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Research Article

Performance Analysis of IEEE 802.11 DCF under Nonsaturation Condition

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Carrier sense multiple access with collision avoidance (CSMA/CA) methods are considered to be attractive MAC protocols for wireless LANs. IEEE 802.11 distributed coordination function (DCF) is a random channel access scheme based on CSMA/CA method and the binary slotted exponential backoff procedure to reduce the packet collision. In this paper, we propose a new analytical model for a nonsaturated IEEE 802.11 DCF network and evaluate its performance. We verify our model using simulations and show that our results agree with the simulations.

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1. Introduction

In recent years, a rapid evolution in wireless local area networks (WLANs) has been witnessed. IEEE 802.11-based medium access control (MAC) protocols have been widely used for WLANs. The IEEE 802.11 MAC defines the contention-based distributed coordination function (DCF) [1, 2]. In order to prevent the interference and confirm a successful transmission, the DCF includes two access techniques: basic and request-to-send/clear-to-send (RTS/CTS) access mechanisms. The basic access mechanism is a two-way handshaking method where the transmitter transmits a data frame and the receiver replies with an acknowledgment (ACK) frame to confirm a successful transmission. In addition to the basic access method, the RTS/CTS mechanism reserves the medium before transmitting a data frame by transmitting an RTS frame and replying with a CTS frame.

Many works on the modeling of IEEE 802.11 DCF have been studied [3–11]. Bianchi [3] was the first to derive a model that incorporates the exponential backoff process inherent to IEEE 802.11 as a bidimensional Markov chain. The bidimensional Markov chain model has become a common method to study the performance of the IEEE 802.11 MAC protocol and its enhancements. In [5–7], the backoff time was assumed to be geometrically distributed with a parameter related to the contention status of the medium. Wu et al. [9] followed the same Markov chain model and considered packet retransmission limits to avoid overestimating the throughput of 802.11 as in [3]. However, in most of the analytical papers in the literature, analyses have been conducted under saturation environment, that is, each station has at least one packet to transmit at each time. There is some previous modeling work for nonsaturation environment. For nonsaturation analysis, Ziouva and Antonakopoulos [12] presented an extension of the saturated Bianchi model [4] for nonsaturation analysis. In the analysis, a station is assumed to be empty immediately after the station starts to transmit a packet. In [13], the authors extended [4] for the nonsaturation traffic condition with post-backoff. In the analysis, the MAC buffer is assumed to be always empty when a buffered packet enters a backoff procedure. This assumption is not realistic even when the traffic load is very low and not bursty. Furthermore, the fact that the distributed interframe space (DIFS) is not the integral multiples of an idle slot time was not taken into account. Tickoo and Sikdar [14] developed a simple model using parameters from the saturated model, but the results appear to exhibit poor accuracy. The model in [15] attempts to integrate Bianchi's model with a queueing model, and requires a solution of a fixed-point equation spanning parameters of both the Markov chain and the queueing model. Alizadeh-Shabdiz and Subramaniam [16] extended the Markov chain of [4] to obtain an M/G/1 queueing model for nonsaturation case. Liaw et al. [17] introduced an idle state, not present in Bianchi's model [4], accounting for the case in which the station buffer is empty after a successful completion of a packet transmission. The probability that there is at least a packet in the buffer after a successful transmission is assumed to be constant and independent of the access delay of the transmitted packet. Based on the Markovian state transition model proposed by Liaw et al. [17], Daneshgaran et al. [18] proposed a linear model of the throughput of the IEEE 802.11 DCF protocol under nonsaturation traffic conditions.

In this paper, we propose a new mathematical model to study nonsaturation performance of the DCF more accurately, and then we show the accuracy of our model via computer simulations. Even though the post-backoff procedure is not considered in our proposed analysis model, our approach can be justified by the mathematical tractability of the problem.

2. Backoff procedure in IEEE 802.11 DCF

The DCF is the fundamental access method of the IEEE 802.11. It is based on the CSMA/CA and a backoff procedure to reduce the collision probability between multiple stations accessing the channel. The CSMA/CA mechanism defines two channel states: idle and busy. If a station senses no transmission on the channel, it considers the channel state as idle; otherwise it considers the channel state as busy. In this section, we focus on the basic access mechanism of IEEE 802.11 DCF. Suppose that there are no pending packets in a station. When the station has first a packet to transmit, it starts its carrier sensing to determine the current state of the channel. If the channel is sensed as idle for a period of time equal to DIFS, the station transmits the first arrived packet immediately. If not, the station must wait until the channel becomes idle and subsequently remains idle for a DIFS period. After that the station has to wait a random backoff interval before it is permitted to transmit its packet.

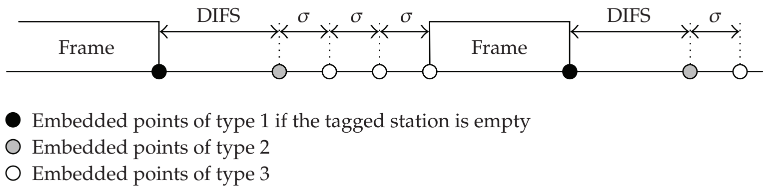

Figure 1: Embedded points.

The remaining backoff interval is indicated by the backoff counter of the station. Initially, the backoff counter is uniformly chosen in a range $[0, \mathrm{CW}_{\min} - 1]$, where $\mathrm{CW}_{\min}$ is known as the minimum contention window size. Whenever the station senses channel as idle during a slot time, the backoff counter is decremented by one. If the channel is sensed as busy, the backoff counter is frozen. After the channel becomes idle again for a period of DIFS, the station resumes the decrement of the backoff counter. When the backoff counter reaches zero, the station transmits its packet and waits for reception of an ACK from the destination after a time interval called short interframe space (SIFS). When the station receives the ACK within the ACK timeout interval, it will immediately perform a backoff procedure, known as post-backoff, even though no additional packets are queued. If the transmission fails, the station doubles the contention window size up to a predefined maximum value $\mathrm{CW}_{\max}$ and repeats the backoff procedure.

3. Markov chain model

We assume the following conditions: (1) a single hop WLAN in which all stations are in the transmission range of each other such that we do not have any hidden terminal, (2) no post-backoff procedure, and (3) ideal channel condition, that is, no capture effect. Since channel conditions are assumed to be ideal, transmission errors are a result of packet collision only. Also, we assume that all stations are identical with respect to their parameters and arrival rates, and that there is no retransmission limit.

To analyze the nonsaturation performance of DCF, we observe the state of a given tagged station at the following 3 types of embedded points (see Figure 1).

