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ASSIGNMENT OF NONLINEAR SAMPLED-DATA DYNAMICS USING GENERALIZED HOLD FUNCTION CONTROL*

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This paper considers the problem of assigning the dynamics of a nonlinear analytic system using nonlinear generalized sampled-data hold function (GSHF) control, in the neighborhood of an equilibrium point. On every sampling interval, the control input consists of a nonlinear time-periodic function applied to the sampled value of the state vector. The nonlinear monodromy map is the state transition map from one sample time to the next. It is shown that this map is arbitrarily assignable by GSHF feedback if and only if the linear part of the system is controllable. Two approaches are proposed to construct a GSHF controller that performs the assignment. The first approach matches the coefficients of the Taylor series expansion of the monodromy map around the equilibrium. The second approach interpolates an optimal control law at several points in the vicinity of the equilibrium. These approaches are illustrated on an example.

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1. INTRODUCTION

The first motivation of this paper is the recent work on generalized sampled-data hold function control (GSHF) of analog systems [1–12]. The basic idea of this method is to let the hold function (i.e., the digital-to-analog converter) of the sampled-data controller be a design parameter. In its simplest form, when applied to a LTI plant, on each sampling interval the control input is obtained by modulating the sampled value of the output by a periodic function [5]. The advantages and disadvantages of GSHF control in achieving robust stability and performance compared to LTI compensation have been documented in the literature [5,9,21,22,23]. Except for [12], this control method has only been applied to linear plants.

The second motivation of this paper is the recent work on feedback linearization of analog nonlinear systems using an analog controller [13, 14]. In view of the widespread use of digital computers to implement feedback control, it is important to analyze the capability that a digital controller has to linearize, or arbitrarily assign, the dynamics of an analog plant.

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This paper considers the problem of assigning the dynamics of a nonlinear analytic system using nonlinear GSHF control, in the neighborhood of an equilibrium point. On every sampling interval, the control input consists of a nonlinear time-periodic function applied to the sampled value of the state vector. The nonlinear monodromy map is the state transition map from one sample time to the next. The original results obtained in this study are as follows. First it is shown in section 2 that the monodromy map is arbitrarily assignable by GSHF feedback if and only if the linear part of the system is controllable. Then in section 3, two approaches are proposed to construct a GSHF controller that performs the assignment. The first approach matches the coefficients of the Taylor series expansion of the monodromy map around the equilibrium. This approach is relatively simple, but it does not guarantee convergence of the formal power series generated for the hold function. Moreover in our experience, it yields domains of attraction that often tend to shrink as more terms are retained in the power series. The second approach interpolates an optimal control law at several points in the vicinity of the equilibrium. It is more computationally demanding than the first one, but in our experience tends to give better attraction regions. These features are illustrated on an example in section 4.

When the desired closed loop monodromy map is linear, the problem of monodromy assignment becomes that of feedback linearization in sampled-data systems. In [15] it was shown that discretization in general destroys feedback linearizability, but that this property may be recovered by using a multirate sampling scheme. This paper is related to [15] because when a nonlinear GSHF controller is implemented using piecewise constant function, it becomes formally equivalent to a multirate controller. This paper is also a generalization of [4] where it is shown that when the plant, the GSHF controller and the desired monodromy are all linear, controllability of the plant is necessary and sufficient for arbitrary assignability.

2. PROBLEM STATEMENT AND AN EXISTENCE RESULT

We consider nonlinear analog autonomous systems of the form

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the input vector, and $f: R^n \times R^m \to R^n$ is analytic and satisfies $f(0,0) = 0$. The control input will be in the form of a nonlinear GSHF static state feedback, that is,

$$\begin{gathered} u(t) = F(t,x(kT)), \ \forall t \in [kT,(k+1)T), \quad k = 0,1,2,..., \\ F(t+T, \xi) = F(t, \xi), \ \forall t \geq 0, \quad \forall \xi \in R^n, \\ F(t,0) = 0, \ \forall t \geq 0, \end{gathered} \tag{2}$$

where $T > 0$ is the sampling period, and the T-periodic hold function F is analytic on $[0, T) \times B_\sigma$, where B_σ is an open neighborhood of the origin. We want to assign the state transition map from one sample time to the next—that is, the closed loop monodromy map—resulting from using the feedback law (2) on system (1). We introduce the following.

