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*Research Article*

# Periodic Solutions of Semilinear Impulsive Periodic System with Time-Varying Generating Operators on Banach Space

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A class of semilinear impulsive periodic systems with time-varying generating operators on Banach space is considered. Using impulsive periodic evolution operator given by us, the $T_0$-periodic $PC$-mild solution is introduced and suitable *Poincaré* operator is constructed. Showing the compactness of *Poincaré* operator and using a new generalized Gronwall inequality with mixed type integral operators given by us, we utilize Leray-Schauder fixed point theorem to prove the existence of $T_0$-periodic $PC$-mild solutions. Our method is an innovation and it is much different from methods of other papers. At last, an example is given for demonstration.



## 1. Introduction

It is well known that impulsive periodic motion is a very important and special phenomenon not only in natural science but also in social science such as climate, food supplement, insecticide population, and sustainable development. No autonomous periodic systems with applications on finite dimensional spaces have been extensively studied. Particularly, no autonomous impulsive periodic systems on finite dimensional spaces are considered and some important results (such as the existence and stability of periodic solutions, the relationship between bounded solution and periodic solution, and robustness by perturbation) are obtained (see [1–5]).

Since the end of last century, many authors including us pay great attention on impulsive systems with time-varying generating operators on infinite dimensional spaces. Particulary, Dr. Ahmed investigated optimal control problems of system governed by artificial heart model, uncertain systems, impulsive system with time-varying generating operators, access control mechanism model, computer network traffic controllers model, and active queue management (AQM) system (see [6–14]). We also gave a series of results for semilinear (strongly nonlinear) impulsive systems with time-varying generating operators and optimal control problems (see [15–18]).

Although, there are some papers on periodic solutions of periodic system with time-varying generating operators on infinite dimensional spaces (see [19–22]), to our knowledge, nonlinear impulsive periodic systems with time-varying generating operators on infinite dimensional (with unbounded operator) have not been extensively investigated. Recently, we consider impulsive periodic system on infinite dimensional spaces. For linear impulsive evolution operator is constructed and $T_0$-periodic $PC$-mild solution is introduced. The existence of periodic solutions and alternative theorem, criteria of Massera type, as well as asymptotical stability and robustness by perturbation are established (see [23–25]).

Herein, we go on studying the semilinear impulsive periodic system with time-varying generating operators

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + f(t,x), \quad t \neq \tau_k, \\ \Delta x(t) &= B_k x(t) + c_k, \quad t = \tau_k, \end{aligned} \tag{1.1}$$

in the parabolic case on infinite dimensional Banach space X, where $\{A(t), t \in [0, T_0]\}$ is a family of closed densely defined linear unbounded operators on X and the resolvent of the unbounded operator $A(t)$ is compact. Time sequence $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k \cdots$, $\lim_{k\to\infty}\tau_k = \infty$, $\tau_{k+\delta} = \tau_k + T_0$, $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$, $k \in \mathbb{Z}_0^+$, $T_0$ is a fixed positive number and $\delta \in \mathbb{N}$ denoted the number of impulsive points between 0 and $T_0$. $f$ is a measurable function from $[0,\infty) \times X$ to $X$ and is $T_0$-periodic in $t$, $B_{k+\delta} = B_k$, $c_{k+\delta} = c_k$. This paper is mainly concerned with the existence of periodic solution for semilinear impulsive periodic system with time-varying generating operators on infinite dimensional Banach space $X$.

In this paper, we use Leray-Schauder fixed point theorem to obtain the existence of periodic solutions for semilinear impulsive periodic system with time-varying generating operators (1.1). First, by virtue of impulsive evolution operator corresponding to linear homogeneous impulsive system with time-varying generating operators, we construct a new *Poincaré* operator $P$ for semilinear impulsive periodic system with time-varying generating operators (1.1), then overcome some difficulties to show the compactness of *Poincaré* operator $P$ which is very important. By a new generalized Gronwall's inequality with mixed-type integral operators given by us, the estimate of fixed point set $\{x = \lambda Px, \lambda \in [0,1]\}$ is established. Therefore, the existence of $T_0$-periodic $PC$-mild solutions for semilinear impulsive periodic system with time-varying generating operators is shown.

In order to obtain the existence of periodic solutions, many authors use Horn's fixed point theorem or Banach fixed point theorem. In [26, 27], by virtue of Horn's fixed point theorem and Banach fixed point theorem, respectively, we also obtain the existence of periodic solutions for impulsive periodic systems. However, the conditions for Horn's fixed point theorem are not easy to be verified sometimes and the conditions for Banach's fixed point theorem are too strong. Here, a new way to show the existence of periodic solutions is given by us, which is much different from our previous works, and other related results in the literature. In addition, the conditions are easier to be verified and more weak compared with some related papers (see [20, 26]). Of course, the new generalized Gronwall's inequality with mixed-type integral operators given by us which can be used in other problems have played an essential role in the study of nonlinear problems on infinite dimensional spaces.

This paper is organized as follows. In Section 2, some results of linear impulsive periodic system with time-varying generating operators and properties of impulsive periodic evolution operator corresponding to homogeneous linear impulsive periodic system with time-varying generating operators are recalled. In Section 3, first, the new generalized Gronwall's inequality with mixed-type integral operator is shown and the $T_0$-periodic $PC$-mild solution for semilinear impulsive periodic system with time-varying generating operators (1.1) is introduced. We construct the suitable *Poincaré* operator $P$ and give the relation between $T_0$-periodic $PC$-mild solution and the fixed point of $P$. After showing the compactness of the *Poincaré* operator $P$ and obtaining the boundedness of the fixed point set $\{x = \lambda Px,\ \lambda \in [0,1]\}$ by virtue of the generalized Gronwall's inequality, we can use Leray-Schauder fixed point theorem to establish the existence of $T_0$-periodic $PC$-mild solutions for semilinear impulsive periodic system with time-varying generating operators. At last, an example is given to demonstrate the applicability of our result.