Type 1: the end of each time period during which the channel is busy due to packet transmissions, regardless of success or collision, if the tagged station is empty at that time.

Type 2: DIFS after each time period during which the channel is busy due to packet transmission, regardless of success or collision.

Type 3: every slot times after the embedded points of type 2 until the channel is sensed to be busy.

We assume that the duration of DIFS is equal to $2\sigma + \sigma_2$ [1], where σ denotes one slot time and σ_2 denotes the duration of SIFS. For notational convenience, we denote SIFS and DIFS as the duration of SIFS and DIFS, respectively.

At each embedded point t, we define a stochastic process $s(t)$ as follows:

$$s(t) = \begin{cases} -2 & \text{if } t \text{ is an embedded type 1;} \\ -1 & \text{if } t \text{ is an embedded point of type 2 or type 3} \\ & \text{at which the backoff counter of the tagged station is not activated yet;} \\ i & \text{if } t \text{ is an embedded point at which the backoff stage is } i \text{ for } 0 \le i \le m, \\ & \text{where } m \text{ is the maximum backoff stage.} \end{cases} \tag{3.1}$$

Since we do not consider post-backoff procedure, the backoff counter of a station is inactivated when the station becomes empty, and reactivated DIFS after the time period during which the channel is sensed to be busy. Let $b(t)$ indicate the following:

(1) when $s(t) = -2$, $b(t) = 0$;

(2) when $s(t) = -1$,

$$b(t) = \begin{cases} 0 & \text{if } 2\sigma \le t_p < \text{DIFS}, \\ 1 & \text{if DIFS} - \sigma \le t_p < 2\sigma, \\ 2 & \text{if } \sigma \le t_p < \text{DIFS} - \sigma, \\ 3 & \text{if DIFS} - 2\sigma \le t_p < \sigma, \\ 4 & \text{if } 0 < t_p < \text{DIFS} - 2\sigma, \\ 5 & \text{if } t_p = 0, \end{cases} \tag{3.2}$$

where t_p denotes the time period elapsed since the first packet arrived at the tagged station which was empty;

(3) when $0 \le s(t) \le m$, $b(t)$ represents the value of the backoff counter at t.

Note that, when $s(t) = -1$, the value $b(t)$ indicates the time at which the first packet enters an empty station. An empty station waits until a packet enters its buffer. Upon the entrance of the packet, the station starts its carrier sensing to determine the current state of the channel. If the channel is idle for a period of time DIFS, the station transmits the packet without experiencing any further backoff procedure. Thus, the transmission epoch of the first packet is affected by the time the packet enters the station. For example, when $s(t) = -1$ and $b(t) = 0$, the packet may be transmitted during the first σ_2 period of the next slot; when $s(t) = -1$ and $b(t) = 1$, the packet may be transmitted during the last σ_1 period of the next slot, where $\sigma_1 = \sigma - \text{SIFS}$; when $s(t) = -1$ and $b(t) = 2$ or 3, the packet may be transmitted during the next slot after one; when $s(t) = -1$ and $b(t) = 4$, the packet may be transmitted during the first σ_2 period of the next slot after two (see Figure 2). The affection is deeper in light traffic conditions than in heavy traffic conditions.

At embedded point t, the state of the tagged station is defined by $(s(t), b(t))$. For example, the state $(-2, 0)$ represents that the tagged station does not have any packet to

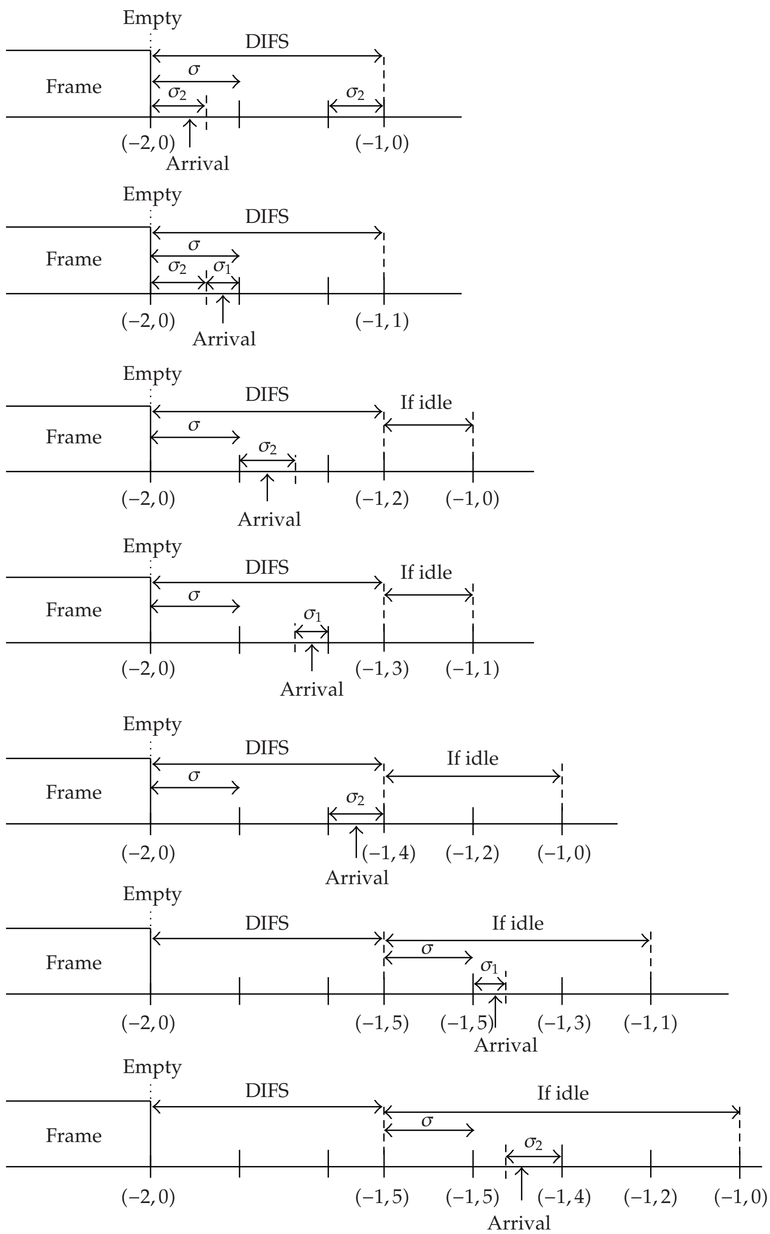

Figure 2: Timing diagram of the embedded points.

transmit immediately after the transmission of a packet at any station in the network. The state $(-1, j)$, $j = 0, 1, \ldots, 5$, represents that the backoff procedure is not activated yet, where j indicates when the first packet arrived after the state $(-2, 0)$. For $i = 0, 1, \ldots, m$, the state (i, j) denotes the backoff state, where the value i is the backoff stage, the value j, $0 \le j \le \mathrm{CW}_i$, is the possible backoff counter value at the backoff stage i, and CW_i is the size of contention window at stage i.

