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Research Article

A New Delay Vector Integral Inequality and Its Application

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A new delay integral inequality is established. Using this inequality and the properties of nonnegative matrix, the attracting sets for the nonlinear functional differential equations with distributed delays are obtained. Our results can extend and improve earlier publications.

1. Introduction

The attracting set of dynamical systems have been extensively studied over the past few decades and various results are reported. It is well known that one of the most researching tools is inequality technique. With the establishment of various differential and integral inequalities [1–12], the sufficient conditions on the attracting sets for different differential systems are obtained [12–16]. However, the inequalities mentioned above are ineffective for studying the attracting sets of a class of nonlinear functional differential equations with distributed delays.

Motivated by the above discussions, in this paper, a new delay vector integral inequality is established. Applying this inequality and the properties of nonnegative matrix, some sufficient conditions ensuring the global attracting set for a class of nonlinear functional differential equations with distributed delays are obtained. The result in [16] is extended.

2. Preliminaries

In this section, we introduce some notations and recall some basic definitions.

E means unit matrix; $\mathbb{R}$ is the set of real numbers. $A \le B (A < B)$ means that each pair of corresponding elements of A and B satisfies the inequality "$\le$ ($<$)". Especially, A is called a nonnegative matrix if $A \ge 0$.

For a nonnegative matrix $A \in \mathbb{R}^{n\times n}$, let $\rho(A)$ denote the spectral radius of A. Then $\rho(A)$ is an eigenvalue of A and its eigenspace is denoted by

$$\Omega_\rho(A) \triangleq \{z \in \mathbb{R}^n \mid Az = \rho(A)z\}, \tag{2.1}$$

which includes all positive eigenvectors of A provided that the nonnegative matrix A has at least one positive eigenvector (see [17]).

$C(X, Y)$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, let $C \triangleq C((-\infty, t_0], \mathbb{R}^n)$, where $t_0 \ge 0$.

Let $\mathbb{R}_+ = [0, +\infty)$. For $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $\varphi \in C$, $\tau(t) \in C[\mathbb{R}, \mathbb{R}_+]$, we define $[x]^+ = (|x_1|, |x_2|, \ldots, |x_n|)^T$, $[A]^+ = (|a_{ij}|)_{n\times n}$, $[\varphi(t)]^+_{\tau(t)} = (\|\varphi_1(t)\|_{\tau(t)}, \|\varphi_2(t)\|_{\tau(t)}, \ldots, \|\varphi_n(t)\|_{\tau(t)})^T$, $\|\varphi_i(t)\|_{\tau(t)} = \sup_{0\le s\le\tau(t)}|\varphi_i(t-s)|$, $i = 1, 2, \ldots, n$. For $\tau(t) = \infty$, we define $[x(t)]^+_{\tau(t)} = [x(t)]^+_{\infty}$.

Definition 2.1 (Xu [4]). $f(t, s) \in UC_t$ means that $f \in C[\mathbb{R}_+ \times \mathbb{R}, \mathbb{R}_+]$ and for any given α and any $\varepsilon > 0$ there exist positive numbers B, T, and A satisfying

$$\int_\alpha^t f(t, s)ds \le B, \quad \int_\alpha^{t-T} f(t, s)ds < \varepsilon, \quad \forall t \ge A. \tag{2.2}$$

Especially, $f \in UC_t$ if $f(t, s) = f(t - s)$ and $\int_0^\infty f(u)du < \infty$.