## 2. Linear impulsive periodic system with time-varying generating operators

In order to study the semilinear impulse periodic system with time-varying generating operators, we first recall some results about linear impulse periodic system with time-varying generating operators here. Let $X$ be a Banach space. $\pounds(X)$ denotes the space of linear operators in $X$; $\pounds_b(X)$ denotes the space of bounded linear operators in $X$. $\pounds_b(X)$ is the Banach space with the usual supremum norm. Define $\widetilde{D} = \{\tau_1, \dots, \tau_\delta\} \subset [0, T_0]$, where $\delta \in \mathbb{N}$ denotes the number of impulsive points between $[0, T_0]$. We introduce $PC([0,T_0];X) \equiv \{x : [0,T_0] \to X \mid x$ is continuous at $t \in [0,T_0] \setminus \widetilde{D},\ x$ is continuous from left and has right-hand limits at $t \in \widetilde{D}\}$ and $PC^1([0,T_0];X) \equiv \{x \in PC([0,T_0];X) \mid \dot{x} \in PC([0,T_0];X)\}$. Set

$$\|x\|_{PC} = \max\left\{\sup_{t\in[0,T_0]}\|x(t+0)\|,\ \sup_{t\in[0,T_0]}\|x(t-0)\|\right\}, \qquad \|x\|_{PC^1} = \|x\|_{PC} + \|\dot{x}\|_{PC}. \tag{2.1}$$

It can be seen that endowed with the norm $\|\cdot\|_{PC}(\|\cdot\|_{PC^1})$, $PC([0,T_0];X)(PC^1([0,T_0];X))$ is a Banach space.

Consider the following homogeneous linear impulsive periodic system with time-varying generating operators:

$$\begin{gathered}\dot{x}(t) = A(t)x(t), \quad t \neq \tau_k,\\ \Delta x(\tau_k) = B_k x(\tau_k), \quad t = \tau_k\end{gathered} \tag{2.2}$$

on Banach space $X$, where $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$, $\{A(t),\ t \in [0,T_0]\}$ is a family of closed densely defined linear unbounded operators on $X$ satisfying the following assumption.

*Assumption A1* (see [28, page 158]). For $t \in [0,T_0]$ one has

($P_1$) The domain $D(A(t)) = D$ is independent of $t$ and is dense in $X$.

($P_2$) For $t \geq 0$, the resolvent $R(\lambda, A(t)) = (\lambda I - A(t))^{-1}$ exists for all $\lambda$ with $\operatorname{Re}\lambda \leq 0$, and there is a constant $M$ independent of $\lambda$ and $t$ such that

$$\|R(\lambda, A(t))\| \leq M(1+|\lambda|)^{-1} \quad \text{for } \operatorname{Re}\lambda \leq 0. \tag{2.3}$$

($P_3$) There exist constants $L > 0$ and $0 < \alpha \leq 1$ such that

$$\|(A(t) - A(\theta))A^{-1}(\tau)\| \leq L|t-\theta|^{\alpha} \quad \text{for } t, \theta, \tau \in [0,T_0]. \tag{2.4}$$

**Lemma 2.1** (see [28, page 159]). *Under the Assumption A1, the Cauchy problem*

$$\dot{x}(t) + A(t)x(t) = 0, \quad t \in (0, T_0] \text{ with } x(0) = x_0 \tag{Eq.1}$$

*has a unique evolution system* $\{U(t,\theta) \mid 0 \le \theta \le t \le T_0\}$ *in* X *satisfying the following properties:*

(1) $U(t,\theta) \in £_b(X)$ *for* $0 \le \theta \le t \le T_0$;

(2) $U(t,r)U(r,\theta) = U(t,\theta)$ *for* $0 \le \theta \le r \le t \le T_0$;

(3) $U(\cdot,\cdot)x \in C(\Delta, X)$ *for* $x \in X$, $\Delta = \{(t,\theta) \in [0,T_0] \times [0,T_0] \mid 0 \le \theta \le t \le T_0\}$;

(4) *For* $0 \le \theta < t \le T_0$, $U(t,\theta) : X \to D$ *and* $t \to U(t,\theta)$ *is strongly differentiable in* X. *The derivative* $(\partial/\partial t)U(t,\theta) \in £_b(X)$ *and it is strongly continuous on* $0 \le \theta < t \le T_0$. *Moreover,*

$$\begin{gathered}\frac{\partial}{\partial t}U(t,\theta) = -A(t)U(t,\theta) \quad \text{for } 0 \le \theta < t \le T_0, \\ \left\| \frac{\partial}{\partial t}U(t,\theta) \right\|_{£_b(X)} = \|A(t)U(t,\theta)\|_{£_b(X)} \le \frac{C}{t-\theta}, \\ \left\|A(t)U(t,\theta)A(\theta)^{-1}\right\|_{£_b(X)} \le C \quad \text{for } 0 \le \theta \le t \le T_0.\end{gathered} \tag{2.5}$$

(5) *For every* $v \in D$ *and* $t \in (0,T_0]$, $U(t,\theta)v$ *is differentiable with respect to* $\theta$ *on* $0 \le \theta \le t \le T_0$

$$\frac{\partial}{\partial \theta}U(t,\theta)v = U(t,\theta)A(\theta)v, \tag{2.6}$$

*and, for each* $x_0 \in X$, *the Cauchy problem* (Eq.1) *has a unique classical solution* $x \in C^1([0,T_0];X)$ *given by*

$$x(t) = U(t,0)x_0, \quad t \in [0,T_0]. \tag{2.7}$$

In addition to Assumption A1, we introduce the following assumptions.

*Assumption A2.* There exits $T_0 > 0$ such that $A(t+T_0) = A(t)$ for $t \in [0,T_0]$.

*Assumption A3.* For $t \ge 0$, the resolvent $R(\lambda, A(t))$ is compact.

Then, we have the following lemma.

**Lemma 2.2.** *Assumptions A1, A2, and A3 hold. Then evolution system* $\{U(t,\theta) \mid 0 \le \theta \le t \le T_0\}$ *in* X *also satisfying the following two properties:*

(6) $U(t+T_0, \theta+T_0) = U(t,\theta)$ *for* $0 \le \theta \le t \le T_0$;

(7) $U(t,\theta)$ *is compact operator for* $0 \le \theta < t \le T_0$.

In order to introduce an impulsive evolution operator and give it's properties, we need the following assumption.

*Assumption B.* For each $k \in \mathbb{Z}_0^+$, $B_k \in £_b(X)$, there exists $\delta \in \mathbb{N}$ such that $\tau_{k+\delta} = \tau_k + T_0$ and $B_{k+\delta} = B_k$.