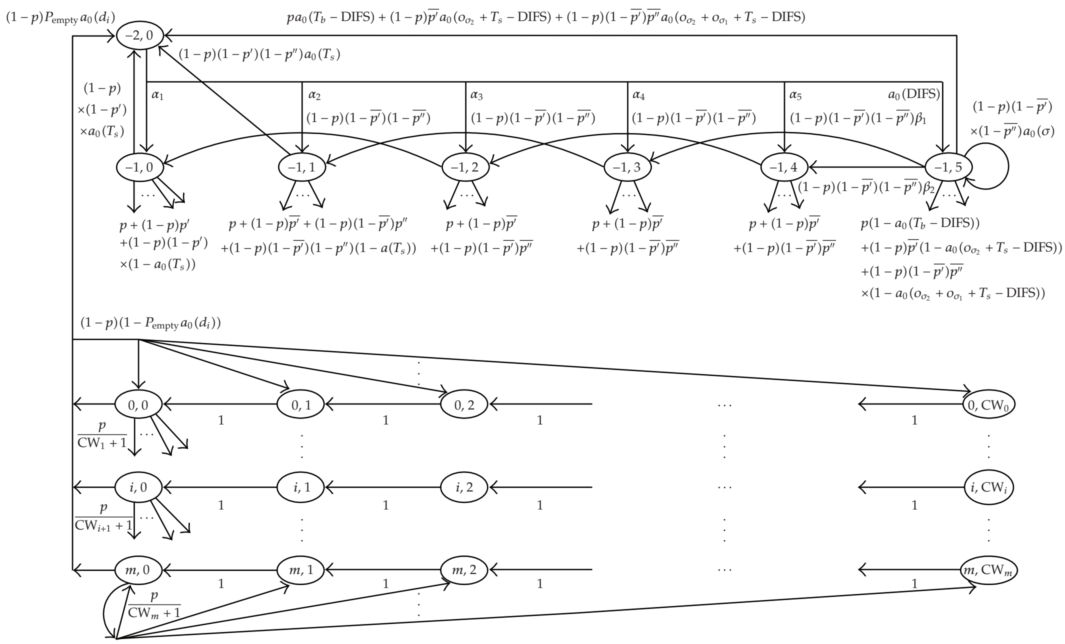

Figure 3: State transition diagram of the proposed IEEE 802.11 DCF model.

The stochastic process $\{(s(t), b(t))\}$ constitutes a 2-dimensional Markov chain, which models the tagged station. The state transition diagram of the Markov chain of the tagged station is presented in Figure 3, where we use the following notations.

(1) α_1, α_2, α_3, α_4, and α_5: probabilities that, after the state of a station becomes $(-2, 0)$, the first packet arrives at the empty station between 0 and σ_2, between σ_2 and σ, between σ and $\sigma + \sigma_2$, between $\sigma + \sigma_2$ and 2σ, and between 2σ and DIFS, respectively. Note that, because the time period between an embedded point with state $(-2, 0)$ and the next embedded point is always DIFS, the elapsed time t_p after the first packet arrivals is $2\sigma \le t_p < \text{DIFS}$ with probability α_1, $\text{DIFS} - \sigma \le t_p < 2\sigma$ with probability α_2, $\sigma \le t_p < \text{DIFS} - \sigma$ with probability α_3, $\text{DIFS} - 2\sigma \le t_p < \sigma$ with probability α_4, and $0 < t_p < \text{DIFS} - 2\sigma$ with probability α_5 at the next embedded point, at which the state of the station becomes $(-1, 0)$, $(-1, 1)$, $(-1, 2)$, $(-1, 3)$, and $(-1, 4)$, respectively (see Figure 2). For example, when packets arrive at the station according to a Poisson process with rate λ, we have

$$\begin{aligned}
\alpha_1 &= 1 - e^{-\lambda\sigma_2}, \\
\alpha_2 &= e^{-\lambda\sigma_2} - e^{-\lambda\sigma}, \\
\alpha_3 &= e^{-\lambda\sigma} - e^{-\lambda(\sigma+\sigma_2)}, \\
\alpha_4 &= e^{-\lambda(\sigma+\sigma_2)} - e^{-2\lambda\sigma}, \\
\alpha_5 &= e^{-2\lambda\sigma} - e^{-\lambda\text{DIFS}}.
\end{aligned} \tag{3.3}$$

(2) β_1 (resp., β_2): probability that, after the state of a station becomes $(-1,5)$, the first packet arrives at the empty station between 0 and σ_1 (resp., σ_1 and σ). When packets arrive at the station according to a Poisson process with rate λ, we have

$$\begin{aligned}\beta_1 &= 1 - e^{-\lambda\sigma_1},\\ \beta_2 &= e^{-\lambda\sigma_1} - e^{-\lambda\sigma}.\end{aligned} \tag{3.4}$$

(3) $a_0(l)$: probability that there are no packet arrivals into a station for time period l. When packets arrive at the station according to a Poisson process with rate λ, we have

$$a_0(l) = e^{-\lambda l}. \tag{3.5}$$

For analytical simplicity, the probability $a_0(l)$ is approximated as $a_0(E[l])$ in this section.

(4) o_l: random variable representing the time period until the first packet arrives, given that at least one packet arrives at a station for time period l. Let $o_{l,k}$, $k \geq 1$, be the random variable representing the time period until the first packet arrives, given that there are k arrivals for time period l. When packets arrive at the station according to a Poisson process with rate λ, the random variable $o_{l,k}$ is the first-order statistic corresponding to a random sample of size k from a uniform distribution over the interval $[0,l]$, and its mean is $l/(k+1)$ [19]. Hence,

$$\overline{o_l} \equiv E[o_l] = \sum_{k=1}^{\infty} \frac{l}{k+1} \frac{(\lambda l)^k e^{-\lambda l}}{k!\left(1 - e^{-\lambda l}\right)} = \frac{1 - (1 + \lambda l)e^{-\lambda l}}{\lambda\left(1 - e^{-\lambda l}\right)}. \tag{3.6}$$

(5) p: probability that, from the tagged station's point of view, at least one of the other stations transmits a packet at the beginning of a slot time.