DEFINITION 2.1. System (1) is said to be *locally nonlinear monodromy assignable* if for every $T > 0$, for every $\Psi: R^n \to R^n$ that is analytic around the origin and satisfies $\Psi(0) = 0$, there exists B_σ, an open neighborhood of the origin, and a hold function (2) that is analytic on $[0, T) \times B_\sigma$, such that whenever $x(kT) \in B_\sigma$, (1)–(2) yield

$$x((k + 1)T) = \Psi(x(kT)). \tag{3}$$

Remark 2.1. The requirement that the hold function be analytic is motivated by the fact that in practice the control law will often be implemented with finite precision—typically with piecewise polynomial functions. Recall that the Stone-Weierstrass Theorem [18] guarantees that every continuous function (hence every analytic function) can be approximated to arbitrary accuracy by piecewise polynomial functions. Hence, this requirement could be relaxed to simple continuity with an increased sophistication of the theory.

We can now state the following existence result.

Proposition 2.1. Let

$$A = \left[\frac{\partial f}{\partial x}\right]_{(x,\,u) = (0,0)}, \quad B = \left[\frac{\partial f}{\partial u}\right]_{(x,\,u) = (0,0)}, \tag{4}$$

characterize the linearized version of (1). System (1) is locally nonlinear monodromy assignable if and only if the pair (A, B) is controllable.

Proof. Controllability of (A,B) is necessary because the operations of linearization and GSHF feedback commute. In other words, the linear part of the closed-loop monodromy map is obtained by applying the linear part of the GSHF feedback (2) to the linear part of the plant (1). If (A,B) has an uncontrollable eigenvalue λ, then under any GSHF feedback (2), the linear part of the closed loop monodromy map will have an eigenvalue $e^{\lambda T}$, hence this map is not arbitrarily assignable.

To prove sufficiency, we use the following idea: For a given sampling period $T > 0$, and a desired monodromy Ψ, we define an optimal control problem for system (1) with boundary conditions

$$x(0) = \xi, \tag{5a}$$

$$x(\mathrm{T}) = \Psi(\xi), \tag{5b}$$

where $\xi \in R^n$ is a parameter. This optimal control problem yields a two-point boundary value problem (TPBVP) whose solution is shown to exist and depend analytically on ξ around $\xi = 0$. This, in turn, produces a control law that solves the assignment problem.

To develop this idea, consider the optimal control problem

$$\min_u J(\xi,u(\cdot)) = \frac{1}{2}\int_0^T u(t)^T u(t)dt, \text{ subject to (1), (5).} \tag{6}$$

For $p \in R^n$, define the Hamiltonian function

$$H(x,p,u) = p^T f(x,u) - \frac{1}{2} u^T u. \tag{7}$$

Also define

$$w(x,p) = \arg\max_u H(x,p,u). \tag{8}$$

Obviously, $w(0,0) = 0$. Moreover, since

$$\frac{\partial H}{\partial u} = \left(\frac{\partial f}{\partial u}\right)^T p - u, \quad \left(\frac{\partial^2 H}{\partial u^2}\right)_{(x,p) = (0,0)} = -I, \tag{9}$$

which is nonsingular, the implicit functions theorem [19] guarantees that $w(x,p)$ exists in a neighborhood of $(x,p) = (0,0)$, is analytic in that neighborhood, and satisfies

$$\left(\frac{\partial w}{\partial x}\right)_{(x,p) = (0,0)} = 0, \quad \left(\frac{\partial w}{\partial p}\right)_{(x,p) = (0,0)} = B^T. \tag{10}$$

The optimal control problem (6) yields the TPBVP [17]

$$\dot{x} = f(x,w(x,p)), \quad \dot{p} = -\left(\frac{\partial f}{\partial x}\right)^T_{(x,w(x,p))} p, \text{ subject to (5).} \tag{11}$$

At $\xi = 0$, (11) admits the obvious trivial solution $x(t) = 0$, $p(t) = 0$, $t \in [0, T]$. We claim that this solution depends analytically on ξ around $\xi = 0$. To prove this, we will show that the *initial conditions* of (11) depend analytically on ξ, then use the standard result on analytic dependence of the solution of an initial value problem with respect to the initial conditions [19].