Consider the following Cauchy's problem

$$\begin{aligned} \dot{x}(t) &= A(t)x(t), \quad t \in [0, T_0] \setminus \widetilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k), \quad k = 1, 2, \ldots, \delta, \\ x(0) &= x_0. \end{aligned} \tag{2.8}$$

For every $x_0 \in X$, $D$ is an invariant subspace of $B_k$, using Lemma 2.1, step by step, one can verify that the Cauchy problem (2.8) has a unique classical solution $x \in PC^1([0, T_0]; X)$ represented by $x(t) = \mathcal{S}(t, 0)x_0$ where $\mathcal{S}(\cdot, \cdot) : \Delta \to \pounds(X)$ given by

$$\mathcal{S}(t, \theta) = \begin{cases} U(t, \theta), & \tau_{k-1} \le \theta \le t \le \tau_k, \\ U(t, \tau_k^+)(I + B_k)U(\tau_k, \theta), & \tau_{k-1} \le \theta < \tau_k < t \le \tau_{k+1}, \\ U(t, \tau_k^+)\left[\prod_{\theta < \tau_j < t}(I + B_j)U(\tau_j, \tau_{j-1}^+)\right](I + B_i)U(\tau_i, \theta), & \\ & \tau_{i-1} \le \theta < \tau_i \le \cdots < \tau_k < t \le \tau_{k+1}. \end{cases} \tag{2.9}$$

The operator $\mathcal{S}(t, \theta)$ $((t, \theta) \in \Delta)$ is called impulsive evolution operator associated with $\{B_k; \tau_k\}_{k=1}^{\infty}$.

The following lemma on the properties of the impulsive evolution operator $\mathcal{S}(t, \theta)((t, \theta) \in \Delta)$ associated with $\{B_k; \tau_k\}_{k=1}^{\infty}$ are widely used in this paper.

**Lemma 2.3** (see [24, Lemma 1]). *Assumptions A1, A2, A3, and B hold. Impulsive evolution operator $\{\mathcal{S}(t, \theta), (t, \theta) \in \Delta\}$ has the following properties.*

(1) *For $0 \le \theta \le t \le T_0$, $\mathcal{S}(t, \theta) \in \pounds_b(X)$, that is, $\sup_{0 \le \theta \le t \le T_0} \|\mathcal{S}(t, \theta)\| \le M_{T_0}$, where $M_{T_0} > 0$.*

(2) *For $0 \le \theta < r < t \le T_0$, $r \neq \tau_k$, $\mathcal{S}(t, \theta) = \mathcal{S}(t, r)\mathcal{S}(r, \theta)$.*

(3) *For $0 \le \theta \le t \le T_0$ and $N \in Z_0^+$, $\mathcal{S}(t + NT_0, \theta + NT_0) = \mathcal{S}(t, \theta)$.*

(4) *For $0 \le t \le T_0$ and $M \in Z_0^+$, $\mathcal{S}(MT_0 + t, 0) = \mathcal{S}(t, 0)[\mathcal{S}(T_0, 0)]^M$.*

(5) *$\mathcal{S}(t, \theta)$ is compact operator for $0 \le \theta < t \le T_0$.*

Here, we note that system (2.2) has a $T_0$-periodic $PC$-mild solution $x$ if and only if $S(T_0, 0)$ has a fixed point. The impulsive evolution operator $\{\mathcal{S}(t, \theta), (t, \theta) \in \Delta\}$ can be used to reduce the existence of $T_0$-periodic $PC$-mild solutions for linear impulsive periodic system with time-varying generating operators to the existence of fixed points for an operator equation. This implies that we can build up the new framework to study the periodic $PC$-mild solutions for the semilinear impulsive periodic system with time-varying generating operators on Banach space.

Now we introduce the $PC$-mild solution of Cauchy's problem (2.8) and $T_0$-periodic $PC$-mild solution of the system (2.2).

*Definition 2.4.* For every $x_0 \in X$, the function $x \in PC([0, T_0]; X)$ given by $x(t) = \mathcal{S}(t, 0)x_0$ is said to be the $PC$-mild solution of the Cauchy problem (2.8).

*Definition 2.5.* A function $x \in PC([0, +\infty); X)$ is said to be a $T_0$-periodic $PC$-mild solution of system (2.2) if it is a $PC$-mild solution of Cauchy's problem (2.8) corresponding to some $x_0$ and $x(t + T_0) = x(t)$ for $t \ge 0$.

Secondly, we recall the following nonhomogeneous linear impulsive periodic system with time-varying generating operators

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + f(t), \quad t \neq \tau_k, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad t = \tau_k, \end{aligned} \tag{2.10}$$

where $f \in L^1([0,T_0];X)$, $f(t+T_0) = f(t)$ for $t \geq 0$ and $c_k$ satisfies the following assumption.

*Assumption C.* For each $k \in \mathbb{Z}_0^+$ and $c_k \in X$, there exists $\delta \in \mathbb{N}$ such that $c_{k+\delta} = c_k$.

In order to study system (2.10), we need to consider the following Cauchy problem

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + f(t), \quad t \in [0,T_0] \setminus \widetilde{D}, \\ \Delta x(\tau_k) &= B_k x(\tau_k) + c_k, \quad k = 1,2,\ldots,\delta, \\ x(0) &= x_0, \end{aligned} \tag{2.11}$$

and introduce the $PC$-mild solution of Cauchy's problem (2.11) and $T_0$-periodic $PC$-mild solution of system (2.10).

*Definition 2.6.* A function $x \in PC([0,T_0];X)$, for finite interval $[0,T_0]$, is said to be a $PC$-mild solution of the Cauchy problem (2.10) corresponding to the initial value $x_0 \in X$ and input $f \in L^1([0,T_0];X)$ if $x$ is given by

$$x(t) = \mathcal{S}(t,0)x_0 + \int_0^t \mathcal{S}(t,\theta)f(\theta)d\theta + \sum_{0 \leq \tau_k < t} \mathcal{S}(t,\tau_k^+)c_k. \tag{Eq.2}$$

*Definition 2.7.* A function $x \in PC([0,+\infty);X)$ is said to be a $T_0$-periodic $PC$-mild solution of system (2.10) if it satisfies the expression (Eq.2) and $x(t+T_0) = x(t)$ for $t \geq 0$.