(6) p' (resp., $\overline{p'}$): conditional probability that one of the stations except the tagged station transmits a packet during the first σ_2 period of a slot time, given that the tagged station has a packet (resp., no packets) available for transmission during the time period and no stations transmit a packet at the beginning of the slot time.

(7) p'' (resp., $\overline{p''}$): conditional probability that one of the stations except the tagged station transmits a packet during the last σ_1 period of a slot time, given that the tagged station has a packet (resp., no packets) available for transmission during the time period and no stations transmit a packet at the beginning of the slot time and during the first σ_2 period of the slot time.

(8) τ: probability that a station transmits a packet at the beginning of a slot time.

(9) τ': probability that a station has a packet available for transmission during the first σ_2 period of a slot time, given that the station did not transmit a packet at the beginning of the slot time.

(10) τ'': probability that a station has a packet available for transmission during the last σ_1 period of a slot time, given that the station did not transmit a packet at the beginning of the slot time and during the first σ_2 period of the slot time.

(11) d_i: random variable representing the time period from the beginning of stage 0 at a station to the completion of a successful transmission from the station, given that the transmission occurs at stage i.

(12) P_{empty}: probability that a station has only one packet when the backoff stage of the station becomes 0.

Since the probability that there are no packet arrivals during the time interval between when a packet becomes the first packet in the buffer and when the packet's transmission is finished is zero under saturation traffic condition, that is, $a_0(d_i) = 0$ for $i = 0, 1, \ldots, m$, the states $(-2, 0)$ and $(-1, j)$ of the Markov chain $\{(s(t), b(t))\}$ are transient states under saturation traffic condition. Thus, the steady-state probabilities of the Markov chain $\{(s(t), b(t))\}$ under saturation traffic condition are the same as those of Bianch's model [4]. Hence, this paper focuses on the stations under saturation traffic conditions.

We consider the number of contending stations as fixed, defined as n. Let l_i be the time period between when a station enters stage i and when the backoff counter of the station becomes 0 at stage i. Note that the backoff counter decreases by one for each idle time slot and is suspended when the channel is busy. We consider the mean time that elapses for one decrement for the backoff counter: the probability that at least one of the channels except the tagged station transmits a packet at the beginning of a slot time is p, and the mean elapsed time for one decrement in this case is $T_b + \sigma$, where T_b denotes the required time for the freezing time due to the busy channel. The time period T_b is obtained as

$$T_b = \frac{(n-1)\tau(1-\tau)^{n-2}}{p} T_s + \left[1 - \frac{(n-1)\tau(1-\tau)^{n-2}}{p}\right] T_c, \tag{3.7}$$

where T_s denotes the required time for the successful transmission of a packet and T_c is the wasting time due to the collision of a transmitted packet. The probability that one of the channels except the tagged station transmits a packet during the first σ_2 period of a slot time is $(1-p)\overline{p'}$, and the mean elapsed time in this case is approximated as $\overline{o_{\sigma_2}} + T_s + \sigma$. The probability that one of the channels except the tagged station transmits a packet during the last σ_1 period of a slot time is $(1-p)(1-\overline{p'})\overline{p''}$, and the mean elapsed time in this case is approximated as $\sigma_2 + \overline{o_{\sigma_1}} + T_s + \sigma$. Thus, the mean $\overline{l_i} \equiv E[l_i]$ of l_i can be obtained as

$$\begin{aligned}
\overline{l_i} &= \frac{\mathrm{CW}_i}{2} \times \Big[p(T_b + \sigma) + (1-p)\overline{p'}\left(\overline{o_{\sigma_2}} + T_s + \sigma\right) \\
&\qquad + (1-p)(1-\overline{p'})\overline{p''}\left(\sigma_2 + \overline{o_{\sigma_1}} + T_s + \sigma\right) + (1-p)(1-\overline{p'})(1-\overline{p''})\sigma\Big] \\
&= \frac{\mathrm{CW}_i}{2} \times \Big[\sigma + pT_b + (1-p)\overline{p'}\left(\overline{o_{\sigma_2}} + T_s\right) + (1-p)(1-\overline{p'})\overline{p''}\left(\sigma_2 + \overline{o_{\sigma_1}} + T_s\right)\Big].
\end{aligned} \tag{3.8}$$

Using $\overline{l_i}$, the mean $\overline{d_i}$ of d_i can be obtained as

$$\overline{d_i} = T_s - \mathrm{DIFS} + iT_c + \sum_{k=0}^{i} \overline{l_k} \tag{3.9}$$

for $0 \le i < m$; assuming no retransmission limit, we obtain

$$\overline{d_m} = T_s - \text{DIFS} + \left(m + \frac{p}{1-p}\right)T_c + \sum_{k=0}^{m-1}\overline{l_k} + \frac{\overline{l_m}}{1-p}. \tag{3.10}$$

4. Mathematical analysis

Let $b_{i,j}$ be the stationary probability of the described Markov chain. The stationary probabilities satisfy the following balance equations:

$$\begin{gathered}
b_{-1,0} = \alpha_1 b_{-2,0} + (1-p)(1-\overline{p'})(1-\overline{p''})b_{-1,2};\\
b_{-1,1} = \alpha_2 b_{-2,0} + (1-p)(1-\overline{p'})(1-\overline{p''})b_{-1,3};\\
b_{-1,2} = \alpha_3 b_{-2,0} + (1-p)(1-\overline{p'})(1-\overline{p''})b_{-1,4};\\
b_{-1,3} = \alpha_4 b_{-2,0} + (1-p)(1-\overline{p'})(1-\overline{p''})\beta_1 b_{-1,5};\\
b_{-1,4} = \alpha_5 b_{-2,0} + (1-p)(1-\overline{p'})(1-\overline{p''})\beta_2 b_{-1,5};\\
b_{-1,5} = a_0(\text{DIFS})b_{-2,0} + (1-p)(1-\overline{p'})(1-\overline{p''})a_0(\sigma)b_{-1,5};\\
b_{i,0} = p^i b_{0,0}, \quad 1 \le i < m;\\
b_{m,0} = \frac{p^m}{1-p}b_{0,0};\\
b_{i,k} = \frac{\text{CW}_i + 1 - k}{\text{CW}_i + 1}b_{i,0}, \quad 0 \le i \le m,\ 0 \le k \le \text{CW}_i.
\end{gathered} \tag{4.1}$$