The system of differential equations (11) together with the initial conditions $x(0) = 0$, $p(0) = 0$, admit the unique trivial solution $x(t) = 0$, $p(t) = 0$, $t \geq 0$. Therefore $x(T)$ depends analytically on $p(0)$ [19]. We will show that this dependence is analytically invertible by showing that the Jacobian matrix $(\partial x(T)/\partial p(0))$ is nonsingular. This Jacobian is evaluated as follows. Linearize (11) around the nominal solution $x(t) = 0$, $p(t) = 0$ to obtain

$$\begin{bmatrix} \delta\dot{x} \\ \delta\dot{p} \end{bmatrix} = \begin{bmatrix} A & BB^T \\ 0 & -A^T \end{bmatrix} \begin{bmatrix} \delta x \\ \delta p \end{bmatrix}. \tag{12}$$

Integrate (12) on $[0, T]$ to obtain

$$\begin{bmatrix} \delta x(T) \\ \delta p(T) \end{bmatrix} = \begin{bmatrix} e^{AT} & W \\ 0 & e^{-A^T T} \end{bmatrix} \begin{bmatrix} \delta x(0) \\ \delta p(0) \end{bmatrix}, \quad W = e^{AT} \int_0^T e^{-At} BB^T e^{-A^T t} dt, \tag{13}$$

indicating that

$$\frac{\partial x(T)}{\partial p(0)} = W. \tag{14}$$

Since (A,B) is controllable, W is invertible. Therefore by the implicit functions theorem [19], the dependence of $x(T)$ on $p(0)$ is analytically invertible, hence $p(0)$ depends analytically on $x(T)$. Now in (5), $x(T)$ depends analytically on ξ. Therefore in (11), all initial conditions depend analytically on ξ in a sufficiently small neighborhood of $\xi = 0$, and so does its solution $x(t)$, $p(t)$. Based on that solution, we define

$$F(t,\xi) = w(x(t), p(t)),\ t \in [0,T),$$

subject to (5), (11),

$$F(t + kT,\xi) = F(t,\xi),\ t \in [0,T),\ k = 1,2,3,... \tag{15}$$

which depends analytically on ξ and t, and performs the monodromy assignment.

Remark 2.2. Without the requirement that the hold function be analytic, the sufficiency part of Proposition 2.1 would be a simple consequence of the Axiom of Choice [20]. Indeed, for a given sampling period T, a given monodromy Ψ and for $\xi \in R^n$, define

$$U_\xi = \{u{:}[0,T] \rightarrow R^m{:}\ (5a) \text{ and } (1) \text{ imply } (5b)\}. \tag{16}$$

When (A,B) is controllable, there exists a neighborhood of the origin B_σ such that U_ξ is not empty whenever $\xi \in B_\sigma$. The problem of finding a GSHF that performs the monodromy assignment is then equivalent to finding a *choice function* for the collection of sets $\{U_\xi,\ \xi \in B_\sigma\}$, by which we mean a systematic rule which to ξ associates one particular element of U_ξ. Such a choice function is guaranteed to exist by the axiom. However, the axiom does not guarantee that we can construct a particular choice function that depends *analytically* on ξ. This is precisely what (15) provides.

3. TWO APPROXIMATE ASSIGNMENT METHODS

3.1 Polynomial Assignment

Since the right-hand side of (1) is analytic, we write it in the form

$$f(x,u) = \sum_{i=1}^{\infty} f_i(x,u), \quad f_i(x,u) = \sum_{k=0}^{i} f_{ik}\, x^{(i-k)} \otimes u^{(k)}, \tag{17}$$

where $f_i(x,u)$ is a homogeneous polynomial in (x,u) with degree i, superscript (j) denotes Kronecker power of order j, and the known matrices f_{ik} have appropriate dimensions.

Similarly we assume that the GSHF control law (2) has the form

$$F(t,x) = \sum_{i=1}^{\infty} F_i(t)x^{(i)}, \quad t\epsilon[0,T). \tag{18}$$

The desired monodromy map also has the form

$$\Psi(x) = \sum_{i=1}^{\infty}\Psi_i\, x^{(i)}, \tag{19}$$

where the known matrices Ψ_i have appropriate dimensions. The problem is to determine the unknown analytic functions $F_i(t)$ in (18) so that the monodromy map resulting from (1), (2), and (18) coincides with (19). We have the following.

