3.22}$$

Similarly, it can be obtained that $\xi_2(t^1) = -\widehat{A}_{i_k 21}\xi_1(t^1) - \widehat{A}_{di_k 21}\xi_1(t^2) - \widehat{A}_{di_k 22}\xi_2(t^2)$. Substituting this into (3.22), we get $\xi_2(t) = (-\widehat{A}_{di_k 22})^2\xi_2(t^2) - \sum_{j=0}^{1}(-\widehat{A}_{di_k 22})^j(\widehat{A}_{i_k 21}\xi_1(t^j) + \widehat{A}_{di_k 21}\xi_1(t^{j+1}))$. Continuing in the same manner and noting that $t^j < t^{j-1}$, then there exists a finite positive integer T_{i_k} such that

$$\xi_2(t) = \left(-\widehat{A}_{di_k 22}\right)^{T_{i_k}}\xi_2\left(t^{T_{i_k}}\right) - \sum_{j_{i_k}=0}^{T_{i_k}-1}\left(-\widehat{A}_{di_k 22}\right)^{j_{i_k}}\left(\widehat{A}_{i_k 21}\xi_1\left(t^{j_{i_k}}\right) + \widehat{A}_{di_k 21}\xi_1\left(t^{j_{i_k}+1}\right)\right), \tag{3.23}$$

where $t^{T_{i_k}} \in (t_{k-1}, t_k]$ and $t^{T_{i_k}} \to t_k$. When $t \in [t_{k-1}, t_k)$, the i_{k-1}th subsystem is activated. Then, following a similar procedure as the above, there exists a finite positive integer $T_{i_{k-1}}$ such that

$$\begin{aligned}\xi_2\left(t^{T_{i_k}}\right) &= \left(-\widehat{A}_{di_{k-1}22}\right)^{T_{i_{k-1}}}\xi_2\left(t^{T_{i_k}+T_{i_{k-1}}}\right)\\&\quad - \sum_{j_{i_{k-1}}=T_{i_k}}^{T_{i_k}+T_{i_{k-1}}-1}\left(-\widehat{A}_{di_{k-1}22}\right)^{j_{i_{k-1}}-T_{i_k}}\left(\widehat{A}_{i_{k-1}21}\xi_1\left(t^{j_{i_{k-1}}}\right) + \widehat{A}_{di_{k-1}21}\xi_1\left(t^{j_{i_{k-1}}+1}\right)\right),\end{aligned} \tag{3.24}$$

where $t^{T_{i_k}+T_{i_{k-1}}} \in (t_{k-2}, t_{k-1}]$ and $t^{T_{i_k}+T_{i_{k-1}}} \to t_{k-1}$. After k-times iterative manipulations, t belongs to $[t_0, t_1)$, and there exists a finite positive integer T_{i_0} such that

$$\begin{aligned}\xi_2\left(t^{T_{i_k}+\cdots+T_{i_1}}\right) &= \left(-\widehat{A}_{di_0 22}\right)^{T_{i_0}}\xi_2\left(t^{T_{i_k}+\cdots+T_{i_0}}\right)\\&\quad - \sum_{j_0=T_{i_k}+\cdots+T_{i_1}}^{T_{i_k}+\cdots+T_{i_0}-1}\left(-\widehat{A}_{di_0 22}\right)^{j_{i_0}-T_{i_k}-\cdots-T_{i_1}}\left(\widehat{A}_{i_0 21}\xi_1\left(t^{j_{i_0}}\right) + \widehat{A}_{di_0 21}\xi_1\left(t^{j_{i_0}+1}\right)\right),\end{aligned} \tag{3.25}$$

where $t^{T_{i_k}+\cdots+T_{i_0}} \in (-d_1 - d_2, t_0]$ and $t^{T_{i_k}+\cdots+T_{i_0}} \to t_0$. By a simple induction, we have

$$\begin{aligned}\xi_2(t) &= \left[\prod_{j=0}^{k}\left(-\widehat{A}_{di_j 22}\right)^{T_{i_j}}\right]\xi_2\left(t^{T_{i_k}+\cdots+T_{i_1}+T_{i_0}}\right) - \sum_{j_{i_k}=0}^{T_{i_k}-1}\left(-\widehat{A}_{di_k 22}\right)^{j_{i_k}}\left(\widehat{A}_{i_k 21}\xi_1\left(t^{j_{i_k}}\right) + \widehat{A}_{di_k 21}\xi_1\left(t^{j_{i_k}+1}\right)\right)\\&\quad - \sum_{p=1}^{k}\left\{\left[\prod_{q=p}^{k}\left(-\widehat{A}_{di_q 22}\right)^{T_{i_q}}\right]\sum_{j_{i_{p-1}}=T_{i_k}+\cdots+T_{i_p}}^{T_{i_k}+\cdots+T_{i_{p-1}}-1}\left(\varphi_1(t) + \varphi_2(t)\right)\right\},\end{aligned} \tag{3.26}$$

where

$$\varphi_1(t) = \left(-\widehat{A}_{di_{p-1}22}\right)^{j_{i_{p-1}}-T_{i_k}-\cdots-T_{i_p}} \widehat{A}_{i_{p-1}21}\xi_1\left(t^{j_{i_{p-1}}}\right),$$
$$\varphi_2(t) = \left(-\widehat{A}_{di_{p-1}22}\right)^{j_{i_{p-1}}-T_{i_k}-\cdots-T_{i_p}} \widehat{A}_{di_{p-1}21}\xi_1\left(t^{j_{i_{p-1}}+1}\right) \tag{3.27}$$

