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Research Article

A Four-Type Decision-Variable MINLP Model for a Supply Chain Network Design

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We propose a mixed integer nonlinear programming model for the design of a one-period planning horizon supply chain with integrated and flexible decisions on location of plants and of warehouses, on levels of production and of inventory, and on transportation models, considering stochastic demand and the ABC classification for finished goods, which is an NP-hard industrial engineering optimization problem. Furthermore, computational implementation of the proposed model is presented through the direct application of the outer approximation algorithm on some randomly generated supply chain data.

1. Introduction

It is known that industrial organizations can obtain significant savings through the optimal design of their supply chain networks. Indeed, the optimal design can contribute to refine logistics objects as well as logistics strategies, improve on the architecture logistics network, and above all, support decision making. However, decision makers have troublesome task when dealing with integrated planning of logistics networks. Since this industrial engineering optimization problem is in general difficult and more specifically NP-hard even for networks with small sizes, trying one by one potential plans is very time consuming, and therefore impractical.

In fact the optimization of an integrated logistics network design is still a challenge, specially if many items, many layers, many logistics components, many different types of decision variables and stochastic demands are being considered.

With respect to the number of different types of decision variables, just a few existing studies have addressed the logistics network design problem considering three or more layers and deterministic demands with four different types using mixed integer linear programming models (MILP) [1, 2]. According to the recent review made in [3], the works of [4, 5] can fit the design optimization of a one-period planning horizon logistics network with stochastic demand with three or more layers, but they involve only decisions on location using MILP models.

Uncertainty of customer demands has also been considered in [6] in order to determine, for example, the optimal network design, transportation and inventory levels of a single-item multiechelon supply chain. In [7], the same authors formulated a bi-criterion MINLP for the optimal design of responsive process supply chains with inventories, considering economic and responsiveness objectives.

Besides the cited references relevant for this work, there exist many works in the literature that address the optimization of logistics network design problem considering diverse aspects; we encourage the reader to see more details in the remarkable review of [3].

In this work, we propose a more realistic mathematical formulation for the design of a one-period logistics network having three layers (suppliers, plants, warehouses and customers), which has many finished products with stochastic demands. The proposed model is flexible and integrates decisions on location of plants and of warehouses, on levels of production and of inventory, and on transportation models. It is formulated as a mixed integer nonlinear programming problem (MINLP) so that it can incorporate decisions on inventory levels in more realistic scale, according to [8] *apud* Croxton and Zinn [9].

Based on the models of Cordeau et al. [10] and of Miranda and Garrido [11], the proposed model innovates in terms of formulating a four-type decision-variable logistics network design problem considering three layers and multi products with stochastic demands, as a MINLP. In relation to the MILP model in [10], the proposed model includes decisions on inventory levels in warehouses based on the stochastic demands of the costumers. Although the MILP model of [11] considers stochastic demand for one product, it involves only decisions on inventory levels, whereas the proposed model considers additionally decisions on location of plants and of warehouses, on production levels, and on transportation models for a multiproduct logistics network. Moreover, the proposed model makes use of the ABC classification for finished products, setting an appropriate level of service for each product depending on its classification. In this case, level of service of a product is given in terms of its stock availability; the higher the ABC classification, the higher is the stock availability.

Furthermore, the results of computational experiments on the proposed model are presented through the direct application of the outer approximation algorithm, proposed by Duran and Grossmann [12], on three randomly generated supply chain data. Geographic information system (GIS) is used to locate and define distances between the nodes of the logistics network (suppliers, plants, warehouses and customers) and optimize them.

Many algorithms have been proposed to optimize integrated logistics networks by making use of particular properties of the models or combining existing techniques. For example, through the exploitation of the separable model, a spatial decomposition algorithm based on Lagragean relaxation and piecewise linear approximation was proposed in [6] to find the optimal network design, transportation and inventory levels of a single-item multiechelon supply chain. In [13], two heuristic methods are proposed to solve approximately a joint supply chain network design and inventory management model.

While the first algorithm introduces a convexification scheme before addressing a MINLP, the second one uses Lagragean relaxation and decomposition technique to deal with the nonconvexity nature of the model.

The work is presented as follows. Section 2 presents the notation and the mathematical formulation for a four-type decision-variable MINLP model in order to find an optimal design of a certain supply chain network. In Section 3, we briefly describe the outer approximation algorithm and the computational experiments realized to solve three instances whose parameters and supply chain components were randomly generated. Final comments are given in Section 4.

2. The Proposed Formulation

Here, we present the proposed MINLP model with four types of decision variables in order to find an optimal design of a more realistic multiproduct supply chain network with three layers (suppliers, plants, warehouses and customers). The proposed mathematical formulation is based on an extension of the MILP model presented in Cordeau et al. [10] for a network design problem with fewer components and fewer layers and deterministic demands. Besides the decisions on facility locations, on production and on transportation addressed by [10], the proposed model includes strategic decisions on inventory levels, as well as more constraints related to potential facilities, production of multi finished products and their transportation along the network. The inventory policy used in this study is stochastic, based on order point and immediate replenishment, with multistorage points.