Lemma 2.2 (Lasalle [18]). *If $M \ge 0$ and $\rho(M) < 1$, then $(E - M)^{-1} \ge 0$.*

3. Delay Integral Inequality

Theorem 3.1. *Let $y(t) \in C[\mathbb{R}, \mathbb{R}_+^{n\times 1}]$ be a solution of the delay integral inequality*

$$y(t) \le G(t, t_0)\varphi(t_0) + \int_{\alpha_1}^t B(t, s)\left[y(s)\right]_{\tau(s)}ds + \int_{\alpha_2}^t \Psi(t, s)\int_{\alpha_3}^s \zeta(s, v)\left[y(v)\right]_{\tau(v)}dv\,ds + D, \quad t > t_0, \tag{3.1}$$

$$y(t) \le \varphi(t), \quad \forall t \in (-\infty, t_0], \tag{3.2}$$

where $G(t, t_0) \in C(\mathbb{R}_+ \times \mathbb{R}_+, \mathbb{R}_+^{n\times n})$, $\varphi(t) \in C[(-\infty, t_0], \mathbb{R}_+^{n\times 1}]$, $D = (d_1, \ldots, d_n)^T \ge 0$, $\lim_{t\to+\infty}(t - \tau(t)) = +\infty$, $\alpha_i \in (-\infty, 0]$ $(i = 1, 2, 3)$. Assume that the following conditions are satisfied:

(H_1) *$G(t, t_0)\varphi(t_0) \to 0$ as $t \to +\infty$, $B(t, u) = (b_{ij}(t, u))_{n\times n}$, $\Psi(t, u) = (\varphi_{ij}(t, u))_{n\times n}$, $\zeta(t, u) = (\zeta_{ij}(t, u))_{n\times n}$, $b_{ij}(t, u)$, $\varphi_{ij}(t, u)$, $\zeta_{ij}(t, u) \in UC_t$, and there exists a nonnegative matrix $\Pi = (\pi_{ij})_{n\times n}$ such that*

$$\int_{\alpha_1}^t B(t, s)ds + \int_{\alpha_2}^t \Psi(t, s)\int_{\alpha_3}^s \zeta(s, v)dv\,ds \le \Pi, \quad \text{for } \forall t \ge t_0 \tag{3.3}$$

(H_2) $\rho(\Pi) < 1$.

If the initial condition satisfies

$$\varphi(t) < k_1 z + (E - \Pi)^{-1} D, \quad t \in (-\infty, t_0], \tag{3.4}$$

where $k_1 \geq 0$, $z > 0$, and $z \in \Omega_\rho(\Pi)$, then there exists a constant $k \geq k_1$ such that

$$y(t) < kz + (E - \Pi)^{-1} D, \quad \textit{for } t \geq t_0. \tag{3.5}$$

Proof. By the condition $\Pi \geq 0$ and Lemma 2.2, together with $\rho(\Pi) < 1$, this implies that $(E - \Pi)^{-1}$ exists and $(E - \Pi)^{-1} \geq 0$.

From $\lim_{t \to +\infty} G(t, t_0)\varphi(t_0) = 0$, there is a $T > t_0$ such that

$$G(t, t_0)\varphi(t_0) \leq \frac{1 - \rho(\Pi)}{2} k_1 z, \quad \text{for } t \geq T. \tag{3.6}$$

By the continuity of $y(t)$, together with (3.2) and (3.4), there exists a constant $k \geq k_1$ such that

$$y(t) < kz + (E - \Pi)^{-1} D, \quad \text{for } t \in (-\infty, T]. \tag{3.7}$$

In the following, we will prove that

$$y(t) < kz + (E - \Pi)^{-1} D, \quad \text{for } t \geq T. \tag{3.8}$$

If this is not true, from (3.7) and the continuity of $y(t)$, then there must be a constant $t_1 > T$ and some integer i such that

$$y_i(t_1) = e^i \left\{ kz + (E - \Pi)^{-1} D \right\}, \tag{3.9}$$

$$y(t) \leq kz + (E - \Pi)^{-1} D, \quad \text{for } t \leq t_1, \tag{3.10}$$

where $e^i = (\underbrace{0, 0, \ldots, 0, 1}_{i}, 0, \ldots, 0)$.