Therefore, from (3.15), (3.21), and (3.26), and noting $t^{T_{i_k}+\cdots+T_{i_0}} \in (-d_1 - d_2, t_0]$, we obtain

$$\|\xi_2(t)\| \le \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5, \tag{3.28}$$

where

$$\Delta_1 = \left[\prod_{j=0}^{k}\left\|\left(\widehat{A}_{di_j22}\right)^{T_{i_j}}\right\|\right]\|x_{t_0}\|_{d_1+d_2},$$
$$\Delta_2 = \widehat{A}_{21}\sum_{j_{i_k}=0}^{T_{i_k}-1}\left\|\left(\widehat{A}_{di_k22}\right)^{j_{i_k}}\right\|\left\|\xi_1\left(t^{j_{i_k}}\right)\right\|,$$
$$\Delta_3 = \widehat{A}_{d21}\sum_{j_{i_k}=0}^{T_{i_k}-1}\left\|\left(\widehat{A}_{di_k22}\right)^{j_{i_k}}\right\|\left\|\xi_1\left(t^{j_{i_k}+1}\right)\right\|, \tag{3.29}$$
$$\Delta_4 = \widehat{A}_{21}\sum_{p=1}^{k}\left\{\left[\prod_{q=p}^{k}\left\|\left(\widehat{A}_{di_q22}\right)^{T_{i_q}}\right\|\right]\sum_{j_{i_{p-1}}=T_{i_k}+\cdots+T_{i_p}}^{T_{i_k}+\cdots+T_{i_{p-1}}-1}\|\varphi_1(t)\|\right\},$$
$$\Delta_5 = \widehat{A}_{d21}\sum_{p=1}^{k}\left\{\left[\prod_{q=p}^{k}\left\|\left(\widehat{A}_{di_q22}\right)^{T_{i_q}}\right\|\right]\sum_{j_{i_{p-1}}=T_{i_k}+\cdots+T_{i_p}}^{T_{i_k}+\cdots+T_{i_{p-1}}-1}\|\varphi_2(t)\|\right\}.$$

Note

$$t_0 \ge t^{T_{i_k}+\cdots+T_{i_0}} = t - \sum_{j=0}^{T_{i_k}+\cdots+T_{i_0}-1} d\left(t^j\right) \ge t - (T_{i_k} + \cdots + T_{i_0})(d_1 + d_2). \tag{3.30}$$

Using (3.19) and the relation $T_a \geq T_a^* = (\ln\beta)/\alpha$, the first term in (3.28) can be estimated as

$$\begin{aligned}
\Delta_1 &= \left[\prod_{j=0}^{k}\left\|\left(e^{(1/2)\alpha(d_1+d_2)}\widehat{A}_{di_j22}\right)^{T_{i_j}}\right\| e^{-(1/2)\alpha(T_{i_k}+\cdots+T_{i_0})(d_1+d_2)}\right]\|x_{t_0}\|_{d_1+d_2} \\
&\leq \left[\prod_{j=0}^{k}\hbar_{i_j}e^{-\eta_{i_j}T_{i_j}}\right]e^{-(1/2)\alpha(t-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&\leq \left[\prod_{j=0}^{k}\hbar_{i_j}e^{-\eta_{i_j}T_{i_j}}\right]e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&:= \chi_1 e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}.
\end{aligned} \tag{3.31}$$

By (3.15), (3.12), (3.21), (3.20) and (3.19), we get

$$\begin{aligned}
\left\|\left(\widehat{A}_{di_k22}\right)^{j_{i_k}}\right\|\left\|\xi_1\left(t^{j_{i_k}}\right)\right\| &\leq \left\|\left(\widehat{A}_{di_k22}\right)^{j_{i_k}}\right\|\sqrt{\frac{\lambda_2}{\lambda_1}}e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t^{j_{i_k}}-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&\leq \sqrt{\frac{\lambda_2}{\lambda_1}}\left\|e^{(1/2)\alpha(d_1+d_2)}\left(\widehat{A}_{di_k22}\right)^{j_{i_k}}\right\|e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t^{j_{i_k}-1}-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&\leq \cdots \leq \sqrt{\frac{\lambda_2}{\lambda_1}}\left\|\left(e^{(1/2)\alpha(d_1+d_2)}\widehat{A}_{di_k22}\right)^{j_{i_k}}\right\|e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t^0-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&\leq \sqrt{\frac{\lambda_2}{\lambda_1}}\left(\hbar_{i_k}e^{-\eta_{i_k}j_{i_k}}\right)e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}.
\end{aligned} \tag{3.32}$$

Then, the second term in (3.28) can be estimated as

$$\begin{aligned}
\Delta_2 &\leq \widehat{A}_{21}\sqrt{\frac{\lambda_2}{\lambda_1}}\left[\sum_{j_{i_k}=0}^{T_{i_k}-1}\hbar_{i_k}e^{-\eta_{i_k}j_{i_k}}\right]e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&\leq \hbar_{i_k}\widehat{A}_{21}\sqrt{\frac{\lambda_2}{\lambda_1}}\frac{e^{\eta_{i_k}}}{e^{\eta_{i_k}}-1}e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&:= \chi_2 e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}.
\end{aligned} \tag{3.33}$$