PROPOSITION 3.1. Let B_σ be a neighborhood of the origin. Suppose the GSHF control law (18) converges in $[0,T) \times B_\sigma$, and yields an analytic closed-loop monodromy map.

Define

$$a_1(t) = e^{At} + \int_0^t e^{A(t-\tau)}BF_1(\tau)d\tau, \ \ t\epsilon[0,T],$$

$$a_j(t) = \int_0^t e^{A(t-\tau)}BF_j(\tau)d\tau + \sum_{i=2}^{j} \int_0^t e^{A(t-\tau)}K_{ij}(\tau)d\tau, \quad t\epsilon[0,T], \quad j \geq 2, \tag{20}$$

where

$$K_{ij}(t) = \sum_{k=1}^{i+1} \sum_{\delta=i-k+1}^{j-k+1} f_{ik}\Bigg[\Bigg(\underset{p_1+p_2+\ldots+p_{i-k+1}=\delta}{\sum_{p_1=1}\sum_{p_2=1} \ldots \sum_{p_{i-k+1}=1}} a_{p_1}(t)\otimes\ldots\otimes a_{p_{i-k+1}}(t)\Bigg)\otimes \Bigg(\underset{q_1+q_2+\cdots+q_{k-1}=j-\delta}{\sum_{q_1=1}\sum_{q_2=1}\ldots\sum_{q_{k-1}=1}} F_{q_1}(t)\otimes\ldots\otimes \mathrm{F}_{qk-1}(t)\Bigg)\Bigg]. \tag{21}$$

Then the actual closed-loop monodromy map has the form

$$\Psi_a(x) = \sum_{i=l}^{\infty} a_i(T)x^{(i)}. \tag{22}$$

Proof. The proof is a lengthy but straightforward calculation. The idea is to use the GSHF feedback (2), (18) on system (1), (17), and on the interval $[kT, (k+1)T]$, let the intersampling behavior be characterized by

$$x(kT+\tau) = \sum_{i=1}^{\infty} a_i(\tau)x^{(i)}(kT), \qquad \tau\epsilon[0,T),$$

$$f_i(x(kT+\tau), u(kT+\tau)) = \sum_{j=1}^{\infty} K_{ij}(\tau)x^{(j)}(kT), \quad \tau\epsilon[0,T). \tag{23}$$

See [12] for details.

Let $\nu \geq 1$ be integer. To assign the first ν coefficients of the Taylor series expansion of the closed-loop monodromy map, we can specify

$$a_i(T) = \Psi_i,\ 1 \leq i \leq \nu. \tag{24}$$

Equations (20), (21), and (24) can be considered as linear equations for the unknown analytic functions $F_i\ (t)$. When the pair (A, B) is controllable, (20), (24) have the obvious solution

$$F_1(t) = B^T e^{A^T(T-t)}\, W_c^{-1}\, (\Psi_1 - e^{AT}),$$

$$F_j(t) = B^T e^{A^T(T-t)} W_c^{-1} \left(\Psi_j - \sum_{i=2}^{j} \int_0^T e^{A(T-\tau)} K_{ij}(\tau) d\tau \right), \qquad j \geq 2,$$

$$W_c = \int_0^T e^{A(T-t)} BB^T e^{A^T(T-t)} dt. \tag{25}$$

It should also be remarked that (20)–(22) can be solved from low order to high order by computing sequentially $F_I(t)$, $a_1(t)$, $F_2(t)$, $a_2(t)$, and so on. In other words, the ith order nonlinearity $F_i(t)$ is introduced in the GSHF to assign the ith order term in the monodromy, without destroying the assignment of the lower-order terms. However, as we assign more terms, we do not guarantee that the formal power series (18) thus generated converges anywhere else than at the origin. In fact, from our experience, such a procedure tends to give quite poor attraction regions, apparently due to the occurrence of large excursions from the origin between sampling times. This leads to the idea of penalizing the intersampling behavior, which is presented in the next subsection.

3.2. Interpolation

The idea of this method is similar to that in the proof of sufficiency in Proposition 2.1—use a control law that optimizes a subsidiary cost function and performs the monodromy assignment through its boundary conditions. In addition here, we assume that the control input is a linear combination of known functions of time with unknown coefficients. We determine the values of these coefficients that would yield optimality for several chosen initial values in the vicinity of the origin in R^n. Then we construct a polynomial function of x that interpolates these values. The combination of this polynomial function of x with the known functions of time yields the GSHF feedback law.