## 3. Periodic solutions of semilinear impulsive periodic system with time-varying generating operators

In order to use Leray-Schauder theorem to show the existence of periodic solutions, we need the following generalized Gronwall's inequality with mixed-type integral operator which is much different from the classical Gronwall's inequality and can be used in other problems (such as discussion on integral-differential equation of mixed type, see [15]). It will play an essential role in the study of nonlinear problems on infinite dimensional spaces.

**Lemma 3.1.** *Let $a \geq 0,\ b \geq 0,\ c \geq 0,\ 0 \leq \lambda_1 \leq 1,\ 0 \leq \lambda_2 < 1$. If $x \in PC([0,T_0];X)$ satisfies*

$$\|x(t)\| \leq a + b\int_0^t \|x(\theta)\|^{\lambda_1} d\theta + c\int_0^{T_0} \|x(\theta)\|^{\lambda_2} d\theta, \quad \forall t \in [0,T_0], \tag{3.1}$$

*then there exists a constant $M^* = M^*(a,b,c,\lambda_2,T_0) > 0$ such that*

$$\|x(t)\| \leq M^*, \quad \forall t \in [0,T_0]. \tag{3.2}$$

*Proof.* Let

$$y(t) = \|x(t)\| + 1, \quad \forall t \in [0, T_0],$$
$$M = \max_{t\in[0,T_0]} \|y(t)\|. \tag{3.3}$$

Then,

$$1 \leq y(t) \leq 1 + a + b\int_0^t \|y(\theta)\| d\theta + cT_0 M^{\lambda_2}, \quad \forall t \in [0, T_0]. \tag{3.4}$$

By Gronwall's inequality, we obtain

$$y(t) \leq (1 + a + cT_0 M^{\lambda_2})e^{bt}, \quad \forall t \in [0, T_0]. \tag{3.5}$$

Thus,

$$M \leq (1 + a + cT_0 M^{\lambda_2})e^{bT_0} \leq (1 + a + cT_0)e^{bT_0} M^{\lambda_2}. \tag{3.6}$$

Therefore,

$$\|x(t)\| \leq M \leq \left((1 + a + cT_0)e^{bT_0}\right)^{1/(1-\lambda_2)} \equiv M^*(a, b, c, \lambda_2, T_0), \quad \forall t \in [0, T_0]. \tag{3.7}$$

This completes the proof. □

Now, we consider the following semilinear impulsive periodic system with time-varying generating operators

$$\dot{x}(t) = A(t)x(t) + f(t, x), \quad t \neq \tau_k,$$
$$\Delta x(t) = B_k x(t) + c_k, \quad t = \tau_k, \tag{3.8}$$

and introduce *Poincaré* operator and study the $T_0$-periodic $PC$-mild solution of system (3.8).

In order to study the system (3.8), we first consider Cauchy's problem

$$\dot{x}(t) = A(t)x(t) + f(t, x), \quad t \in [0, T_0] \setminus \widetilde{D},$$
$$\Delta x(\tau_k) = B_k x(\tau_k) + c_k, \quad k = 1, 2, \ldots, \delta,$$
$$x(0) = \overline{x}. \tag{3.9}$$

By virtue of the expression of the $PC$-mild solution of the Cauchy problem (2.11), we can introduce the $PC$-mild solution of the Cauchy problem (3.9).

*Definition 3.2.* A function $x \in PC([0, T_0]; X)$ is said to be a $PC$-mild solution of the Cauchy problem (3.9) corresponding to the initial value $\overline{x} \in X$ if $x$ satisfies the following integral equation:

$$x(t) = \mathcal{S}(t, 0)\overline{x} + \int_0^t \mathcal{S}(t, \theta) f(\theta, x(\theta)) d\theta + \sum_{0\leq\tau_k<t} \mathcal{S}(t, \tau_k^+) c_k \quad \text{for } t \in [0, T_0]. \tag{3.10}$$

Now, we introduce the $T_0$-periodic $PC$-mild solution of system (3.8).

*Definition 3.3.* A function $x \in PC([0,+\infty);X)$ is said to be a $T_0$-periodic $PC$-mild solution of system (3.8) if it is a $PC$-mild solution of Cauchy's problem (3.9) corresponding to some $\overline{x}$ and $x(t+T_0) = x(t)$ for $t \geq 0$.

In order to prove the existence of the $PC$-mild solution of Cauchy's problem (3.9), we need the following assumption.

*Assumption F.* (F1): $f : [0,\infty) \times X \to X$ is measurable for $t \geq 0$ and for any $x, y \in X$ satisfying $\|x\|, \|y\| \leq \rho$ there exists a positive constant $L_f(\rho) > 0$ such that

$$\left\| f(t,x) - f(t,y) \right\| \leq L_f(\rho)\|x - y\|. \tag{3.11}$$

(F2): There exists a positive constant $M_f > 0$ such that

$$\left\| f(t,x) \right\| \leq M_f(1 + \|x\|) \quad \forall x \in X. \tag{3.12}$$

(F3): $f(t,x)$ is $T_0$-periodic in $t$. That is, $f(t+T_0,x) = f(t,x),\ t \geq 0$.

Then, we have the following theorem.