Similarly, the third term in (3.28) can be bounded by

$$\begin{aligned}
\Delta_3 &\leq \hbar_{i_k}e^{(1/2)\alpha(d_1+d_2)}\widehat{A}_{d21}\sqrt{\frac{\lambda_2}{\lambda_1}}\frac{e^{\eta_{i_k}}}{e^{\eta_{i_k}}-1}e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2} \\
&:= \chi_3 e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}.
\end{aligned} \tag{3.34}$$

In addition, following a similar deduction as that in (3.32), we obtain

$$\|\varphi_1(t)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}}\left(e^{(1/2)\alpha(d_1+d_2)}\right)^{T_{i_k}+\cdots+T_{i_p}}\left(\hbar_{i_{p-1}}e^{-\eta_{i_{p-1}}(j_{i_{p-1}}-T_{i_k}-\cdots-T_{i_p})}\right)e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}. \tag{3.35}$$

Then, considering this and (3.19), the fourth term in (3.28) can be estimated as

$$\begin{aligned}
\Delta_4 &\le \widehat{A}_{21}\sqrt{\frac{\lambda_2}{\lambda_1}}\sum_{p=1}^{k}\left\{\left[\prod_{q=p}^{k}\left\|\left(e^{(1/2)\alpha(d_1+d_2)}\widehat{A}_{di_q22}\right)^{T_{i_q}}\right\|\right]\sum_{j_{i_{p-1}}=T_{i_k}+\cdots+T_{i_p}}^{T_{i_k}+\cdots+T_{i_{p-1}}-1}\hbar_{i_{p-1}}e^{-\eta_{i_{p-1}}(j_{i_{p-1}}-T_{i_k}-\cdots-T_{i_p})}\right\}\\
&\quad\times e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}\\
&\le \widehat{A}_{21}\sqrt{\frac{\lambda_2}{\lambda_1}}\sum_{p=1}^{k}\left\{\hbar_{i_{p-1}}\left[\prod_{q=p}^{k}\hbar_{i_q}e^{-\eta_{i_q}T_{i_q}}\right]\frac{e^{\eta_{i_{p-1}}}}{e^{\eta_{i_{p-1}}}-1}\right\}e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}\\
&:= \chi_4 e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}.
\end{aligned} \tag{3.36}$$

Similarly, the fifth term in (3.28) can be bounded by

$$\begin{aligned}
\Delta_5 &\le e^{(1/2)\alpha(d_1+d_2)}\widehat{A}_{d21}\sqrt{\frac{\lambda_2}{\lambda_1}}\sum_{p=1}^{k}\left\{\hbar_{i_{p-1}}\left[\prod_{q=p}^{k}\hbar_{i_q}e^{-\eta_{i_q}T_{i_q}}\right]\frac{e^{\eta_{i_{p-1}}}}{e^{\eta_{i_{p-1}}}-1}\right\}e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}\\
&:= \chi_5 e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}.
\end{aligned} \tag{3.37}$$

Therefore, using (3.31) and (3.33)–(3.37), $\|\xi_2(t)\|$ can be estimated as

$$\|\xi_2(t)\| \le (\chi_1+\chi_2+\chi_3+\chi_4+\chi_5)e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_{d_1+d_2}. \tag{3.38}$$

Combining (3.15), (3.12) and (3.38) yields that system (2.4) is exponentially stable for any switching sequence $\mathcal{S}$ with average dwell time $T_a \ge T_a^* = (\ln\beta)/\alpha$. This completes the proof. □

Remark 3.2. In terms of LMIs, Theorem 3.1 presents a delay-range-dependent exponential admissibility condition for the switched singular systems with interval time-varying delay. It is noted that this condition is obtained by using the integral inequality (Lemma 2.4); no additional free-weighting matrices are introduced to deal with the cross-term. Therefore, the condition proposed here involves much less decision variables than those obtained by using the free-weighting matrices method [16, 19, 21, 22] if the same Lyapunov function is chosen.

Remark 3.3. Equation (3.26) plays an important role in analyzing the exponential stability of the algebraic subsystem, which can be seen as a generalization of the iterative equation in [37] for nonswitched singular time-delay system to switched case.

Remark 3.4. If $\beta = 1$ in $T_a \geq T_a^* = (\ln\beta)/\alpha$, which leads to $P_{i11} \equiv P_{j11}$, $Q_{il} \equiv Q_{jl}$, $Z_{il} \equiv Z_{jl}$, $l = 1, 2$, for all $i, j \in \mathcal{I}$, and $T_a^* = 0$, then system (2.4) possesses a common Lyapunov function and the switching signals can be arbitrary.

In the following, we are to deal with the design problem of DOF controller for the SSTD system (2.1). Applying the DOF controller (2.5) to system (2.1) gives the following closed-loop system

$$\overline{E}\dot{\eta}(t) = \overline{A}_{\sigma(t)}\eta(t) + \overline{A}_{d\sigma(t)}\eta(t - d(t)), \tag{3.39}$$

where $\eta(t) = [x^T(t)x_c^T(t)]^T$, and

$$\begin{gathered}\overline{E} = \begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix}, \qquad \overline{A}_{\sigma(t)} = \begin{bmatrix} A_{\sigma(t)} + B_{\sigma(t)}D_{c\sigma(t)}C_{\sigma(t)} & B_{\sigma(t)}C_{c\sigma(t)} \\ B_{c\sigma(t)}C_{\sigma(t)} & A_{c\sigma(t)} \end{bmatrix} \\ \overline{A}_{d\sigma(t)} = \begin{bmatrix} A_{d\sigma(t)} + B_{\sigma(t)}D_{c\sigma(t)}C_{d\sigma(t)} & 0 \\ B_{c\sigma(t)}C_{d\sigma(t)} & 0 \end{bmatrix}.\end{gathered} \tag{3.40}$$

The following Theorem presents a sufficient condition for solvability of the DOF controller design problem for system (2.1).