The proposed formulation was also developed based on the work of Miranda and Garrido [11], that considers the one-period supply chain design problem with two layers, one product with stochastic demand, and decision only on inventory levels, while the proposed model considers stochastic demand for all finished products in the logistics network and strategic decisions on location of facilities, on production and on transportation, which are integrated to decision on inventory levels.

In general perspective, the proposed model deals with location-allocation of facilities in three layers. The model treats production levels in each designed plant. It also treats modes of transportation between each origin-destination pair of the network. Inventory costs are considered in order to support decision on allocation of warehouses and on amount of items to be stored. The model is multi item with one-period planning horizon and indivisible demand. It does not consider any interaction between similar facilities nor routing of the products.

2.1. Notation

We present the notation used hereafter for sets, parameters and decision variables in this study. As one should notice, we used most of the notation of Cordeau et al. [10].

Sets

C: Set of costumers

C^f: set of costumers of finished product f

D: set of potential destinations ($D = C \cup P \cup W$)

D^k: set of potential destinations for commodity k

D^r: set of potential destinations for raw material r

F: set of finished products

F^r: set of finished products that require raw material r

K: set of commodities ($K = F \cup R$)

M_{od}: set of transportation modes between o and d

M_{od}^k: set of transportation modes for commodity k between o and d

O: set of origins ($O = P \cup S \cup W$)

O^k: set of potential origins for commodity k

O^r: set of potential origins for raw material r

P: set of potential plant locations

P^f: set of potential plant locations that assembly product f

R: set of raw materials

S: set of potential suppliers

S^r: set of potential suppliers of raw material r

W: set of potential warehouse locations

W^f: set of potential warehouse locations that store product f.

Parameters

a_c^f: Demand of customer c for product f

b^{rf}: amount of raw material r required in product f

c_o: fixed cost of selecting origin o

c_o^k : fixed cost of assigning commodity k to origin o

c_{od}^k: fixed cost of providing commodity k to destination d from origin o

c_{od}^m: fixed cost of using transportation mode m from origin o to d

c_{od}^{km}: unitary cost of providing commodity k to d from o using transportation mode m

CPI_w^f: handling cost of product f in warehouse w

CP_w^f: fixed cost of getting product f from warehouse w

D_w^f: demand of warehouse w for finished product f

d_c^f: demand mean value for product f by customer c

g^{fm}: amount of capacity required by one unity of product f in mode m; similarly, we have the description for g^{km}

g_{od}^m: capacity of transportation of mode m from o to d; similarly, we have the description for g_{wc}^m

IC_w^f: cost of storing product f in warehouse w

LT_w^f: lead time to replenish product f from warehouse w

n: number of segments of data time unit with respect to the fixed planning horizon time unit

N_1: maximum number of warehouses in a logistics network

N_2: maximum number of plants in a logistics network

q_o^k: maximum amount of commodity k shipped from o

q_{od}^k: maximum amount of commodity k shipped from o to d

TH: monetary updating factor

u_o: capacity of origin o

u_o^k: amount of capacity required by one unit of commodity k at origin o

v_c^f: demand variance value for product f by customer c

$Z_{w_{1-\alpha}}^f$: the standard normal probability that warehouse w, with level of service $1-\alpha$, should cover demand for product f during lead times, according to the ABC classification.

Decision Variables

X_{wc}^{fm}: Amount of product f provided by warehouse w to costumer c using transportation mode m; similarly, we have the description for X_{od}^{km}

U_o: indicate if origin o is selected

V_o^k: indicate if commodity k is assigned to origin o

Y_{wc}^f: indicate if warehouse w provides product f to costumer c; similarly, we have the description for Y_{od}^k

Z_{wc}^m: indicate if transportation mode m is selected to serve from warehouse w to costumer c; similarly, we have the description for Z_{od}^m.

2.2. *Mathematical Model*

The proposed model is the following:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{o\in O}\left(c_o U_o + \sum_{d\in D}\sum_{m\in M_{od}} c_{od}^m Z_{od}^m\right) \\
& + \sum_{k\in K}\sum_{o\in O^k}\left[c_o^k V_o^k + \sum_{d\in D^k}\left(c_{od}^k Y_{od}^k + \sum_{m\in M_{od}^k} c_{od}^{km} X_{od}^{km}\right)\right] \\
& + \frac{TH}{n}\sum_{f\in F}\sum_{w\in W}\left(CPI_w^f \sum_{c\in C}\sum_{m\in M} X_{wc}^{fm}\right) \\
& + TH\sum_{f\in F}\sum_{w\in W}\left[\sqrt{\frac{2\, CP_w^f IC_w^f}{n}}\left(\sum_{c\in C}\sum_{m\in M} X_{wc}^{fm}\right)^{1/2}\right. \\
& \qquad \left. + IC_w^f\sqrt{LT_w^f}\, Z_{w_{1-\alpha}}^m\left(\sum_{c\in C} v_c^f\ Y_{wc}^f\right)^{1/2}\right]
\end{aligned}
\tag{2.1}
$$