Using (3.1), (3.3), (3.6), (3.10), and $\rho(\Pi) < 1$, we obtain that

$$\begin{aligned}
y_i(t_1) &= e^i\{y(t_1)\} \\
&\le e^i\left\{G(t_1,t_0)\varphi(t_0) + \int_{\alpha_1}^{t_1} B(t_1,s)\left[y(s)\right]_{\tau(s)}ds\right. \\
&\qquad \left. + \int_{\alpha_2}^{t_1}\Psi(t_1,s)\int_{\alpha_3}^{s}\zeta(s,v)\left[y(v)\right]_{\tau(v)}dv\,ds + D\right\} \\
&\le e^i\left\{\frac{1-\rho(\Pi)}{2}k_1 z + \int_{\alpha_1}^{t_1} B(t_1,s)\left[kz + (E-\Pi)^{-1}D\right]ds\right. \\
&\qquad \left. + \int_{\alpha_2}^{t_1}\Psi(t_1,s)\int_{\alpha_3}^{s}\zeta(s,v)\left[kz + (E-\Pi)^{-1}D\right]dv\,ds + D\right\} \\
&\le e^i\left\{\frac{1-\rho(\Pi)}{2}kz + \Pi\left[kz + (E-\Pi)^{-1}D\right] + (E-\Pi)(E-\Pi)^{-1}D\right\} \\
&\le e^i\left\{\frac{1-\rho(\Pi)}{2}kz + \rho(\Pi)kz + (\Pi + E - \Pi)(E-\Pi)^{-1}D\right\} \\
&\le e^i\left\{\frac{1+\rho(\Pi)}{2}kz + (E-\Pi)^{-1}D\right\} \\
&< e^i\left\{kz + (E-\Pi)^{-1}D\right\}.
\end{aligned} \tag{3.11}$$

This contradicts the equality in (3.9), and so (3.8) holds. The proof is complete. □

4. Applications

The delay integral inequality obtained in Section 3 can be widely applied to study the attracting set of the nonlinear functional differential equations. To illustrate the theory, we consider the following differential equation with distributed delays

$$\begin{gathered}
\dot{x}(t) = A(t)x(t) + F\left(t, x_t, \int_{-\infty}^{t} g(t,s,x(s))ds\right), \quad t \ge t_0, \\
x(t) = \varphi(t), \quad -\infty < t \le t_0,
\end{gathered} \tag{4.1}$$

where $x = (x_1,\ldots,x_n)^T \in \mathbb{R}^n$, $g \in C[\mathbb{R}\times\mathbb{R}\times\mathbb{R}^n,\mathbb{R}^n]$, $F \in C[\mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n,\mathbb{R}^n]$, $g(t,s,0) \equiv 0$, $F(t,0,0) \equiv 0$, $x_t = x(t-\tau(t))$, $\tau(t) \in C[\mathbb{R},\mathbb{R}_+]$, $t_0 > 0$, $\lim_{t\to+\infty}(t-\tau(t)) = +\infty$. We always assume that for any $\varphi \in C$, the system (4.1) has at least one solution through (t_0, φ) denoted by $x(t, t_0, \varphi)$ or simply $x(t)$ if no confusion should occur.

In order to study the attracting set, we rewrite (4.1) as

$$x(t) = \Phi(t, t_0)\varphi(t_0) + \int_{t_0}^{t} \Phi(t, s) F\left(s, x_s, \int_{-\infty}^{s} g(s, v, x(v)) dv\right) ds, \quad t \geq t_0,$$
$$x(t) = \varphi(t), \quad -\infty < t \leq t_0, \tag{4.2}$$

where $\Phi(t, t_0)$ be a fundamental matrix of the linear equation $\dot{x}(t) = A(t)x(t)$.