Theorem 3.5. *For prescribed scalars $\alpha > 0$, $\gamma > 0$, $d_1 \geq 0$, $d_2 > 0$ and $0 \leq \mu < 1$, if for each $i \in \mathcal{I}$, and given scalars ξ_{i1} and ξ_{i2}, there exist matrices Υ_i, $\widehat{A}_{ci}$, $\widehat{B}_{ci}$, $\widehat{C}_{ci}$, $\widehat{D}_{ci}$, Q_{il11}, Q_{il12}, Q_{il22}, Z_{il11}, Z_{il12}, Z_{il22}, $l = 1, 2$, and R_i and U_i of the following form*

$$R_i = \begin{bmatrix} R_{i11} & 0 \\ R_{i21} & R_{i22} \end{bmatrix}, \qquad U_i = \begin{bmatrix} U_{i11} & 0 \\ U_{i21} & U_{i22} \end{bmatrix} \tag{3.41}$$

with $R_{i11} \in \mathbf{R}^{r\times r}$, $U_{i11} \in \mathbf{R}^{r\times r}$, $R_{i11} > 0$, $U_{i11} > 0$, $R_{i22} \in \mathbf{R}^{(n-r)\times(n-r)}$, $U_{i22} \in \mathbf{R}^{(n-r)\times(n-r)}$, and R_{i22} and U_{i22} being invertible, such that

$$\begin{bmatrix} \Upsilon_{i11} & \Upsilon_{i12} & \Upsilon_{i13} & \Upsilon_{i14} & \Upsilon_{i15} & 0 & c_1E^TZ_{i111}E & c_1E^TZ_{i112}E \\ * & \Upsilon_{i22} & \Upsilon_{i23} & \Upsilon_{i24} & \Upsilon_{i25} & 0 & c_1E^TZ_{i112}^TE & c_1E^TZ_{i122}E \\ * & * & \Upsilon_{i33} & \Upsilon_{i34} & \Upsilon_{i35} & 0 & 0 & 0 \\ * & * & * & \Upsilon_{i44} & \Upsilon_{i45} & 0 & 0 & 0 \\ * & * & * & * & \Upsilon_{i55} & \Upsilon_{i56} & c_2E^TZ_{i211}E & c_2E^TZ_{i212}E \\ * & * & * & * & * & \Upsilon_{i66} & c_2E^TZ_{i212}^TE & c_2E^TZ_{i222}E \\ * & * & * & * & * & * & \Upsilon_{i77} & \Upsilon_{i78} \\ * & * & * & * & * & * & * & \Upsilon_{i88} \end{bmatrix} < 0, \tag{3.42}$$

$$\overline{Q}_{il} = \begin{bmatrix} Q_{il11} & Q_{il12} \\ * & Q_{il22} \end{bmatrix} > 0, \quad \overline{Z}_{il} = \begin{bmatrix} Z_{il11} & Z_{il12} \\ * & Z_{il22} \end{bmatrix} > 0, \quad l = 1, 2, \tag{3.43}$$