$$\text{subject to} \quad \sum_{s\in S^r}\sum_{m\in M_{sp}^r} X_{sp}^{rm} - \sum_{f\in F^r}\sum_{w\in W^f}\sum_{m\in M_{pw}^f} b^{rf} X_{pw}^{fm} = 0, \quad r \in R,\ p \in P, \tag{2.2}$$

$$\sum_{p\in P^f}\sum_{m\in M_{pw}^f} X_{pw}^{fm} - \sum_{c\in C}\sum_{m\in M_{wc}^f} X_{wc}^{fm} = 0, \quad f \in F,\ w \in W^f, \tag{2.3}$$

$$\sum_{w\in W^f}\sum_{m\in M_{wc}^f} X_{wc}^{fm} = a_c^f, \quad f \in F,\ c \in C^f, \tag{2.4}$$

$$\sum_{k\in K}\sum_{d\in D^k}\sum_{m\in M_{od}^k} u_o^k X_{od}^{km} - u_o U_o \le 0, \quad o \in O, \tag{2.5}$$

$$\sum_{d\in D^k}\sum_{m\in M_{od}^k} X_{od}^{km} - q_o^k V_o^k \le 0, \quad k \in K,\ o \in O^k, \tag{2.6}$$

$$\sum_{m\in M_{od}^{km}} X_{od}^{km} - q_{od}^k Y_{od}^k \le 0, \quad k \in K,\ o \in O^k,\ d \in D^k, \tag{2.7}$$

$$\sum_{k\in K} g^{km} X_{od}^{km} - g_{od}^m Z_{od}^m \le 0, \quad o \in O,\ d \in D,\ m \in M_{od}, \tag{2.8}$$

$$\sum_{f\in F} u_w^f g^{fm} X_{wc}^{fm} - g_{wc}^m Z_{wc}^m \le 0, \quad w \in W,\ c \in C,\ m \in M_{wc}, \tag{2.9}$$

$$\sum_{w\in W} U_w \le N_1, \tag{2.10}$$

$$\sum_{p\in P} U_p \le N_2, \tag{2.11}$$

$$\sum_{w\in W} Y_{wc}^f = 1, \quad f \in F,\ c \in C, \tag{2.12}$$

$$X_{od}^{km} \in \mathbb{R}_+, \quad k \in K,\ o \in O^k,\ d \in D^k,\ m \in M_{od}^k, \tag{2.13}$$

$$U_o \in \{0,1\}, \quad o \in O, \tag{2.14}$$

$$V_o^k \in \{0,1\}, \quad k \in K,\ o \in O^k, \tag{2.15}$$

$$Y_{od}^k \in \{0,1\}, \quad k \in K,\ o \in O^k,\ d \in D^k, \tag{2.16}$$

$$Z_{od}^m \in \{0,1\}, \quad k \in K,\ o \in O^k,\ d \in D^k,\ m \in M_{od}. \tag{2.17}$$

The objective function (2.1) aims to model decisions on facilities location, on production, on transportation and on inventory, minimizing the corresponding costs. It results from incorporating inventory costs addressed in model [11] into model [10], in a total of 5 big terms (displayed in 5 lines). The first two big terms of the sum (2.1) represent the fixed and variable costs related to the decisions of location and allocation for the considered logistics network, while the last three big terms represent the fixed and variable costs related to the decisions on inventory levels. Recall that $\sum_{w\in W}\sum_{m\in M_{pw}} X_{pw}^{fm}$ represents the total amount of product f manufactured at plant p during the planning horizon time.

The cost of transportation of product f between plant p and warehouse w, that appeared in model [11], is now represented by parameter CPI_w^f in terms of handling costs. Observe that parameter $Z_{w_{1-\alpha}}^m$, showed in the term that models safety stock cost in (2.1), now reflects the level of service of each potential warehouse according to the ABC classification of finished products considered in the network. Recall that level of service is given in terms of stock availability. Also, notice that the objective function model considers the possibility of adjusting the data in case the data time unit corresponds to the planning horizon time unit divided by n.

After gathering model [10] with parts of model [11] related to inventory, we could reduce the number of constraints and variables using the fact that the economic order quantity of product f for warehouse w is given by

$$Q_w^f = \sqrt{\frac{2\ CP_w^f D_w^f}{IC_w^f}}, \tag{2.18}$$

where the demand of warehouse w for product f is given by

$$D_w^f = \frac{1}{n}\sum_{c\in C}\sum_{m\in M} X_{wc}^{fm} = \sum_{c\in C} d_c^f Y_{wc}^f, \tag{2.19}$$

which is introduced into the objective function so that its final version becomes the expression (2.1).