From state transition diagram, the expression for $p_{0,0}$ is given by

$$\begin{aligned}
b_{0,0} &= \{p + (1-p)p' + (1-p)(1-p')(1-a_0(T_s))\}b_{-1,0}\\
&\quad + \{p + (1-p)\overline{p'} + (1-p)(1-\overline{p'})p'' + (1-p)(1-\overline{p'})(1-p'')(1-a_0(T_s))\}b_{-1,1}\\
&\quad + \{p + (1-p)\overline{p'} + (1-p)(1-\overline{p'})\overline{p''}\}(b_{-1,2} + b_{-1,3} + b_{-1,4})\\
&\quad + \{p(1-a_0(T_b - \text{DIFS})) + (1-p)\overline{p'}(1-a_0(o_{\sigma_2} + T_s - \text{DIFS}))\\
&\qquad + (1-p)(1-\overline{p'})\overline{p''}(1-a_0(\sigma_2 + o_{\sigma 1} + T_s - \text{DIFS}))\}b_{-1,5}\\
&\quad + \sum_{i=0}^{m}(1-p)(1-P_{\text{empty}}a_0(d_i))p_{i,0}\\
&= \{p + (1-p)p' + (1-p)(1-p')(1-a_0(T_s))\}b_{-1,0}\\
&\quad + \{p + (1-p)\overline{p'} + (1-p)(1-\overline{p'})p'' + (1-p)(1-\overline{p'})(1-p'')(1-a_0(T_s))\}b_{-1,1}\\
&\quad + \{p + (1-p)\overline{p'} + (1-p)(1-\overline{p'})\overline{p''}\}(b_{-1,2} + b_{-1,3} + b_{-1,4})
\end{aligned}$$

$$
\begin{aligned}
&+\left\{p\left(1-a_0\left(T_b-\mathrm{DIFS}\right)\right)+(1-p)\overline{p'}\left(1-a_0\left(o_{\sigma_2}+T_s-\mathrm{DIFS}\right)\right)\right.\\
&\quad\left.+(1-p)\left(1-\overline{p'}\right)\overline{p''}\left(1-a_0\left(\sigma_2+o_{\sigma 1}+T_s-\mathrm{DIFS}\right)\right)\right\}b_{-1,5}\\
&+b_{0,0}-(1-p)P_{\mathrm{empty}}\left[\sum_{i=0}^{m-1}a_0\left(d_i\right)p^i+\frac{p^m}{1-p}a_0\left(d_m\right)\right]b_{0,0}.
\end{aligned}
\tag{4.2}
$$

Thus,

$$
\begin{aligned}
b_{0,0}=&\left[\left\{p+(1-p)p'+(1-p)\left(1-p'\right)\left(1-a_0\left(T_s\right)\right)\right\}b_{-1,0}\right.\\
&+\left\{p+(1-p)\overline{p'}+(1-p)\left(1-\overline{p'}\right)p''+(1-p)\left(1-\overline{p'}\right)\left(1-p''\right)\left(1-a_0\left(T_s\right)\right)\right\}b_{-1,1}\\
&+\left\{p+(1-p)\overline{p'}+(1-p)\left(1-\overline{p'}\right)\overline{p''}\right\}\left(b_{-1,2}+b_{-1,3}+b_{-1,4}\right)\\
&+\left\{p\left(1-a_0\left(T_b-\mathrm{DIFS}\right)\right)+(1-p)\overline{p'}\left(1-a_0\left(o_{\sigma_2}+T_s-\mathrm{DIFS}\right)\right)\right.\\
&\quad\left.\left.+(1-p)\left(1-\overline{p'}\right)\overline{p''}\left(1-a_0\left(\sigma_2+o_{\sigma 1}+T_s-\mathrm{DIFS}\right)\right)\right\}b_{-1,5}\right]\\
&\times\frac{1}{(1-p)P_{\mathrm{empty}}\left[\sum_{i=0}^{m-1}a_0\left(d_i\right)p^i+\left(p^m/(1-p)\right)a_0\left(d_m\right)\right]}.
\end{aligned}
\tag{4.3}
$$

Hence, each of the stationary probabilities can be expressed in terms of $b_{-2,0}$, which can be obtained from the normalization condition

$$
b_{-2,0}+\sum_{k=0}^{5}b_{-1,k}+\sum_{i=0}^{m}\sum_{k=0}^{\mathrm{CW}_i}b_{i,k}=1.
\tag{4.4}
$$

Each of τ, τ', and τ'' can be expressed as a function of the stationary probabilities. Since the probability that an embedded point is the beginning of a slot time is $1-b_{-2,0}$ and the probability that a station transmits a packet at the embedded time is $\sum_{i=0}^{m}b_{i,0}$, the probability τ is given by

$$
\tau=\frac{\sum_{i=0}^{m}b_{i,0}}{1-b_{-2,0}}.
\tag{4.5}
$$

Since $(1-r)r'$ is the probability that a station has a packet available for transmission during the first σ_2 period of a slot time and that is given by $b_{-1,0}/(1-b_{-2,0})$, the probability τ' is given by

$$
\tau'=\frac{b_{-1,0}}{\left(1-b_{-2,0}\right)(1-\tau)}.
\tag{4.6}
$$

Since $(1-\tau)(1-\tau')\tau''$ is the probability that a station has a packet available for transmission during the last σ_1 period of a slot time and that is given by $b_{-1,1}/(1-b_{-2,0})$, the probability τ'' is given by

$$
\tau''=\frac{b_{-1,1}}{\left(1-b_{-2,0}\right)(1-\tau)\left(1-\tau'\right)}.
\tag{4.7}
$$

Having obtained τ, τ', and τ'', the probabilities p, p', $\overline{p'}$, p'', and $\overline{p''}$ can be determined as follows:

$$
\begin{aligned}
p &= 1-(1-\tau)^{n-1},\\
p' &= \sum_{i=1}^{n-1}\frac{i}{i+1}\binom{n-1}{i}(\tau')^{i}(1-\tau')^{n-i-1},\\
\overline{p'} &= 1-(1-\tau')^{n-1},\\
p'' &= \sum_{i=1}^{n-1}\frac{i}{i+1}\binom{n-1}{i}(\tau'')^{i}(1-\tau'')^{n-i-1},\\
\overline{p''} &= 1-(1-\tau'')^{n-1}.
\end{aligned}
\tag{4.8}
$$