where $\Upsilon_{i11} = \mathrm{Sym}\{\xi_{i1}A_i+\xi_{i1}B_i\widehat{D}_{ci}C_i+\widehat{B}_{ci}C_i\}+\sum_{l=1}^{2}Q_{il11}+\alpha E^TR_i-c_1E^TZ_{i111}E$, $\Upsilon_{i12} = \widehat{A}_{ci}+\xi_{i1}B_i\widehat{C}_{ci}+\xi_{i2}A_i^T+\xi_{i2}C_i^T\widehat{D}_{ci}^TB_i^T+C_i^T\widehat{B}_{ci}^T+\sum_{l=1}^{2}Q_{il12}-c_1E^TZ_{i112}E$, $\Upsilon_{i13} = -\xi_{i1}I+R_i^T+\xi_{i1}A_i^T+\xi_{i1}C_i^T\widehat{D}_{ci}^TB_i^T+C_i^T\widehat{B}_{ci}^T$, $\Upsilon_{i14} = -Y_i+\xi_{i2}A_i^T+\xi_{i2}C_i^T\widehat{D}_{ci}^TB_i^T+C_i^T\widehat{B}_{ci}^T$, $\Upsilon_{i15} = \Upsilon_{i35} = \xi_{i1}A_{di}+\xi_{i1}B_i\widehat{D}_{ci}C_{di}+\widehat{B}_{ci}C_{di}$, $\Upsilon_{i22} = \mathrm{Sym}\{\widehat{A}_{ci}+\xi_{i2}B_i\widehat{C}_{ci}\}+\sum_{l=1}^{2}Q_{il22}+\alpha E^TU_i-c_1E^TZ_{i122}E$, $\Upsilon_{i23} = -\xi_{i2}I+\widehat{A}_{ci}^T+\xi_{i1}\widehat{C}_{ci}^TB_i^T$, $\Upsilon_{i24} = -Y_i+\widehat{A}_{ci}^T+\xi_{i2}\widehat{C}_{ci}^TB_i^T+U_i^T$, $\Upsilon_{i25} = \Upsilon_{i45} = \xi_{i2}A_{di}+\xi_{i2}B_i\widehat{D}_{ci}C_{di}+\widehat{B}_{ci}C_{di}$, $\Upsilon_{i33} = -2\xi_{i1}I+d_1^2Z_{i111}+d_2^2e^{\alpha d_1}Z_{i211}$, $\Upsilon_{i34} = -\xi_{i2}I-Y_i+d_1^2Z_{i112}+d_2^2e^{\alpha d_1}Z_{i212}$, $\Upsilon_{i44} = -\mathrm{Sym}\{Y_i\}+d_1^2Z_{i122}+d_2^2e^{\alpha d_1}Z_{i222}$, $\Upsilon_{i55} = -(1-\mu)e^{-\alpha(d_1+d_2)}Q_{i211}-c_2E^TZ_{i211}E$, $\Upsilon_{i56} = -(1-\mu)e^{-\alpha(d_1+d_2)}Q_{i212}-c_2E^TZ_{i212}E$, $\Upsilon_{i66} = -(1-\mu)e^{-\alpha(d_1+d_2)}Q_{i222}-c_2E^TZ_{i222}E$, $\Upsilon_{i77} = -(1-\mu)e^{-\alpha d_1}Q_{i111}-c_1E^TZ_{i111}E-c_2E^TZ_{i211}E$, $\Upsilon_{i78} = -(1-\mu)e^{-\alpha d_1}Q_{i112}-c_1E^TZ_{i112}E-c_2E^TZ_{i212}E$, $\Upsilon_{i88} = -(1-\mu)e^{-\alpha d_1}Q_{i122}-c_1E^TZ_{i122}E-c_2E^TZ_{i222}E$. *Then, there exists a DOF controller in the form of* (2.5)*, such that system* (3.39) *is exponentially admissible for any switching sequence* $\mathcal{S}$ *with average dwell time* $T_a \geq T_a^* = \ln\beta/\alpha$*, where* $\beta \geq 1$ *satisfies*

$$R_{i11} \leq \beta R_{j11}, \quad U_{i11} \leq \beta U_{j11}, \quad \overline{Q}_{il} \leq \beta\overline{Q}_{jl}, \quad \overline{Z}_{il} \leq \beta\overline{Z}_{jl}, \quad l = 1,2, \quad \forall i,j \in \mathcal{I}. \tag{3.44}$$

Moreover, a desired DOF controller realisation is given by

$$A_{ci} = Y_i^{-1}\widehat{A}_{ci}, \quad B_{ci} = Y_i^{-1}\widehat{B}_{ci}, \quad C_{ci} = \widehat{C}_{ci}, \quad D_{ci} = \widehat{D}_{ci}, \quad \forall i \in \mathcal{I}. \tag{3.45}$$

Proof. From Theorem 3.1, we known that system (3.39) is exponentially admissible for any switching sequence $\mathcal{S}$ with average dwell time $T_a \geq T_a^* = (\ln\beta)/\alpha$, where $\beta \geq 1$ satisfying (3.3), if for each $i \in \mathcal{I}$, there exist matrices $\overline{Q}_{il} > 0$, $\overline{Z}_{il} > 0$, $l = 1,2$, and $\overline{P}_i$ with the form of (3.1) such that the inequality (3.2) with E, A_i and A_{di} instead of $\overline{E}$, $\overline{A}_i$ and $\overline{C}_i$, respectively, holds. By decomposing Φ_i in (3.2), we obtain that for each $i \in \mathcal{I}$

$$\Phi_i = \Pi_i\Lambda_i\Pi_i^T < 0, \tag{3.46}$$

where $\overline{J}_i$ is any invertible matrix with compatible dimension, and

$$\Pi_i = \begin{bmatrix} I & 0 & 0 \\ \overline{A}_i & \overline{A}_{di} & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T, \qquad \Lambda_i = \begin{bmatrix} \Lambda_{i11} & \overline{P}_i^T-\overline{J}_i+\overline{A}_i^T\overline{J}_i^T & \overline{J}_i\overline{A}_{di} & c_1\overline{E}^T\overline{Z}_{i1}\overline{E} \\ * & -\overline{J}_i-\overline{J}_i^T+\overline{U}_i & \overline{J}_i\overline{A}_{di} & 0 \\ * & * & \Lambda_{i33} & c_2\overline{E}^T\overline{Z}_{i2}\overline{E} \\ * & * & * & \Lambda_{i44} \end{bmatrix} \tag{3.47}$$

with $\Lambda_{i11} = \mathrm{Sym}\{\overline{J}_i\overline{A}_i\}+\sum_{l=1}^{2}\overline{Q}_{il}+\alpha\overline{E}_i^T\overline{P}_i-c_1\overline{E}^T\overline{Z}_{i1}\overline{E}$, $\Lambda_{i33} = -(1-\mu)e^{-\alpha(d_1+d_2)}\overline{Q}_{i2}-c_2\overline{E}^T\overline{Z}_{i2}\overline{E}$, and $\Lambda_{i44} = -e^{-\alpha d_1}\overline{Q}_{i1}-c_1\overline{E}^T\overline{Z}_{i1}\overline{E}-c_2\overline{E}^T\overline{Z}_{i2}\overline{E}$. Hence, $\Phi_i < 0$ holds if