As one can verify, the constraints (2.2)–(2.8) and (2.13)–(2.17) are exactly the same as introduced in model [10]. The group of constraints (2.2) ensures that the total amount of raw material r shipped by a supplier to plant p is equal to the amount required by all products made at this plant, while constraints (2.3) assure that all finished products that enter a warehouse must leave it. Demands constraints are imposed by (2.4). Global capacity limits on suppliers, plants and warehouses are given by constraints (2.5). Constraints (2.6) limit the total amount of a given raw material that is purchased from a particular supplier or limit the number of units of a finished product that are made in a particular plant. If origin o is selected to provide the commodity k to destination d, the constraints (2.7) guarantee this transportation. Capacity constraint for each transportation model is given in (2.8). In order to deal with the possibility of considering the flow of stock-keeping units of products (SKU) in the network, besides the flow of products units, we introduce the factor constraint (2.9). This factor enables that SKU of products can flow from warehouses to costumers through transportation modes with equivalent occupancy. The constraints (2.10) and (2.11) impose an upper bound on the number of open and potential warehouses and plants, respectively N_1 and N_2, in the studied supply chain. The constraint (2.12) assures that only one warehouse can provide a specific finished product to a costumer. Finally, the considered decision variables are defined in constraints (2.13)–(2.17).

As we can see, the proposed model (2.1)–(2.17) is a mixed integer nonlinear programming problem with a nonlinear objective function and linear constraints. Mixed integer nonlinear programming are more appropriate to model supply chain network design problems which include location, transportation and inventory costs than mixed integer linear programming, because, according to Ballou [8] *apud* Croxton and Zinn [9], in reality the relation between the number of warehouses and inventory is non linear.

Nevertheless, the proposed model has some limitations. For instance, the model considers storage only in warehouses. Another limiting aspect of the model is the fact that a unique supplier can not satisfy the demand of each costumer for all products. Discounts on quantity are not considered for acquisition nor transportation of products.

3. Methodology and Computational Tests

Among the existing methodologies that can solve a general mixed integer nonlinear programming problem, like

$$\begin{aligned} \text{(MINLP) minimize} \quad & f(x,y) \\ \text{subject to} \quad & g_i(x,y) \leq 0, \quad i = 1,\ldots,p, \\ & h_j(x,y) = 0, \quad j = 1,\ldots,q, \\ & x \in X \subseteq \mathbb{R}^n, \quad y \in Y \subseteq \mathbb{Z}_+^m, \end{aligned} \tag{3.1}$$

where $f : X \times Y \to \mathbb{R}$, $g_i : X \times Y \to \mathbb{R}$ $(i = 1,\ldots,q)$ and $h_j : X \times Y \to \mathbb{R}$ $(j = 1,\ldots,q)$, we choose the outer approximation (OA) algorithm proposed by Duran and Grossmann in [12]. It consists in solving an alternate sequence of nonlinear programming subproblems and linear relaxed versions of mixed integer linear programming master problems. If by assumption (1) X is a nonempty, convex and compact set, Y is finite, (2) f and g_i, $i = 1,\ldots,q$, are convex and differentiable in $X \times Y$, (3) h_j, $j = 1,\ldots,q$, is linear function in $X \times Y$, and (4) certain constraint qualification is satisfied for the nonlinear programming subproblems, which results from the relaxation of the integrality of y in MINLP, then OA algorithm stops in a finite number of iterations at a global optimal solution. Otherwise, it reports an infeasible solution.

One of the advantages of OA method is the fact that it generally requires relatively few cycles or major iterations with less computational effort. The potential of the OA method is showed in [12], where the authors compared the performance of OA method with a standard branch & bound procedure and with the generalized Benders decomposition (GBD) method on a set of four test MINLP problems.

Since the objective function (1) is not convex, which contradicts assumption (2), there is no (theoretical) guarantee that the OA algorithm will find the global optimum. But, in practice, OA can find global optima of some nonconvex MINLP problems.

3.1. Computational Experiments

We test the proposed model on three randomly generated instances of a certain supply chain network design. Some of the data originated from an earlier work of Monteiro [14]. The remaining data were randomly generated in order to get supply chains with balanced costs. (We skip these details due to the limited space.)

The OA algorithm as well as the instances data were implemented in AIMMS 3.8. The nonlinear programming subproblems generated by OA algorithm were solved by applying MINOS 5.5, since, according to [15], it has good performance when dealing with nonlinear problems with linear constraints, such as the proposed model. With reliability, CPLEX 11 were applied to solve MILP subproblems generated by OA algorithm. The parameters in AIMMS were initially set such that the OA algorithm would select automatically (without the user interference) the starting point for each run. We ran the computational experiments in a notebook Core2Duo, with 2 GHz processor and 2 Gb RAM for all instances.