Let $h_{i,j}$ denote the mean sojourn time at state (i,j). Then,

$$
\begin{aligned}
h_{-2,0} &= \mathrm{DIFS},\\
h_{-1,0} &= pT_b+(1-p)p'\left(o_{\sigma_2}+T_s\right)+(1-p)(1-p')a_0\left(T_s\right)\left(o_{\sigma_2}+T_s-\mathrm{DIFS}\right)\\
&\quad+(1-p)(1-p')\left(1-a_0\left(T_s\right)\right)\left(o_{\sigma_2}+T_s\right),\\
h_{-1,1} &= pT_b+(1-p)\overline{p'}\left(o_{\sigma_2}+T_s\right)+(1-p)(1-\overline{p'})p''\left(\sigma_2+o_{\sigma_1}+T_s\right)\\
&\quad+(1-p)(1-\overline{p'})(1-p'')a_0\left(T_s\right)\left(\sigma_2+o_{\sigma_1}+T_s-\mathrm{DIFS}\right)\\
&\quad+(1-p)(1-\overline{p'})(1-p'')\left(1-a_0\left(T_s\right)\right)\left(\sigma_2+o_{\sigma_1}+T_s\right),\\
h_{-1,2} &= pT_b+(1-p)\overline{p'}\left(o_{\sigma_2}+T_s\right)+(1-p)(1-\overline{p'})\overline{p''}\left(\sigma_2+o_{\sigma_1}+T_s\right)+(1-p)(1-\overline{p'})(1-\overline{p''})\sigma,\\
h_{-1,k} &= h_{-1,2},\quad k=3,4,\\
h_{-1,5} &= pa_0\left(T_b-\mathrm{DIFS}\right)\left[T_b-\mathrm{DIFS}\right]+p\left[1-a_0\left(T_b-\mathrm{DIFS}\right)\right]T_b\\
&\quad+(1-p)\overline{p'}a_0\left(o_{\sigma_2}+T_s-\mathrm{DIFS}\right)\left[o_{\sigma_2}+T_s-\mathrm{DIFS}\right]\\
&\quad+(1-p)\overline{p'}\left[1-a_0\left(o_{\sigma_2}+T_s-\mathrm{DIFS}\right)\right]\left[o_{\sigma_2}+T_s\right]\\
&\quad+(1-p)(1-\overline{p'})\overline{p''}a_0\left(\sigma_2+o_{\sigma_1}+T_s-\mathrm{DIFS}\right)\left[\sigma_2+o_{\sigma_1}+T_s-\mathrm{DIFS}\right]\\
&\quad+(1-p)(1-\overline{p'})\overline{p''}\left[1-a_0\left(\sigma_2+o_{\sigma_1}+T_s-\mathrm{DIFS}\right)\right]\left[\sigma_2+o_{\sigma_1}+T_s\right]\\
&\quad+(1-p)(1-\overline{p'})(1-\overline{p''})\sigma,\\
h_{i,0} &= pT_c+(1-p)T_s-(1-p)P_{\mathrm{empty}}a_0\left(d_i\right)\mathrm{DIFS},\quad 0\le i\le m,\\
h_{i,k} &= h_{-1,2},\quad 0\le i\le m,\ 1\le k\le \mathrm{CW}_i.
\end{aligned}
\tag{4.9}
$$

The probability P_{empty} can be determined based on the fact that, by PASTA theorem and Burke's theorem [20], the stationary distributions of the tagged station at an arbitrary time point and immediately after an arbitrary completion time point of successful transmission are identical. The probability that the tagged station is empty at an arbitrary time point is

$$\begin{aligned}
&\Big[b_{-2,0}\{1-a_0(\text{DIFS})\}\\
&\quad + b_{-1,5}\Big\{1-pa_0\big(T_b-\text{DIFS}\big)-(1-p)\overline{p'}a_0\big(o_{\sigma_2}+T_s-\text{DIFS}\big)\\
&\qquad -(1-p)(1-\overline{p'})\overline{p''}a_0\big(\sigma_2+o_{\sigma_1}+T_s-\text{DIFS}\big)-(1-p)(1-\overline{p'})(1-\overline{p''})a_0(\sigma)\Big\}\Big]\\
&\quad\times\frac{1}{\lambda\big[b_{-2,0}h_{-2,0}+\sum_{k=0}^{5}b_{-1,k}\,h_{-1,k}+\sum_{i=0}^{m}\sum_{k=0}^{\text{CW}_i}b_{i,k}h_{i,k}\big]},
\end{aligned} \tag{4.10}$$

and the probability that the tagged station is empty immediately after an arbitrary completion time point of successful transmission at the tagged station is

$$\frac{(1-p)P_{\text{empty}}\sum_{i=0}^{m}a_0(d_i)b_{i,0}+(1-p)(1-p')a_0(T_s)b_{-1,0}+(1-p)(1-\overline{p'})(1-p'')a_0(T_s)b_{-1,1}}{b_{0,0}+(1-p)(1-p')b_{-1,0}+(1-p)(1-\overline{p'})(1-p'')b_{-1,1}}. \tag{4.11}$$

Thus,

$$P_{\text{empty}}=\frac{\text{Num. of } P_{\text{empty}}}{(1-p)\sum_{i=0}^{m}a_0(d_i)b_{i,0}}, \tag{4.12}$$

where

$$\begin{aligned}
\text{Num. of } P_{\text{empty}} = {}& \frac{1}{\lambda\big[b_{-2,0}h_{-2,0}+\sum_{k=0}^{5}b_{-1,k}\,h_{-1,k}+\sum_{i=0}^{m}\sum_{k=0}^{\text{CW}_i}b_{i,k}h_{i,k}\big]}\\
&\times\Big[b_{-2,0}\{1-a_0(\text{DIFS})\}\\
&\quad + b_{-1,5}\Big\{1-pa_0\big(T_b-\text{DIFS}\big)-(1-p)\overline{p'}a_0\big(o_{\sigma_2}+T_s-\text{DIFS}\big)\\
&\qquad -(1-p)(1-\overline{p'})\overline{p''}a_0\big(\sigma_2+o_{\sigma_1}+T_s-\text{DIFS}\big)\\
&\qquad -(1-p)(1-\overline{p'})(1-\overline{p''})a_0(\sigma)\Big\}\Big]\\
&\times\big[b_{0,0}+(1-p)(1-p')b_{-1,0}+(1-p)(1-\overline{p'})(1-p'')b_{-1,1}\big]\\
&-(1-p)(1-p')a_0(T_s)b_{-1,0}-(1-p)(1-\overline{p'})(1-p'')a_0(T_s)b_{-1,1}.
\end{aligned} \tag{4.13}$$

From (4.1)–(4.12), the stationary probability $b_{i,k}$ can be found by a numerical method.

The system throughput S, the fraction of time used for successful payload transmission, can be expressed as

$$S=\frac{\text{Num. of } S}{\text{Den. of } S}, \tag{4.14}$$

Table 1: System parameters.