More specifically, we assume that the control input of (21) has the form

$$u(t) = \sum_{i=1}^{N} v_i(t)\alpha_i = V(t)\alpha, \quad t\epsilon[kT,(k+1)T), \tag{26}$$

where $N >> m \times n$ is a chosen integer, $v_i{:}R \to R^m$ are known, linearly independent T-periodic functions of time, and the coefficient vector $\alpha \in R^N$ is unknown. For the spanning functions v_i we have used piecewise polynomial splines with small support.

Next we choose a subsidiary cost function $g{:}R^n \times R^m \to R{:}(x,u) \to g(x,u)$ that will penalize the intersampling behavior. For the cost function g we have effectively used a quadratic positive definite function. We also choose M points $\xi^1,\ldots,\xi^M$ in the vicinity of the origin of R^n. Then for each of these points ξ^k, we solve the optimization problem

$$\min_\alpha \int_0^T g(x(t),u(t))dt,$$

subject to

$$\dot{x}(t) = f(x(t),u(t)), \qquad x(0) = \xi^k, \qquad x(T) = \Psi(\xi^k), \qquad u(t) = \sum_{i=1}^{N} v_i(t)\alpha_i. \tag{27}$$

This is now a constrained parameter optimization problem, which we solve using the gradient projection technique. See [12, 16] for explicit expressions of the gradient of the cost function and constraints in (27) with respect to the unknown vector α.

Let α^k, $1 \leq k \leq M$ be the optimizers of problem (27). (The existence and uniqueness of these optimizers can be guaranteed under mild technical conditions on f and g, using an argument similar to that in the proof of Proposition 2.1 [12]). The next step in the method is to find a polynomial function $\pi{:}R^n \to R^N$ which interpolates the α^k, that is, which satisfies

$$\pi(\xi^k) = \alpha^k, \qquad 1 \leq k \leq M. \tag{28}$$

As expected from an interpolation problem, this step leads to linear algebraic equations for the coefficients of the polynomial π [12]. Finally, the GSHF is given by

$$F(t,x) = V(t)\pi(x), \quad t\epsilon[0,T), \quad F(t+kT,\xi) = F(t,\xi), \quad t\epsilon[0,T), \quad k = 1,2,3,\ldots. \tag{29}$$

The resulting control law therefore has the property that whenever $x(kT)$ coincides with one of the points ξ^j, on the interval $[kT, (k + 1)T]$, the input not only satisfies the boundary conditions (23), but also optimizes the integral of the cost function $g(x,u)$ subject to the constraint (26). However, note that the interpolation method does not guarantee local asymptotic stability of the origin when the linear part of the desired monodromy is asymptotically stable, whereas the polynomial assignment method does.

4. EXAMPLES

To illustrate briefly the methods of section 3 (see [12] for details), we will perform the approximate feedback linearization of the second order single input nonlinear system [15]

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = u + ux_2^2, \tag{30}$$

which, according to Proposition 2.1, is locally nonlinear monodromy assignable around the origin. Let the sampling period $T = 1$. The desired linear closed-loop monodromy map has the form (23), (19) where

$$\Psi_1 = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.3 \end{bmatrix},$$

$$\Psi_k = 0, \; k \geq 2. \tag{31}$$

Example 1 We first use the polynomial assignment method of section 3.1. Using (20), (21), (24), and (25), the first five terms of the controller are

$$F_1(t) = [6t-3 \quad -4.2t + 1.4],$$

$$F_3(t) = [0.771t - 0.386 \quad -3.96t + 1.98 \quad 7.092t - 3.546 \quad -2.652t + 1.001],$$

$$F_5(t) = [0.099t - 0.05 \quad -0.949t + 0.501 \quad 3.811t-2.121 \quad -7.678t + 4.387$$
$$7.276t - 4.095 \quad -1.816t + 0.828]$$

$$F_2(t) = F_4(t) = 0, \; \forall t\in[0,1], \tag{32}$$

where, for instance, $x^{(5)}(t)$ is defined as $x^{(5)} = [x_1^5 \; x_1^4x_2 \; x_1^3x_2^2 \; x_1^2x_2^3 \; x_1x_2^4 \; x_2^5]^T$, pertaining to $F_5(t)$. The other Kronecker powers of $x(t)$ are similarly defined. There are no even-order terms in this controller. Based on the above results, we define three GSHF controllers obtained by truncating (18) to order 1,3, and 5, respectively. Thus Controller 1 uses $F_1(t)$ only and is linear, Controller 2 uses $F_1(t)$ and $F_3(t)$, while Controller 3 uses $F_1(t)$, $F_3(t)$, and $F_5(t)$. There is no significant difference between the responses of these three controllers when the initial conditions have norm less than ⅓. However, if we check the phase plane for the attraction regions (see Fig. 1), those of Controllers 2 and 3 are much smaller than that of Controller 1.