For the experiments on the proposed model (2.1)–(2.17), we choose year as the time unit of the planning horizon for the design of the network. In this case, as the expected demand data values were generated in months as well as the lead times of products replenishments, we set $n = 12$ (number of segments of the data time unit with respect to the fixed planning horizon time unit). Also, we set $N_1 = N_2 = 4$, and fixed $TH = 11.25$ based on recent Brazilian taxes.

First Computational Test

The first instance was randomly generated to present the following supply chain characteristics:

(1) a network with 3 echelons or layers composed by 7 suppliers, 6 plants, 6 warehouses and 20 costumers is considered.

(2) It has a total of 7 distinct raw materials and 3 different finished products.

(3) There are 2 transportation modes (TR1 and TR2) with different charges.

(4) Each supplier has a minimum and a maximum quantity limit of inputs to offer the manufacturers. The freight in this echelon is the cost of transportation plus the cost of purchase. There are two options of transportation from suppliers to plants, which depends on the capacity of transportation mode; in one case the supplier is in charge of the cost of transportation; in the other case the plant is in charge of it.

(5) Each plant has a fixed maintenance cost as well as a product allocation cost. The freight between a plant and a warehouse depends on the distance and transportation server.

(6) Each warehouse has an annual fixed maintenance cost and allocation cost for each type of product. There is also a handling cost by item. (The ordering cost is included in the objective function (2.1).)

(7) The distribution process considers a unique supplier by product for each costumer.

(8) Each costumer has a specific demand for each product, with mean and variance values based on the monthly historical demand. (A month has 20 working days.)

(9) The third product (PR3) is in Class A of the classification ABC. Its lead time lasts 2 days, the other products have lead time equals to 3 days.

(10) The product PR3 is available in stock 95% when a order is placed, while other products are available 85%.

(11) All three products have corresponding $u_o^k = 1$ and $g^{km} = 1$. This means that one unit of a product has equivalent unit in both transportation modes.

Thus, considering all the characteristics of the supply chain, the first randomly generated instance for problem (2.1)–(2.17) has 1,525 real variables and 1,293 binary variables, and 1,444 functional constraints. An optimal solution was found by the implemented OA algorithm in 8,013.87 seconds, with 12 calls to MINOS and 12 calls to CPLEX. (As mentioned early, there is no guarantee that this optimal solution is global.)

Table 1: Costs for the optimal network design of instance 1.

Optimal costs (in $)	
Acquisition and transportation costs (supplier-plant)	452,793.80
Transportation costs (warehouse-costumer)	241,901.01
Transportation costs (plant-warehouse)	119,486.43
Carrying costs (warehouses)	79,036.14
Maintenance costs (plants)	169,880.00
Maintenance costs (warehouses)	59,234.00
Allocation costs (products-plants)	19,377.01
Allocation costs (products-warehouses)	8,289.17
Other allocation costs	3,503.14
Total cost	1,153,500.70

Table 2: Inventory control information for products in warehouse 1 (WH1) of instance 1.

Inventory information	WH1		
	PR1	PR2	PR3
Order point	562	697	529
Order quantity	576	588	611
Lead-time (in month)	0.15	0.15	0.10
Demand mean value	3339	4125	4262
Security stock	62	79	103

Both routines realized 12 and 6,888,781 iterations, respectively. The amount of memory used by AIMMS was 96.7 MB. The costs related to the optimal design of the logistics network associated to the first instance are shown in Table 1. The information related to inventory control of the optimal design for the finished products of instance 1 is presented in Table 2. In Figure 1, the optimal logistics network flow of the products from warehouse WH1 to the customers in instance 1 is illustrated.

Second Computational Test

The second instance has the same characteristics as the first one with more network components. Corresponding random data was generated to have a supply chain structure with 6 finished products, 12 raw materials, 10 suppliers, 40 costumers, 8 plants, 8 warehouses, and still with 2 transportation modes.

For this instance, the sixth product (PR6) is stored in packages of 6 units. The SKU of PR6 has a volume of 2.4 in both transportation modes. For the remaining products, one unit of a product has equivalent unit in both transportation modes. With respect to classification ABC, we have that PR3 belongs to class A, PR2 and PR6 are in class B, and PR1, PR4 and PR5 are in class C. The availability in stock is 95%, 85%, and 70% for products in the classes A, B and C, respectively. The lead time for products in the classes A, B and C lasts 2, 3 and 4 days, respectively.