$$\Lambda_i < 0. \tag{3.48}$$

Let $\overline{E} = \mathrm{diag}\{E,E\}$. For each $i \in \mathcal{I}$, define

$$\overline{P}_i = \mathrm{diag}\{R_i, U_i\}. \tag{3.49}$$

By (2.3) and (3.41), we have

$$E^T R_i = R_i^T E \geq 0, \qquad E^T U_i = U_i^T E \geq 0, \tag{3.50}$$

which combining (3.49) yields

$$\overline{E}^T \overline{P}_i = \overline{P}_i^T \overline{E} \geq 0. \tag{3.51}$$

Then, from (3.41), (3.44) and (3.51), it can be deduced that

$$\overline{E}^T \overline{P}_i \leq \beta \overline{E}^T \overline{P}_j, \quad \forall i, j \in \mathcal{I}. \tag{3.52}$$

Denote

$$\overline{J}_i = \begin{bmatrix} \xi_{i1} I & Y_i \\ \xi_{i2} I & Y_i \end{bmatrix}. \tag{3.53}$$

Substituting (3.43), (3.49) and (3.53) into (3.48), and defining

$$\widehat{A}_{ci} = Y_i A_{ci}, \quad \widehat{B}_{ci} = Y_i B_{ci}, \quad \widehat{C}_{ci} = C_{ci}, \quad \widehat{D}_{ci} = D_{ci}, \quad \forall i \in \mathcal{I} \tag{3.54}$$

we can easily obtain (3.42). This completes the proof. □

Remark 3.6. Note that condition Φ_i of Theorem 3.1 involves some product terms between the Lyapunov matrices and the system matrices, which complicates the DOF control synthesis problem. To solve this problem, in the proof of Theorem 3.5, we have made a decoupling between the Lyapunov matrices and the system matrices by introducing a slack matrix $\overline{J}_i$ in condition Λ_i. Compared with the variable change method used in [39, 40], the decoupling technique proposed here simplifies the DOF controller design problem greatly, which decreases the conservatism in some sense.

Remark 3.7. Scalars ξ_{i1} and ξ_{i2}, $i \in \mathcal{I}$, in Theorem 3.5 are tuning parameters which need to be specified first. The optimal values of these parameters can be found by applying some optimization algorithms such as the program fminsearch in the optimization toolbox of MATLAB, the branch-and-band algorithm [41].

Remark 3.8. It is noted that in this paper, the derivative matrix E is assumed to be switch-mode-independent. If E is also switch-mode-dependent, then E is changed to E_i, $i \in \mathcal{I}$. In this case, the transformation matrices P and Q should become P_i and Q_i so that $P_i E_i Q_i = \operatorname{diag}\{I_{r_i}, 0\}$, and the state of the transformed system becomes $\tilde{x}(t) = Q_i^{-1}(t) = [\tilde{x}_{i1}^T(t) \quad \tilde{x}_{i2}^T(t)]^T$ with $\tilde{x}_{i1}^T(t) \in \mathbf{R}^{r_i}$ and $\tilde{x}_{i1}^T(t) \in \mathbf{R}^{n-r_i}$, which implies that there does not exist one common state space coordinate basis for all subsystems. Then, some assumptions for E_i (e.g., E_i, $i \in \mathcal{I}$, have the same right zero subspace [27]) should be made so that Q_i remains the same; in this case, the method presented in this paper is also valid. How to investigate the general SSTD system with E being switch-mode-dependent is an interesting problem for future work via other approaches.

Table 1: Comparison of allowable upper bound $\tilde{d}_2$ for different d_1 in Example 4.1.

Methods		$\tilde{d}_2$		Number of variables
Lemma 1 [21]	1.130 ($d_1 = 0.1$)	1.099 ($d_1 = 0.3$)	1.084 ($d_1 = 0.7$)	84
Theorem 1 [22]	1.130 ($d_1 = 0.1$)	1.099 ($d_1 = 0.3$)	1.084 ($d_1 = 0.7$)	84
Theorem 3.1	1.134 ($d_1 = 0.1$)	1.133 ($d_1 = 0.3$)	1.133 ($d_1 = 0.7$)	30

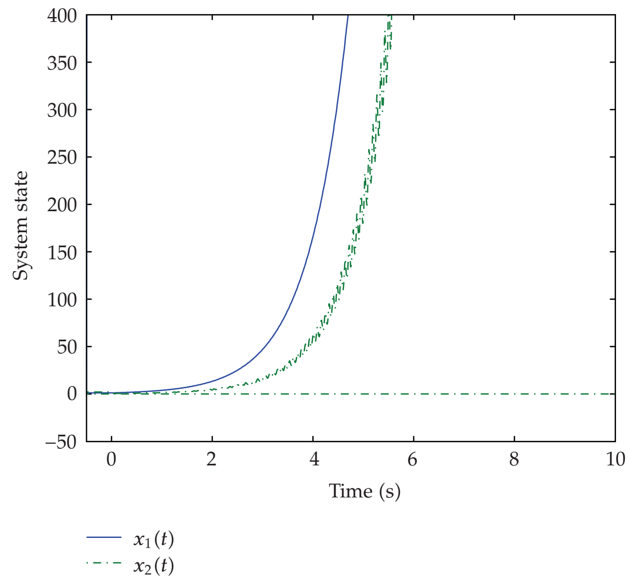

Figure 1: State trajectories of the open-loop subsystem 1.