**Theorem 3.4.** *Assumptions A1, F(F1), and F(F2) hold. Cauchy's problem* (3.9) *has a unique PC-mild solution given by*

$$x(t,\overline{x}) = \mathcal{S}(t,0)\overline{x} + \int_0^t \mathcal{S}(t,\theta) f(\theta, x(\theta,\overline{x}))d\theta + \sum_{0 \leq \tau_k < t} \mathcal{S}(t,\tau_k^+)c_k. \tag{3.13}$$

*Proof.* Under the Assumptions A1, F(F1), and F(F2), using the similar method of Theorem 5.3.3 (see [28, page 169]), Cauchy's problem

$$\begin{gathered} \dot{x}(t) = A(t)x(t) + f(t,x), \quad t \in [s,\tau], \\ x(s) = \overline{x} \in X \end{gathered} \tag{3.14}$$

has a unique mild solution

$$x(t) = U(t,s)\overline{x} + \int_s^t U(t,\theta) f(\theta, x(\theta))d\theta. \tag{3.15}$$

In general, for $t \in (\tau_k, \tau_{k+1}]$, Cauchy's problem

$$\begin{gathered} \dot{x}(t) = A(t)x(t) + f(t,x), \quad t \in (\tau_k, \tau_{k+1}], \\ x(\tau_k) = x_k \equiv (I + B_k)x(\tau_k) + c_k \in X \end{gathered} \tag{3.16}$$

has a unique $PC$-mild solution

$$x(t) = U(t,\tau_k)x_k + \int_{\tau_k}^t U(t,\theta) f(\theta, x(\theta))d\theta. \tag{3.17}$$

Combining all of solutions on $[\tau_k, \tau_{k+1}]$ $(k = 1,\ldots,\delta)$, one can obtain the $PC$-mild solution of the Cauchy problem (3.9) given by

$$x(t,\overline{x}) = \mathcal{S}(t,0)\overline{x} + \int_0^t \mathcal{S}(t,\theta) f(\theta, x(\theta,\overline{x}))d\theta + \sum_{0 \leq \tau_k < t} \mathcal{S}(t,\tau_k^+)c_k. \tag{3.18}$$

□

In order to study the periodic solutions of the system (3.8), we define *Poincaré* operator from $X$ to $X$ as following:

$$P(\overline{x}) = x(T_0, \overline{x}) = \mathcal{S}(T_0, 0)\overline{x} + \int_0^{T_0} \mathcal{S}(T_0, \theta) f(\theta, x(\theta, \overline{x}))d\theta + \sum_{0 \le \tau_k < T_0} \mathcal{S}(T_0, \tau_k^+)c_k, \tag{3.19}$$

where $x(\cdot, \overline{x})$ denote the $PC$-mild solution of the Cauchy problem (3.9) corresponding to the initial value $x(0) = \overline{x}$. We note that a fixed point of $P$ gives rise to a periodic solution.

**Lemma 3.5.** *System* (3.8) *has a $T_0$-periodic $PC$-mild solution if and only if $P$ has a fixed point.*

*Proof.* Suppose $x(\cdot) = x(\cdot + T_0)$, then $x(0) = x(T_0) = P(x(0))$. This implies that $x(0)$ is a fixed point of $P$. On the other hand, if $Px_0 = x_0$, $x_0 \in X$, then for the $PC$-mild solution $x(\cdot, x_0)$ of the Cauchy problem (3.9) corresponding to the initial value $x(0) = x_0$, we can define $y(\cdot) = x(\cdot + T_0, x_0)$, then $y(0) = x(T_0, x_0) = Px_0 = x_0$. Now, for $t > 0$, we can use the (2), (3), and (4) of Lemma 2.3 and Assumptions A2, B, C, F(F3) to obtain

$$\begin{aligned}
y(t) &= x(t + T_0, x_0) \\
&= \mathcal{S}(t + T_0, T_0)\mathcal{S}(T_0, 0)x_0 + \int_0^{T_0} \mathcal{S}(t + T_0, T_0)\mathcal{S}(T_0, \theta) f(\theta, x(\theta, x_0))d\theta \\
&\quad + \sum_{0 \le \tau_k < T_0} \mathcal{S}(t + T_0, T_0)\mathcal{S}(T_0, \tau_k^+)c_k \\
&\quad + \int_{T_0}^{t+T_0} \mathcal{S}(t + T_0, \theta) f(\theta, x(\theta, x_0))d\theta + \sum_{T_0 \le \tau_{k+\delta} < t+T_0} \mathcal{S}(t + T_0, \tau_{k+\delta}^+)c_k \\
&= \mathcal{S}(t, 0)\left\{\mathcal{S}(T_0, 0)x_0 + \int_0^{T_0} \mathcal{S}(T_0, \theta) f(\theta, x(\theta, x_0))d\theta + \sum_{0 \le \tau_k < T_0} \mathcal{S}(T_0, \tau_k^+)c_k\right\} \\
&\quad + \int_0^{t} \mathcal{S}(t + T_0, s + T_0) f(s + T_0, x(s + T_0, x_0))ds + \sum_{0 \le \tau_k < t} \mathcal{S}(t, \tau_k^+)c_k \\
&= \mathcal{S}(t, 0)y(0) + \int_0^{t} \mathcal{S}(t, s) f(s, y(s, y(0)))ds + \sum_{0 \le \tau_k < t} \mathcal{S}(t, \tau_k^+)c_k.
\end{aligned} \tag{3.20}$$

This implies that $y(\cdot, y(0))$ is a $PC$-mild solution of Cauchy's problem (3.9) with initial value $y(0) = x_0$. Thus, the uniqueness implies that $x(\cdot, x_0) = y(\cdot, y(0)) = x(\cdot + T_0, x_0)$, so that $x(\cdot, x_0)$ is a $T_0$-periodic. □

Next, we show that $P$ defined by (3.19) is a continuous and compact operator.

**Lemma 3.6.** *Assumptions A1, A3, F(F1), and F(F2) hold. Then, $P$ is a continuous and compact operator.*

*Proof.* (1) Show that $P$ is a continuous operator on $X$.

Let $\overline{x}, \overline{y} \in \Xi \subset X$, where $\Xi$ is a bounded subset of $X$. Suppose $x(\cdot, \overline{x})$ and $x(\cdot, \overline{y})$ are the $PC$-mild solutions of Cauchy's problem (3.9) corresponding to the initial value $\overline{x}$ and $\overline{y} \in X$, respectively, given by

$$x(t,\overline{x}) = \mathcal{S}(t,0)\overline{x} + \int_0^t \mathcal{S}(t,\theta)f(\theta,x(\theta,\overline{x}))d\theta + \sum_{0\leq\tau_k<t}\mathcal{S}(T_0,\tau_k^+)c_k,$$
$$x(t,\overline{y}) = \mathcal{S}(t,0)\overline{y} + \int_0^t \mathcal{S}(t,\theta)f(\theta,x(\theta,\overline{y}))d\theta + \sum_{0\leq\tau_k<t}\mathcal{S}(T_0,\tau_k^+)c_k. \tag{3.21}$$