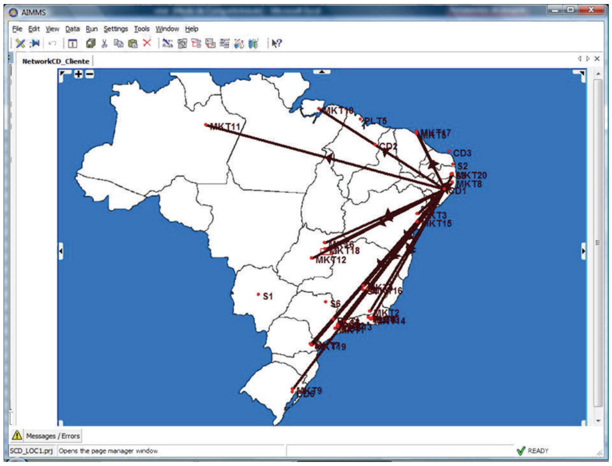

Figure 1: Optimal flow map of the finished products of instance 1.

The second computational test with problem (2.1)–(2.17) has 6,529 real variables and 4,366 binary variables, and 5,061 functional constraints, corresponding to the data of the second instance. An optimal solution was found by the implemented OA algorithm in 10,383.69 sec, with 3 calls to MINOS and 3 calls to CPLEX. Both routines realized 3 and 3,228,522 iterations, respectively. The amount of memory used by AIMMS was 101.0 MB. The costs related to the optimal design of the logistics network associated to the second instance are shown in Table 3. For the products of instance 2, the information related to inventory control of the optimal design is presented in Table 4. In Figure 2, the optimal logistics network flow of the products from warehouses WH4 and WH5 to the customers in instance 2 is illustrated.

Third Computational Test

Consider that the logistics network of instance 3 is structured as instance 1. The data of instance 3 was randomly generated so that its network has a total of 10 distinct finished products and 15 different raw materials. It also has 60 costumers, and a demand for each product that varies from 300 to 5900 units. The products PR7 and PR9 are stored in packages of 6 and 10 units, respectively. In this case, the occupancy in terms of transportation rate (g^{km}) for the new 4 products PR7, PR8, PR9 and PR10 is 2.8, 1.1, 3.0 and 1.2, respectively. We still have 2 transportation modes through all the logistics network. Products PR7 and PR10 belong to class A of the ABC classification, both have lead time of 2 days and 95% of stock availability. Products PR3, PR6 and PR8 are in class B. Each product in class B has lead time equals to 3 days and 85% stock availability. The remaining products are in class C, each of them has lead time of 4 days and 70% stock availability.

The third computational test with instance 3 of problem (2.1)–(2.17) has 13,281 real variables and 8,142 binary variables, and 10,107 functional constraints. An optimal solution was found by the implemented OA algorithm in 15,342.75 sec, with 2 calls to MINOS and 2 calls to CPLEX. Both routines realized 2 and 2,175,477 iterations, respectively. The amount of memory used by AIMMS was 131.2 MB. The costs related to the optimal design of the logistics network associated to the third instance are shown in Table 5. The information related to

Table 3: Costs for the optimal network design of instance 2.

Optimal costs (in $)	
Acquisition and transportation costs (supplier-plant)	1,187,714.69
Transportation costs (warehouse-costumer)	432,554.07
Transportation costs (plant-warehouse)	66,743.19
Carrying costs (warehouses)	212.484.98
Maintenance costs (plants)	218,698.00
Maintenance costs (warehouses)	129,296.00
Allocation costs (products-plants)	45,697.42
Allocation costs (products-warehouses)	16,691.82
Other allocation costs	18,067.32
Total cost	2,327,947.48

Table 4: Inventory control information for products in warehouses 4 and 5 (WH4 and WH5) of instance 2.

Inventory information	WH4			WH5		
	PR1	PR2	PR3	PR4	PR5	PR6
Order point	1,113	1,037	754	773	882	1,634
Order quantity	651	743	777	593	612	983
Lead-time (in month)	0.20	0.15	0.10	0.20	0.20	0.15
Demand mean value	5,353	6,332	6,387	3,718	4,238	10,030
Security stock	43	87	115	30	34	130

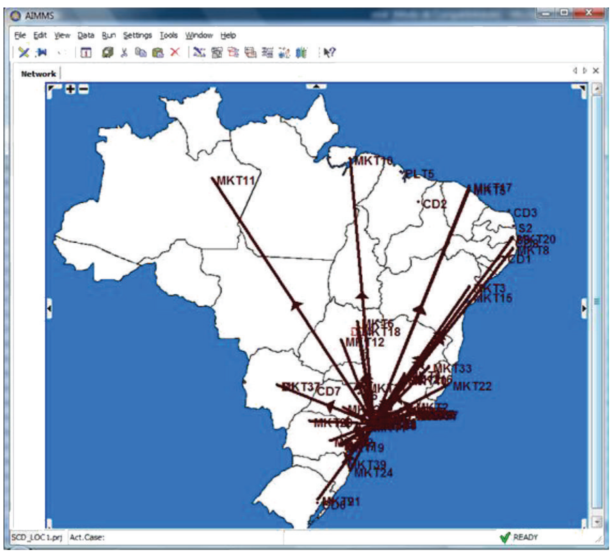

Figure 2: Optimal flow map of the finished products of instance 2.