Example 2 We now apply the interpolation approach of section 3.2. The subsidiary cost function is chosen as

$$g(x,u) = \frac{1}{2}(x_1^2 + 2x_2^2 + u^2). \tag{33}$$

The input function is assumed to be a continuously differentiable linear combination of third degree piecewise polynomials on the intervals [0, 1/6], [1/6, 1/3], [1/3, 2/3], [2/3, 1]. Let M = 5. The 5 initial conditions for which we solve problem (27) form the initial

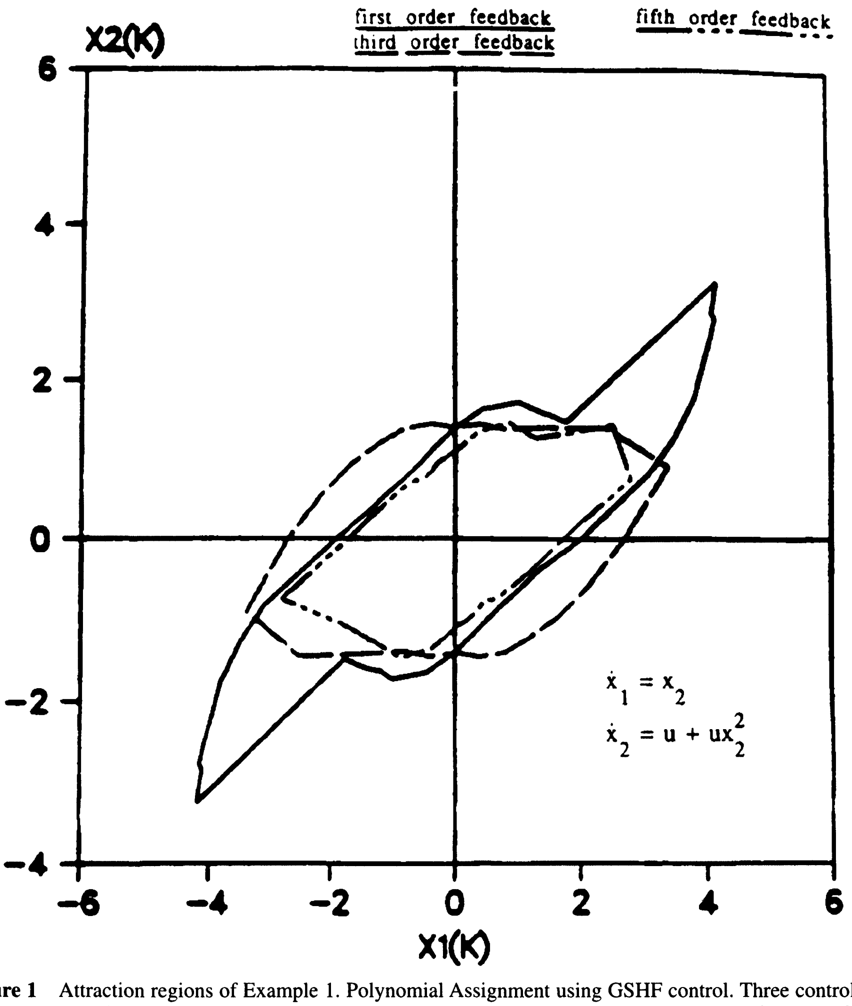

Figure 1 Attraction regions of Example 1. Polynomial Assignment using GSHF control. Three controllers are compared: (1) First-order feedback controller; (2) Third-order feedback controller; (3) Fifth-order feedback controller.

condition set

$$I_0 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ -4 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \end{bmatrix}, \begin{bmatrix} -5 \\ 2 \end{bmatrix} \right\}. \tag{34}$$

The attraction region of the closed-loop sampled-data system is shown in Fig. 2. In that figure, the attraction region obtained in Example 1 is also shown for comparison. Clearly for system (30) with desired monodromy (31), the interpolation method of section 3.2 improves dramatically the attraction region compared with the polynomial approach of section 3.1. The comparative features of Examples 1 and 2 are typical of many experiments we performed.