4. Numerical Examples

In this section, some numerical examples are presented to demonstrate the effectiveness of the proposed methods.

Example 4.1. Consider the switched system (2.4) with $E = I$, $N = 2$ (e.g., there are two subsystems) and the following parameters, which are borrowed from [21]:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -0.7 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \tag{4.1}$$

For $\mu = 0.4$, $\alpha = 0.5$ and $\beta = 1.1$, employing the LMIs in [21, 22] and those in Theorem 3.1 yields an allowable upper bound $\tilde{d}_2$ (in this paper $\tilde{d}_2 = d_1 + d_2$) of the delay $d(t)$ that guarantees the stability of system (2.4). Table 1 shows the values of the upper bound for various d_1 and the number of involved variables by using different methods. It is easily seen from Table 1 that Theorem 3.1 of this paper not only provides better results than those criteria in [21, 22] but also reduces the computational overhead to some extent.

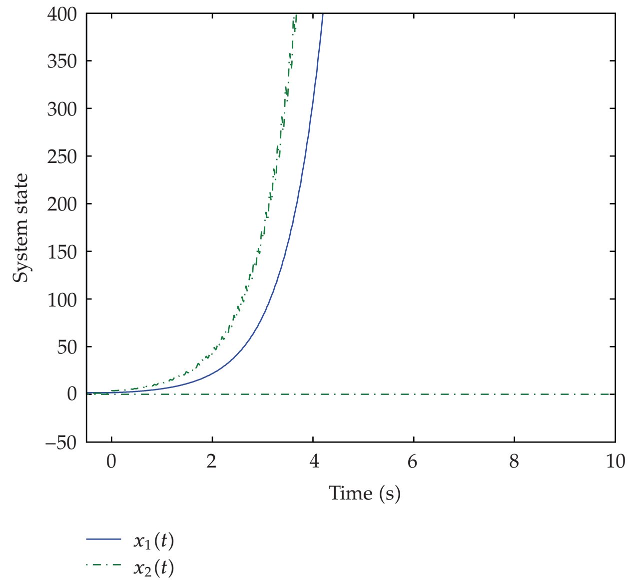

Figure 2: State trajectories of the open-loop subsystem 2.

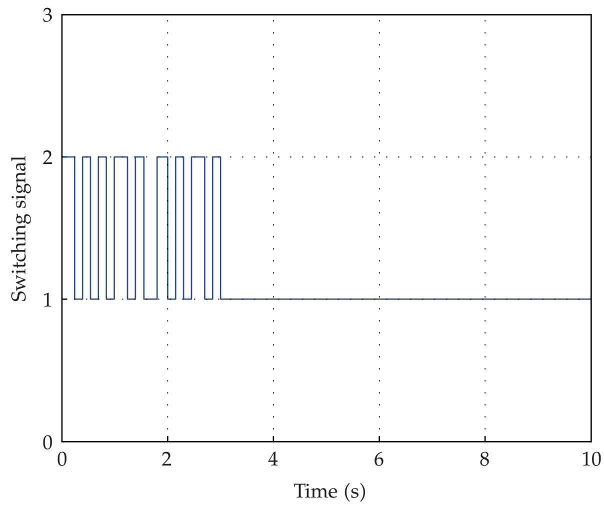

Figure 3: Switching signal with the average dwell time $T_a > 0.13$.

Example 4.2. Consider the switched system (2.4) with $N = 2$ and the related parameters are given as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0.73 & 0 \\ 0 & -1 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.5 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.4 & 0 \\ -0.1 & -1 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \tag{4.2}$$

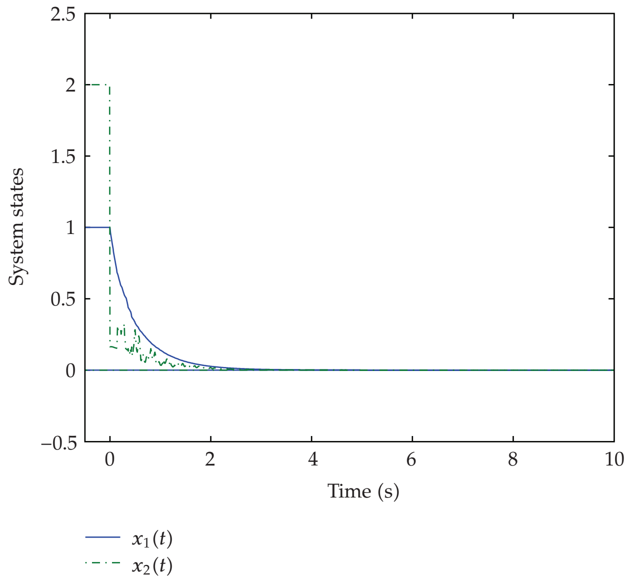

Figure 4: State trajectories of the closed-loop system under DOF control.