MAC header	272 bits
PHY header	128 bits
ACK length	112 bits + PHY header
Channel bit rate	1 Mbit/s
Propagation delay	1 μs
Slot time	50 μs
SIFS	28 μs
DIFS	128 μs
Initial contention window CW	31
Maximum contention window CW	1023
Maximum backoff stage m	5 or 7

with

$$\begin{aligned}\text{Num. of } S &= \left[P_{\text{tr}}P_s + (1-P_{\text{tr}})P_1 + (1-P_{\text{tr}})(1-P_1)P_2\right]E[\text{Payload}],\\ \text{Den. of } S &= \sigma + P_{\text{tr}}\left[P_sT_s + (1-P_s)T_c\right] + (1-P_{\text{tr}})\left[P_1\left(o_{\sigma_2}+T_s\right) + (1-P_1)P_2\left(\sigma_2 + o_{\sigma_1} + T_s\right)\right],\end{aligned} \tag{4.15}$$

where the meaning of P_{tr}, P_s, P_1, and P_2 is as follows: P_{tr} is the probability that there is at least one transmission at the beginning of a slot time, with n stations contending for the channel, each transmitting with probability τ. Thus,

$$P_{\text{tr}} = 1 - (1-\tau)^n. \tag{4.16}$$

The probability P_s is the conditional probability that a packet transmission occurring on the channel at the beginning of a slot time is successful. This event corresponds to the case in which exactly one station transmits at that time. Thus,

$$P_s = \frac{n\tau(1-\tau)^{n-1}}{P_{\text{tr}}}. \tag{4.17}$$

The probability P_1 is the conditional probability that a packet transmission during the first σ_2 period of a slot time is successful, given that there are no transmission at the beginning of the slot time. This event corresponds to the case in which at least one station is in state $(-1,0)$ at the beginning of the slot time, given that all the stations are in state $(-1,0)$ or state (i,j) for $j \neq 0$ at the beginning of the slot time. Thus,

$$P_1 = 1 - (1-\tau')^n. \tag{4.18}$$

The probability P_2 is the conditional probability that a packet transmission during the last σ_1 period of a slot time is successful, given that there are no transmission at the beginning of the

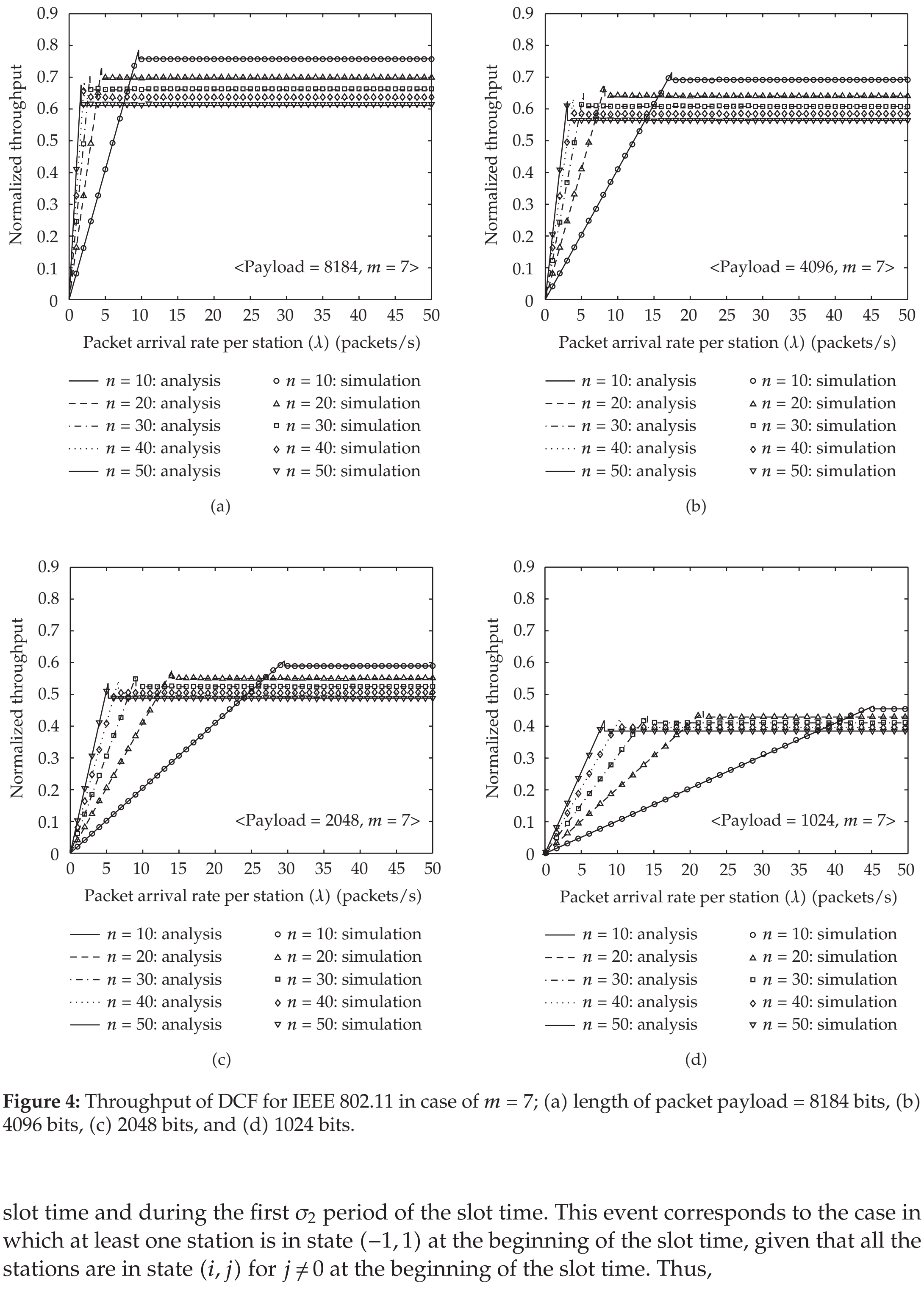

Figure 4: Throughput of DCF for IEEE 802.11 in case of $m = 7$; (a) length of packet payload = 8184 bits, (b) 4096 bits, (c) 2048 bits, and (d) 1024 bits.