Table 5: Costs for the optimal network design of instance 3.

Optimal costs (in $)	
Acquisition and transportation costs (supplier-plant)	3,765,730.62
Transportation costs (warehouse-costumer)	2,949,963.50
Transportation costs (plant-warehouse)	1,228,168.87
Carrying costs (warehouses)	633,424.34
Maintenance costs (plants)	331,910.00
Maintenance costs (warehouses)	114,297.00
Allocation costs (products-plants)	101,683.07
Allocation costs (products-warehouses)	46,628.50
Other allocation costs	23,881.93
Total cost	9,195,687.82

Table 6: Inventory control information for products in warehouse 2 (WH2) of instance 3.

	Products	Order point	Order quantity	Lead-time (in month)	Demand mean value	Security stock
WH2	PR2	875	651	0.20	834	41
	PR6	1,133	797	0.15	1,027	106
	PR7	818	744	0.10	702	116
	PR8	825	619	0.15	734	91
	PR10	745	745	0.10	611	134

Table 7: Inventory control information for products in warehouse 1 (WH1) of instance 3.

	Products	Order point	Order quantity	Lead-time (in month)	Demand mean value	Security stock
WH1	PR1	1,558	865	0.20	1,510	48
	PR2	903	602	0.20	864	39
	PR3	1,381	865	0.15	1,284	97
	PR4	1,136	762	0.20	1,100	36
	PR5	1,293	696	0.20	1,252	41
	PR6	1,336	931	0.15	1,218	118
	PR7	369	532	0.10	294	75
	PR8	1,295	839	0.15	1,183	112
	PR9	1,686	857	0.20	1,628	58
	PR10	1,124	947	0.10	935	189

inventory control of the optimal design for the products of instance 3 is presented in Tables 6 and 7. In Figure 3, the optimal logistics network flow of the products from warehouses WH4 and WH5 to the customers in instance 3 is illustrated.

3.2. *Computational Analysis*

The summary of the computational experiments with 3 different instances is presented in Table 8. We observe that the OA algorithm realized more iterations to solve instance 1 than

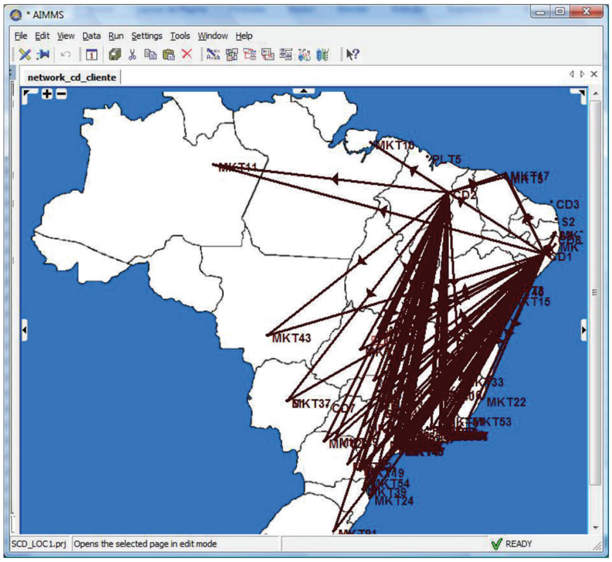

Figure 3: Optimal flow map of the finished products of instance 3.

Table 8: Computational results for all instances.

	Instance 1	Instance 2	Instance 3
Total variables	2,818	10,895	21,423
Binary variables	1,293	4,366	8,142
Functional constraints	1,444	5,061	10,107
Calls to NLP and MILP solvers	12	3	2
Iterations for NLP solver	12	3	2
Iterations for MILP solver	6,888,781	3,228,522	2,175,477
Total time (sec)	8,013.87	10,383.49	15,342.75
Used memory (MB)	96.7	101.0	131.2

to solve the others. One possible explanation is the fact that the OA algorithm might have generated many infeasible nonlinear subproblems, which in turn is an example of a real drawback of this algorithm. In Table 9, we observe that as the supply chain structure becomes more complex in number of components, the majority of the costs increases.

4. Final Comments

We have proposed a new integrated and flexible mathematical formulation for the design of a supply chain network that ultimately shall support decision makers of diverse fields and markets. The proposed model is based on existing formulation from the literature which was extended to include not only facility locations, production, and transportation, but also inventory levels in warehouses based on the stochastic demand of customers, for a more realistic perspective. Although the proposed model has an objective function with a non convex term, we decided to apply the outer approximation algorithm to obtain an optimal solution, because empirical evidences have shown that the outer approximation algorithm can solve a MINLP problem in less computational effort.