Thus, we obtain

$$\|x(t,\overline{x})\| \leq M_{T_0}\|\overline{x}\| + M_{T_0}M_fT_0 + M_{T_0}\sum_{0\leq\tau_k<T_0}\|c_k\| + M_{T_0}\int_0^t\|x(\theta,\overline{x})\|d\theta,$$
$$\|x(t,\overline{y})\| \leq M_{T_0}\|\overline{y}\| + M_{T_0}M_fT_0 + M_{T_0}\sum_{0\leq\tau_k<T_0}\|c_k\| + M_{T_0}\int_0^t\|x(\theta,\overline{y})\|d\theta. \tag{3.22}$$

By Gronwall's inequality with impulse [5, Lemma 1.7.1], one can verify that there exist constants $M_1^*$ and $M_2^* > 0$ such that

$$\|x(t,\overline{x})\| \leq M_1^*, \qquad \|x(t,\overline{y})\| \leq M_2^*. \tag{3.23}$$

Let $\rho = \max\{M_1^*, M_2^*\} > 0$, then $\|x(\cdot,\overline{x})\|$, $\|x(\cdot,\overline{y})\| \leq \rho$ which imply that they are locally bounded. By Assumption F(F1), we obtain

$$\begin{aligned}\|x(t,\overline{x}) - x(t,\overline{y})\| &\leq \|\mathcal{S}(t,0)\|\|\overline{x}-\overline{y}\| + \int_0^t\|\mathcal{S}(t,\theta)\|f(\theta,x(\theta,\overline{x})) - f(\theta,x(\theta,\overline{y}))\|d\theta \\ &\leq M_{T_0}\|\overline{x}-\overline{y}\| + M_{T_0}L_f(\rho)\int_0^t\|x(\theta,\overline{x}) - x(\theta,\overline{y})\|d\theta.\end{aligned} \tag{3.24}$$

By Gronwall's inequality with impulse [5, Lemma 1.7.1] again, one can verify that there exists constant $M > 0$ such that

$$\|x(t,\overline{x}) - x(t,\overline{y})\| \leq MM_{T_0}\|\overline{x}-\overline{y}\| \equiv L\|\overline{x}-\overline{y}\|, \quad \forall t \in [0,T_0], \tag{3.25}$$

which implies that

$$\|P(\overline{x}) - P(\overline{y})\| = \|x(T_0,\overline{x}) - x(T_0,\overline{y})\| \leq L\|\overline{x}-\overline{y}\|. \tag{3.26}$$

Hence, $P$ is a continuous operator on $X$.

(2) Verifies that $P$ takes a bounded set into a precompact set in $X$.

Let $\Gamma$ be a bounded subset of X. Define $K = P\Gamma = \{P(\overline{x}) \in X \mid \overline{x} \in \Gamma\}$.

For $0 < \varepsilon < t \leq T_0$, define

$$K_\varepsilon = P_\varepsilon\Gamma = \mathcal{S}(T_0,T_0-\varepsilon)\{x(T_0-\varepsilon,\overline{x}) \mid \overline{x} \in \Gamma\}. \tag{3.27}$$

Next, we show that $K_\varepsilon$ is precompact in X. In fact, for $\overline{x} \in \Gamma$ fixed, we have

$$\begin{aligned}\|x(T_0-\varepsilon,\overline{x})\| &= \left\|\mathcal{S}(T_0-\varepsilon,0)\overline{x} + \int_0^{T_0-\varepsilon}\mathcal{S}(T_0-\varepsilon,\theta)f(\theta,x(\theta,\overline{x}))d\theta + \sum_{0\leq\tau_k<T_0-\varepsilon}\mathcal{S}(T_0-\varepsilon,\tau_k^+)c_k\right\| \\ &\leq M_{T_0}\|\overline{x}\| + M_{T_0}M_fT_0 + \int_0^{T_0}\|x(\theta,\overline{x})\|d\theta + M_{T_0}\sum_{0\leq\tau_k<T_0}\|c_k\| \\ &\leq M_{T_0}\|\overline{x}\| + M_{T_0}M_fT_0 + T_0\rho + M_{T_0}\sum_{0\leq\tau_k<T_0}\|c_k\|.\end{aligned} \tag{3.28}$$

This implies that the set $\{x(T_0-\varepsilon,\overline{x}) \mid \overline{x} \in \Gamma\}$ is bounded.

By (5) of Lemma 2.3, $\mathcal{S}(T_0, T_0 - \varepsilon)$ is a compact operator. Thus, $K_\varepsilon$ is precompact in $X$. On the other hand, for arbitrary $\overline{x} \in \Gamma$,

$$P_\varepsilon(\overline{x}) = \mathcal{S}(T_0, 0)\overline{x} + \int_0^{T_0-\varepsilon} \mathcal{S}(T_0, \theta) f(\theta, x(\theta, \overline{x})) d\theta + \sum_{0 \le \tau_k < T_0 - \varepsilon} \mathcal{S}(T_0, \tau_k^+) c_k. \tag{3.29}$$

Thus, combined with (3.19), we have

$$\begin{aligned} \|P_\varepsilon(\overline{x}) - P(\overline{x})\| &\le \left\| \int_0^{T_0-\varepsilon} \mathcal{S}(T_0, \theta) f(\theta, x(\theta)) d\theta - \int_0^{T_0} \mathcal{S}(T_0, \theta) f(\theta, x(\theta)) d\theta \right\| \\ &\quad + \left\| \sum_{0 \le \tau_k < T_0 - \varepsilon} \mathcal{S}(T_0, \tau_k^+) c_k - \sum_{0 \le \tau_k < T_0} \mathcal{S}(T_0, \tau_k^+) c_k \right\| \\ &\le \int_{T_0-\varepsilon}^{T_0} \|\mathcal{S}(T_0, \theta)\| \|f(\theta, x(\theta))\| d\theta + M_{T_0} \sum_{T_0 - \varepsilon \le \tau_k < T_0} \|c_k\| \\ &\le 2M_{T_0} M_f (1 + \rho)\varepsilon + M_{T_0} \sum_{T_0 - \varepsilon \le \tau_k < T_0} \|c_k\|. \end{aligned} \tag{3.30}$$

It is showing that the set $K$ can be approximated to an arbitrary degree of accuracy by a precompact set $K_\varepsilon$. Hence, $K$ itself is precompact set in $X$. That is, $P$ takes a bounded set into a precompact set in $X$. As a result, $P$ is a compact operator. □

In order to use Leary-Schauder fixed pointed theorem to examine that the operator $P$ has a fixed point, we have to make the Assumption F(F2) a little strong as following.