and $d_1 = 0.1$, $d_2 = 0.2$, $\mu = 0.4$ and $\alpha = 0.5$. It can be verified that both of the above two subsystems are stable. Let $\beta = 1$; it can be found that there is no feasible solution to this case, which implies that there is no common Lyapunov function for the above two subsystems (see Remark 3.4). Now, we consider the average dwell time scheme, and set $\beta = 1.2$. Solving the LMIs (3.2) gives the following solutions:

$$P_1 = \begin{bmatrix} 19.7719 & 0 \\ 17.9611 & 819.3011 \end{bmatrix}, \quad Q_{11} = \begin{bmatrix} 0.7438 & -1.2478 \\ -1.2478 & 68.3107 \end{bmatrix}, \quad Q_{12} = 10^3 \times \begin{bmatrix} 0.0002 & 0.0102 \\ 0.0102 & 1.0735 \end{bmatrix},$$
$$Z_{11} = \begin{bmatrix} 624.4425 & 0.8810 \\ 0.8810 & 379.5848 \end{bmatrix}, \quad Z_{12} = \begin{bmatrix} 332.4246 & -0.4902 \\ -0.4902 & 373.4311 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 23.3711 & 0 \\ -26.9375 & 642.4532 \end{bmatrix}, \quad Q_{21} = \begin{bmatrix} 0.7501 & -0.8269 \\ -0.8269 & 66.5062 \end{bmatrix}, \quad Q_{22} = \begin{bmatrix} 0.2201 & 8.9316 \\ 8.9316 & 917.3497 \end{bmatrix},$$
$$Z_{21} = \begin{bmatrix} 560.8384 & 0.7643 \\ 0.7643 & 379.5975 \end{bmatrix}, \quad Z_{22} = \begin{bmatrix} 294.2097 & -0.4003 \\ -0.4003 & 373.4858 \end{bmatrix} \tag{4.3}$$

which means that the above switched system is exponentially admissible. Moreover, by further analysis, it can be found that the allowable minimum of β is $\beta_{\min} = 1.046$ when $\alpha = 0.5$; in this case $T_a^* = (\ln \beta_{\min})/\alpha = 0.0899$.

Example 4.3. Consider the switched system (2.1) with $N = 2$ and

$$
\begin{gathered}
E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\
A_1 = \begin{bmatrix} 0.9 & 0 \\ 1 & -5 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.5 & 0.1 \\ 1 & 0.1 \end{bmatrix}, \\
B_1 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix}, \qquad C_{d1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \\
A_2 = \begin{bmatrix} 0.5 & 0.1 \\ 2 & -5 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0.2 & 0.5 \\ 1.5 & 0.1 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}, \\
C_2 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \qquad C_{d2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},
\end{gathered}
\tag{4.4}
$$

and $d(t) = 0.3 + 0.2\sin(1.5t)$. A simple calculation yields $d_1 = 0.1$, $d_2 = 0.4$ and $\mu = 0.3$. By simulation, it can be checked that both of the above two subsystems with $u(t) = 0$ are unstable, and the state responses of the corresponding open-loop systems are shown in Figures 1 and 2, respectively, with the initial condition given by $\phi(t) = [1 \quad 2]^T$, $t \in [-0.5, 0]$. In view of this, our goal is to design a DOF control $u(t)$ in the form of (2.5) such that the closed-loop system is exponentially admissible.

Set $\alpha = 0.5$, $\beta = 1.05$ (thus $T_a \geq T_a^* = (\ln\beta)/\alpha = 0.0976$), and choose $\xi_{11} = 0.9255$, $\xi_{12} = 0.0067$, $\xi_{13} = 0.9811$, $\xi_{14} = 0.0016$. Solving the LMIs (3.41)–(3.44), the corresponding gain matrices of the DOF controller are computed as

$$
\begin{gathered}
A_{c1} = \begin{bmatrix} -26.3829 & -0.4700 \\ -0.8920 & -0.8529 \end{bmatrix}, \qquad A_{c2} = \begin{bmatrix} -67.7480 & 0.6423 \\ 9.5961 & -0.8803 \end{bmatrix}, \qquad B_{c1} = \begin{bmatrix} 0.6811 \\ -0.2272 \end{bmatrix}, \\
B_{c2} = \begin{bmatrix} 4.0654 \\ -1.1668 \end{bmatrix}, \qquad C_{c1} = \begin{bmatrix} 33.6391 & 0.1324 \end{bmatrix}, \qquad C_{c2} = \begin{bmatrix} 23.4740 & -0.5180 \end{bmatrix}, \\
D_{c1} = -22.4825, \qquad D_{c2} = -17.9093
\end{gathered}
\tag{4.5}
$$

To show the effectiveness of the obtained DOF controller, giving a random switching signal with the average dwell time $T_a \geq 0.13$ as shown in Figure 3, we get the state trajectories of the closed-loop system as shown in Figure 4, for the given initial condition $\phi(t) = [1 \quad 2]^T$, $t \in [-0.5, 0]$. It is clear that the designed controller is feasible and ensures the stability of the closed-loop system despite the switching and the time-varying delay.

5. Conclusions

In this paper, the problems of exponential admissibility and DOF control for a class of continuous-time switched singular systems with interval time-varying delay have been investigated. A class of switching signals has been identified for the switched singular time-delay systems to be exponentially admissible under the average dwell time scheme. The DOF controller has been designed, and the corresponding solvability condition has been established by using the LMI technique. Numerical examples have been provided to illustrate the effectiveness of the proposed methods.

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