Table 9: Cumulative costs for the network design for all instances.

Costs description	Instance 1	Instance 2	Instance 3
Acquisition and transportation costs (supplier-plant)	452,793.80	1,187,714.69	3,765,730.62
Transportation costs (warehouse-costumer)	241,901.01	432,554.07	2,949,963.50
Transportation costs (plant-warehouse)	119,486.43	66,743.19	1,228,168.87
Carrying costs (warehouses)	79,036.14	212.484.98	633,424.34
Maintenance costs (plants)	169,880.00	218,698.00	331,910.00
Maintenance costs (warehouses)	59,234.00	129,296.00	114,297.00
Allocation costs (products-plants)	19,377.01	45,697.42	101,683.07
Allocation costs (products-warehouses)	8,289.17	16,691.82	46,628.50
Other allocation costs	3,503.14	18,067.32	23,881.93
Total costs	1,153,500.70	2,327,947.48	9,195,687.82

The integrated analysis of the decision variables, related to suppliers selection, level of production, transportation modes and level of stocks, in a model for one-period, can offer reduced logistics costs, which shows the important contribution of this study.

On the other hand, flexibility of the model allows one to easily either reduce or increase the logistics network complexity in number of components and of layers. Moreover, one can modify the proposed model to consider costs associated to stocks in transit, backlogging, variability in lead time per product, just to mention a few.

Finally, a supply chain network design model that introduces the ABC classification for finished products should support decision on choosing a plan that gives importance to products whose profit contributions are higher.

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References

[1] M. T. Melo, S. Nickel, and F. S. Saldanha Da Gama, "Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning," *Computers and Operations Research*, vol. 33, no. 1, pp. 181–208, 2006.

[2] W. Wilhelm, D. Liang, B. Rao, D. Warrier, X. Zhu, and S. Bulusu, "Design of international assembly systems and their supply chains under NAFTA," *Transportation Research Part E: Logistics and Transportation Review*, vol. 41, no. 6, pp. 467–493, 2005.

[3] M. T. Melo, S. Nickel, and F. Saldanha-da-Gama, "Facility location and supply chain management—a review," *European Journal of Operational Research*, vol. 196, no. 2, pp. 401–412, 2009.

[4] M. I.G. Salema, A. P. Barbosa-Povoa, and A. Q. Novais, "An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty," *European Journal of Operational Research*, vol. 179, no. 3, pp. 1063–1077, 2007.

[5] Tjendera Santoso, Shabbir Ahmed, Marc Goetschalckx, and Alexander Shapiro, "A stochastic programming approach for supply chain network design under uncertainty," *European Journal of Operational Research*, vol. 167, no. 1, pp. 96–115, 2005.

[6] F. You and I. E. Grossmann, “Integrated multi-echelon supply chain design with inventories under uncertainty: MINLP models, computational strategies,” *AIChE Journal*, vol. 56, no. 2, pp. 419–440, 2010.
[7] F. You and I. E. Grossmann, “Balancing responsiveness and economics in process supply chain design with multi-echelon stochastic inventory,” *AIChE Journal*, vol. 57, no. 1, pp. 178–192, 2011.
[8] R. H. Ballou, *Business Logistics/Supply Chain Management*, Bookman, So Paulo, Brazil, 5th edition, 2006.
[9] K. L. Croxton and W. Zinn, “Inventory considerationsin network design,” *Journal of Business Logistics*, vol. 26, no. 1, pp. 149–168, 2005.
[10] J.-F. Cordeau, F. Pasin, and M. M. Solomon, “An integrated model for logistics network design,” *Annals of Operations Research*, vol. 144, pp. 59–82, 2006.
[11] P. A. Miranda and R. A. Garrido, “Incorporating inventory control decisions into a strategic distribution network design model with stochastic demand,” *Transportation Research Part E: Logistics and Transportation Review*, vol. 40, no. 3, pp. 183–207, 2004.
[12] M. A. Duran and I. E. Grossmann, “An outer-approximation algorithm for a class of mixed-integer nonlinear programs,” *Mathematical Programming*, vol. 36, no. 3, pp. 307–339, 1986.
[13] F. You and I. E. Grossmann, “Mixed-integer nonlinear programming models and algorithms for large-scale supply chain design with stochastic inventory management,” *Industrial and Engineering Chemistry Research*, vol. 47, no. 20, pp. 7802–7817, 2008.
[14] M. M. Monteiro, *Modelagem da Cadeia Logística de Suprimentos dos Materiais Comuns e de Subsistência da Marinha do Brasil*, M. Sc. Industrial Engineering Dissertation, UFF, 2002.
[15] L. T. Biegler and I. E. Grossmann, “Retrospective on optimization,” *Computers and Chemical Engineering*, vol. 28, no. 8, pp. 1169–1192, 2004.