(F2′): there exist constant $N_f > 0$ and $0 < \lambda < 1$ such that

$$\|f(t, x)\| \le N_f \left(1 + \|x\|^\lambda\right) \quad \forall x \in X. \tag{3.31}$$

Now, we can give the main results in this paper.

**Theorem 3.7.** *Assumptions A1, A2, A3, B, C, F(F1), F(F2′), and F(F3) hold. Then system* (3.8) *has a $T_0$-periodic PC-mild solution on* $[0, +\infty)$.

*Proof.* By (5) of Lemma 2.3, $\mathcal{S}(T_0, 0)$ is a compact operator on infinite dimensional space $X$. Thus, $\mathcal{S}(T_0, 0) \neq \alpha I,\ \alpha \in \mathbb{R}$. Then, there exists $\beta > 0$ such that

$$\| [\sigma \mathcal{S}(T_0, 0) - I]\overline{x}\| \ge \beta \|\overline{x}\| \quad \text{for } \sigma \in [0, 1]. \tag{3.32}$$

In fact, define $\Pi_\sigma = I - \sigma \mathcal{S}(T_0, 0),\ \sigma \in [0, 1]$, and

$$\Pi_\sigma : [0, 1] \longrightarrow \pounds_b(X), \qquad h(\sigma) = \|\Pi_\sigma\| : [0, 1] \longrightarrow \mathbb{R}^+. \tag{3.33}$$

It is obvious that $h \in C([0, 1]; \mathbb{R}^+)$. Thus, there exist $\sigma_* \in [0, 1]$ and $\beta > 0$ such that

$$h(\sigma_*) = \min\{h(\sigma) \mid \sigma \in [0, 1]\} \ge \beta > 0. \tag{3.34}$$

If not, there exits $\overline{\sigma} \in [0, 1]$ such that $h(\overline{\sigma}) = 0$. We can assert that $\overline{\sigma} \neq 0$ unless $h(\overline{\sigma}) = 1$. Thus, for $\overline{\sigma} \in (0, 1]$

$$\mathcal{S}(T_0, 0) = \frac{1}{\overline{\sigma}} I \quad \text{where } \frac{1}{\overline{\sigma}} \ge 1, \tag{3.35}$$

which is a contradiction with $\mathcal{S}(T_0, 0) \neq \alpha I,\ \alpha \in \mathbb{R}$.

By Theorem 3.4, for fixed $\overline{x} \in X$, the Cauchy problem (3.9) corresponding to the initial value $x(0) = \overline{x}$ has the $PC$-mild solution $x(\cdot, \overline{x})$. By Lemma 3.6, the operator $P$ defined by (3.19), is compact.

According to Leray-Schauder fixed point theory, it suffices to show that the set $\{\overline{x} \in X \mid \overline{x} = \sigma P\overline{x},\ \sigma \in [0,1]\}$ is a bounded subset of X. In fact, let $\overline{x} \in \{\overline{x} \in X \mid \overline{x} = \sigma P\overline{x},\ \sigma \in [0,1]\}$, we have

$$\begin{aligned} \beta\|\overline{x}\| &\le \left\| [\sigma \mathcal{S}(T_0,0) - I]\overline{x} \right\| \\ &= \sigma \int_0^{T_0} \|\mathcal{S}(T_0,\theta)\| \|f(\theta, x(\theta,\overline{x}))\| d\theta + \sigma \sum_{0\le\tau_k<T_0} \|\mathcal{S}(T_0,\tau_k^+)\| \|c_k\|. \end{aligned} \tag{3.36}$$

By Assumption F(F2′),

$$\begin{aligned} \|\overline{x}\| &\le \frac{\sigma}{\beta}\int_0^{T_0} \|\mathcal{S}(T_0,\theta)\| \|f(\theta, x(\theta,\overline{x}))\| d\theta + \frac{\sigma}{\beta} \sum_{0\le\tau_k<T_0} \|\mathcal{S}(T_0,\tau_k^+)\| \|c_k\| \\ &\le \frac{\sigma}{\beta} M_{T_0}\left( N_f T_0 + N_f \int_0^{T_0} \|x(\theta,\overline{x})\|^{\lambda} d\theta + \sum_{0\le\tau_k<T_0} \|c_k\| \right). \end{aligned} \tag{3.37}$$

For $t \in [0, T_0]$, we obtain

$$\begin{aligned} \|x(t,\overline{x})\| &\le M_{T_0}\|\overline{x}\| + M_{T_0}N_f T_0 + M_{T_0}N_f \int_0^{t} \|x(\theta,\overline{x})\|^{\lambda} d\theta + M_{T_0} \sum_{0\le\tau_k<t} \|c_k\| \\ &\le \frac{\sigma}{\beta} M_{T_0}^2\left( N_f T_0 + N_f \int_0^{T_0} \|x(\theta,\overline{x})\|^{\lambda} d\theta + \sum_{0\le\tau_k<T_0} \|c_k\| \right) \\ &\quad + M_{T_0}N_f T_0 + M_{T_0}N_f \int_0^{t} \|x(\theta,\overline{x})\|^{\lambda} d\theta + M_{T_0} \sum_{0\le\tau_k<t} \|c_k\| \\ &\le M_{T_0}N_f T_0 + \frac{\sigma}{\beta} M_{T_0}^2 N_f T_0 + \frac{\sigma}{\beta} M_{T_0}^2 \sum_{0\le\tau_k<T_0} \|c_k\| + M_{T_0} \sum_{0\le\tau_k<T_0} \|c_k\| \\ &\quad + M_{T_0}N_f \int_0^{t} \|x(\theta,\overline{x})\|^{\lambda} d\theta + \frac{\sigma}{\beta} M_{T_0}^2 N_f \int_0^{T_0} \|x(\theta,\overline{x})\|^{\lambda} d\theta. \end{aligned} \tag{3.38}$$

By Lemma 3.1, there exists $M^* > 0$ such that

$$\|x(t,\overline{x})\| \le M^* \quad \text{for } t \in [0, T_0]. \tag{3.39}$$

This implies that $\|x(0,\overline{x})\| = \|\overline{x}\| \le M^*$ for all $\overline{x} \in \{\overline{x} \in X \mid \overline{x} = \sigma P\overline{x}, \sigma \in [0,1]\}$.