slot time and during the first σ_2 period of the slot time. This event corresponds to the case in which at least one station is in state $(-1, 1)$ at the beginning of the slot time, given that all the stations are in state (i, j) for $j \neq 0$ at the beginning of the slot time. Thus,

$$P_2 = 1 - (1 - \tau'')^n. \tag{4.19}$$

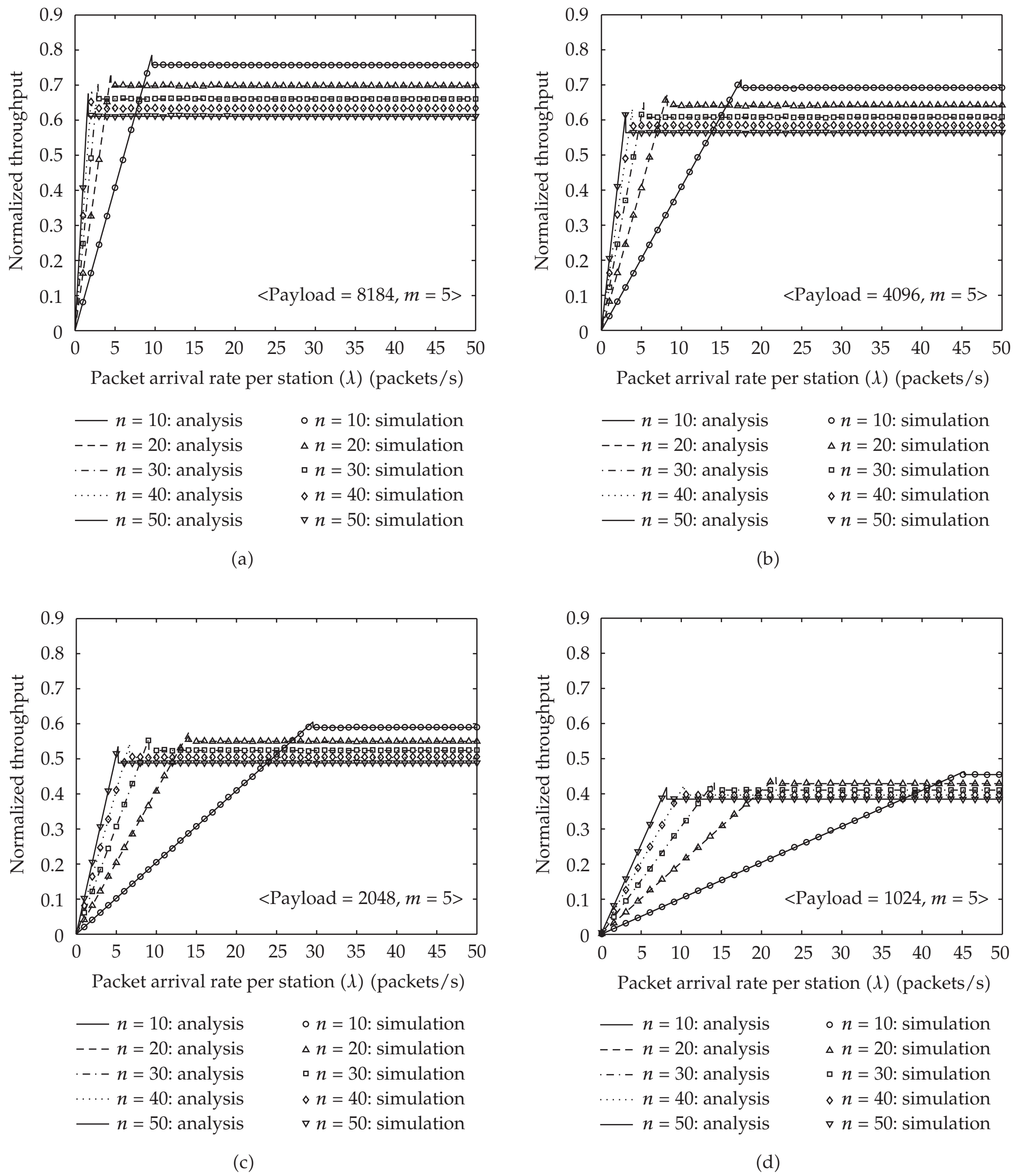

Figure 5: Throughput of DCF for IEEE 802.11 in case of $m = 5$; (a) length of packet payload = 8184 bits, (b) 4096 bits, (c) 2048 bits, and (d) 1024 bits.

5. Numerical examples

In this section, we evaluate the performance of IEEE 802.11 DCF under different maximum backoff stages and different length of packet payload conditions through simulation and analytical results. We have developed a C++ simulator modeling both the DCF protocol details in IEEE 802.11 and the backoff procedures of a specific number of independent transmitting stations. The simulation also takes into account real operations of each transmitting station. In our experiments, we use the parameter setting in Table 1 [4]. Since we assume that no hidden terminals exist, this section deals only with the basic access method without RTS/CTS. We let the packet arrivals to any station be a Poisson process with the same rate λ (packets/s).

Figure 4 shows the throughput of DCF with $m = 7$ as the packet arrival rate λ varies. For given n, the throughout linearly increases as λ increases until the network is saturated. It drastically decreases just before a saturation point, and then maintains constant even though λ increases. Since traffic intensity increases as n increases, the saturation point decreases. In addition, the throughput for large n more sharply increases than that for small n. For all values of n, the maximum throughput is shown to be higher than the saturation throughput. The difference between the maximum throughput and the saturation throughput increases as n increases. For 8184 bits of packet payload and $n = 10$, 20, 30, 40, and 50, the normalized maximum throughputs are about 0.78, 0.73, 0.70, 0.68, and 0.67, respectively, and the normalized saturation throughputs are about 0.76, 0.70, 0.66, 0.63, and 0.61, respectively. In case of the payload of 1024 bits, the normalized maximum throughputs are about 0.463, 0.447, 0.434, 0.425, and 0.418, respectively, and the normalized saturation throughputs are about 0.455, 0.429, 0.411, 0.396, and 0.385 for $n = 10$, 20, 30, 40, and 50, respectively.

For $m = 5$, the throughputs of DCF for IEEE 802.11 with different lengths of packet payload are shown in Figure 5. The general trends of throughputs with $m = 5$ are similar to the case with $m = 7$. In addition, the results show that the normalized maximum throughputs and the normalized saturation throughputs are nearly equal to those with $m = 7$, because the maximum contention window size is enough to effectively resolve collisions under given parameters.

6. Conclusions

We proposed a mathematical model to evaluate the performance of the IEEE 802.11 DCF protocol under unsaturation conditions. Even though the proposed model does not consider the post-backoff procedure, its results are shown to be very close to the simulation results. MAC throughput of DCF linearly increases as packet arrival rate increases until the network is saturated. When the network is saturated, MAC throughput becomes constant for various packet arrival rates.

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