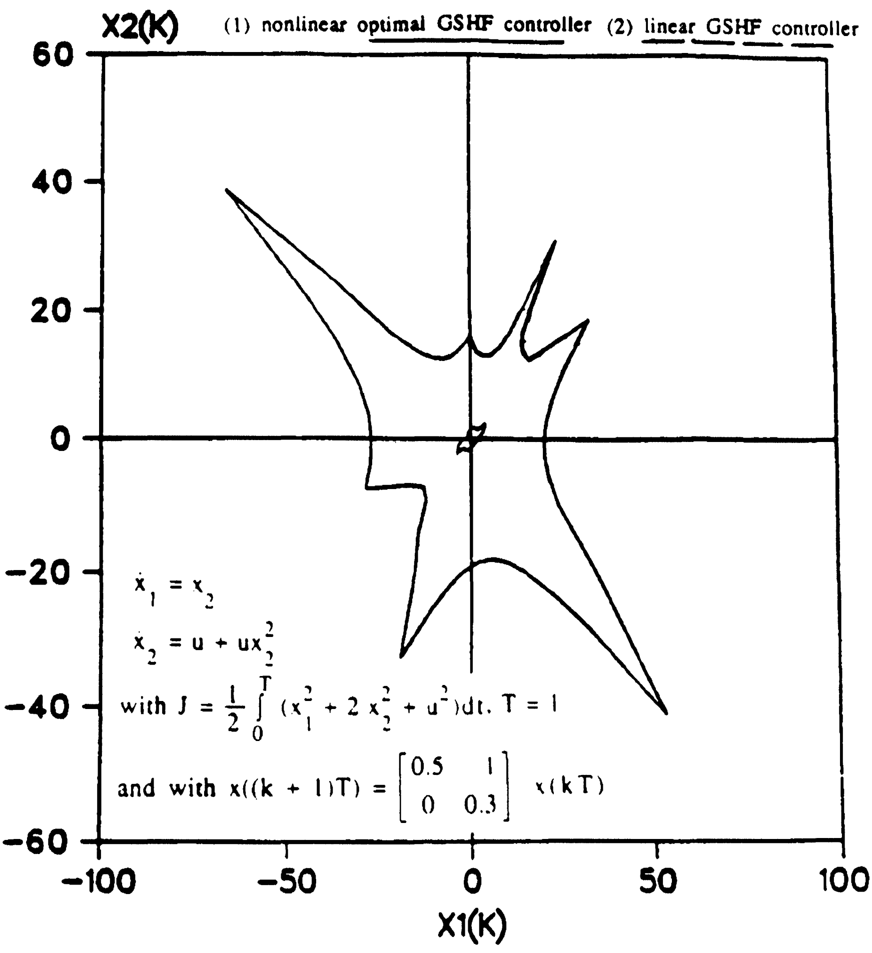

Figure 2 Attraction regions of the closed-loop nonlinear system in Example 2. Two controllers are compared: (1) Nonlinear GSHF controller for approximate monodromy assignment and optimal intersample behavior; (2) First-order controller in Example 1.

4. CONCLUSIONS

This paper has considered the problem of assigning the state transition map from one sample time to the next (i.e., the monodromy map) in a nonlinear analytic system using GSHF control. The basic premise of this approach is that the hold function (i.e., the digital-to-analog converter) is the design parameter. It is shown that the monodromy map is arbitrarily assignable by GSHF feedback if and only if the linear part of the system is controllable. Two approaches have been proposed to construct a GSHF controller that performs the assignment. Their features, which we have found typical in many experiments, have been illustrated on an example.

The main limitation of this work is obviously its emphasis on sample time dynamics. Even though the interpolation method of section 3.2 provides a way of ensuring intersampling performance, our experience suggests that it is in general difficult to prevent large excursions from the origin between sampling times. Another limitation is the assumption that the system has no disturbance, which may be quite detrimental in GSHF control. Furthermore, our reliance on an analytic model of the dynamic system raises the question of robustness of our method against structured and unstructured uncertainties. These issues will be studied in the future.

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