Thus, by Leray-Schauder fixed point theory, there exits $x_0 \in X$ such that $Px_0 = x_0$. By Lemma 3.5, we know that the $PC$-mild solution $x(\cdot, x_0)$ of Cauchy's problem (3.9) corresponding to the initial value $x(0) = x_0$, is just $T_0$-periodic. Therefore $x(\cdot, x_0)$ is a $T_0$-periodic $PC$-mild solution of system (3.8). □

## 4. Application

In this section, an example is given to illustrate our theory. Consider the following problem:

$$\begin{aligned}
&\frac{\partial}{\partial t}x(t,y) = \sin t\Delta x(t,y) + \sqrt{3x^{2/3}(t,y)+2} + \sin(t,y), \quad y \in \Omega,\ t \in (0,2\pi] \setminus \left\{\frac{1}{2}\pi, \pi, \frac{3}{2}\pi\right\}, \\
&\Delta x(\tau_i, y) = x(\tau_i + 0, y) - x(\tau_i - 0, y) \\
&\qquad = \begin{cases} 0.05Ix(\tau_i, y), & i = 1, \\ -0.05Ix(\tau_i, y), & i = 2, \\ 0.05Ix(\tau_i, y), & i = 3, \end{cases} \quad y \in \Omega,\ \tau_i = \frac{i}{2}\pi,\ i = 1,2,3, \\
&x(t,y)|_{y\in\partial\Omega} = 0, \quad t > 0, \\
&\quad x(0,y) = x(2\pi, y),
\end{aligned} \tag{4.1}$$

where $\Omega \subset R^3$ is bounded domain and $\partial\Omega \in C^3$.

Define $X = L^2(\Omega)$, $D(A) = H^2(\Omega) \bigcap H_0^1(\Omega)$, and $A(t)x = -\sin t(\partial^2 x/\partial y_1^2 + \partial^2 x/\partial y_2^2 + \partial^2 x/\partial y_3^2)$ for $x \in D(A)$, which satisfies Assumptions A1, A2, and A3. Define $x(\cdot)(y) = x(\cdot, y)$, $\sin(\cdot)(y) = \sin(\cdot, y)$, $f(\cdot, x(\cdot))(y) = \sqrt{3x^{2/3}(\cdot,y)+2} + \sin(\cdot, y)$ and

$$B_i = \begin{cases} 0.05I, & i = 3m-2, \\ -0.05I, & i = 3m-1,\ \tau_i = \dfrac{i\pi}{2},\ i, m \in \mathbb{N}. \\ 0.05I, & i = 3m. \end{cases} \tag{4.2}$$

It is obvious that $\tau_{i+3} = \tau_i + 2\pi$, $B_{i+3} = B_i \in £_b(L^2(\Omega))$ which satisfy the Assumption B. For any $x, z \in$ X satisfying $1 \le \|x\|, \|z\| \le \rho$,

$$\begin{aligned}
\|f(t,x) - f(t,z)\| &\le \left\| \sqrt{3x^{2/3}+2} - \sqrt{3z^{2/3}+2} \right\| \\
&\le \frac{(\|x\|^{1/3} + \|z\|^{1/3})\|x^{1/3} - z^{1/3}\|}{\sqrt{3\|x\|^{2/3}+2} + \sqrt{3\|z\|^{2/3}+2}} \\
&\le \frac{\rho^{1/3}}{\sqrt{2}}\|x - z\|.
\end{aligned} \tag{4.3}$$

Meanwhile,

$$\begin{aligned}
&\|f(t,x)\| \le \left\|\sqrt{3x^{2/3}+2}\right\| + \|\sin t\| \le \sqrt{3\|x\|^{2/3}+2} + 1 \le 3\left(1 + \|x\|^{2/3}\right), \\
&f(\cdot + 2\pi, x) = \sqrt{3x^{2/3}+2} + \sin(\cdot + 2\pi) = \sqrt{3x^{2/3}+2} + \sin(\cdot) = f(\cdot, x).
\end{aligned} \tag{4.4}$$

These imply that Assumptions F(F1), F(F2′), and F(F3) hold. It comes from

$$\begin{aligned}
\mathcal{S}(2\pi, 0) &= U(2\pi, \tau_3)(I + B_3)U(\tau_3, \tau_2)(I + B_2)U(\tau_2, \tau_1)(I + B_1)U(\tau_1, 0) \\
&= (0.95^2 \cdot 1.05)U(2\pi, 0)
\end{aligned} \tag{4.5}$$

that $\mathcal{S}(2\pi,0)\neq \alpha I,\ \alpha>1$. In fact, if $(0.95^2\cdot 1.05)T(2\pi)=\alpha I$, then $U(2\pi,0)=(\alpha/(0.95^2\cdot 1.05))I$ cannot be compact in $L^2(\Omega)$, which is a contradiction with $U(t,\theta)$which is compact operator for $0\leq\theta<t\leq T_0$. Thus problem (4.1) can be rewritten as

$$\dot{x}(t)=A(t)x(t)+f(t,x),\quad t\in(0,2\pi]\setminus\left\{\frac{1}{2}\pi,\pi,\frac{3}{2}\pi\right\},$$
$$\Delta x\left(\frac{i}{2}\pi\right)=B_i x\left(\frac{i}{2}\pi\right),\quad i=1,2,3, \tag{4.6}$$
$$x(0)=x(2\pi).$$

It satisfies all the assumptions given in Theorem 3.7, our results can be used to problem (4.1). That is, problem (4.1) has a $2\pi$-periodic $PC$-mild solution $x_{2\pi}(\cdot,y)\in PC_{2\pi}([0+\infty);L^2(\Omega))$, where

$$PC_{2\pi}\big([0,+\infty);L^2(\Omega)\big)\equiv\big\{x\in PC\big([0,+\infty);L^2(\Omega)\big)\mid x(t)=x(t+2\pi),\ t\geq 0\big\}. \tag{4.7}$$

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