Throughout this section, we suppose the following:

($\mathbb{A}_1$) $[F(t, x_t, \int_{-\infty}^{t} g(t, s, x(s)) ds)]^+ \leq B(t)[x(t)]_{\tau(t)}^+ + \int_{-\infty}^{t} \zeta(t-s)[x(s)]_{\tau(s)}^+ ds + J(t)$, where $B(t) \in C[\mathbb{R}, \mathbb{R}_+^{n\times n}]$, $\zeta(t-u) = \zeta(t, u) \in C[\mathbb{R} \times \mathbb{R}, \mathbb{R}_+^{n\times n}]$, $J(t) = (J_1(t), \ldots, J_n(t))^T \geq 0$, $J_i(t) \in C[\mathbb{R}, \mathbb{R}_+]$, $i = 1, 2, \ldots, n$,

($\mathbb{A}_2$) $[\Phi(t, s)]^+ B(s) = (w_{ij}(t, s))_{n\times n}$, $[\Phi(t, s)]^+ = (\phi_{ij}(t, s))_{n\times n}$, $\zeta(t-s) = (\zeta_{ij}(t-s))_{n\times n}$. $w_{ij}(t, s)$, $\phi_{ij}(t, s) \in UC_t$. $\int_0^{+\infty} \zeta_{ij}(s) ds < +\infty$, $(i, j = 1, 2, \ldots, n)$. $\lim_{t\to+\infty} \Phi(t, t_0) = 0$. There are two matrices $\Pi = (\pi_{ij})_{n\times n} \geq 0$ and $D = (d_1, \ldots, d_n)^T \geq 0$ such that

$$\int_{t_0}^{t} [\Phi(t, s)]^+ B(s) ds + \int_{t_0}^{t} [\Phi(t, s)]^+ \int_{-\infty}^{s} \zeta(s, v) dv\, ds \leq \Pi,$$
$$\int_{t_0}^{t} [\Phi(t, s)]^+ J(s) ds \leq D, \quad \text{for } \forall t \geq t_0, \tag{4.3}$$

($\mathbb{A}_3$) $\rho(\Pi) < 1$.

Definition 4.1. The set $S \subset C$ is called a global attracting set of (4.1), if for any initial value $\varphi \in C$, the solution $x_t(t_0, \varphi)$ converges to S as $t \to +\infty$. That is,

$$\operatorname{dist}(x_t, S) \longrightarrow 0 \quad \text{as } t \longrightarrow +\infty, \tag{4.4}$$

where $\operatorname{dist}(\phi, S) = \inf_{\varphi\in S} d(\phi, \varphi)$, $d(\phi, \varphi)$ is the distance of ϕ to φ in $\mathbb{R}^n$.

Theorem 4.2. *Assume that* ($\mathbb{A}_1$)–($\mathbb{A}_3$) *hold. Then* $S = \{\varphi \in C \mid [\varphi]_\tau^+ \leq [E - \Pi]^{-1} D\}$ *is a global attracting set of* (4.1).

Proof. It follows from ($\mathbb{A}_1$)-($\mathbb{A}_2$) and (4.2) that

$$[x(t)]^+ \leq [\Phi(t, t_0)]^+ [\varphi(t_0)]^+ + \int_{t_0}^{t} [\Phi(t, s)]^+ B(s)[x(s)]_{\tau(s)}^+ ds$$
$$+ \int_{t_0}^{t} [\Phi(t, s)]^+ \int_{-\infty}^{s} \zeta(s-v)[x(v)]_{\tau(v)}^+ dv\, ds + D, \quad t \geq t_0. \tag{4.5}$$

For the initial conditions $x(t) = \varphi(t), -\infty < t \le t_0$, where $\varphi \in C$, we have

$$[x(t)]^+ \le k_0 z, \qquad -\infty < t \le t_0, \tag{4.6}$$

where $z \in \Omega_\rho(\Pi),\ z > 0,\ k_0 = \|\varphi\| / \min_{1\le i\le n} z_i \ge 0,\ \|\varphi\| = \max_{1\le i\le n}\|\varphi_i(t_0)\|_\infty$, and so

$$[x(t)]^+ \le k_0 z + (E - \Pi)^{-1} D, \quad -\infty < t \le t_0. \tag{4.7}$$

By (4.5)-(4.7), $(\mathbb{A}_1)$–$(\mathbb{A}_3)$ and Theorem 3.1, then there exists a constant $k > k_0$ such that

$$[x(t)]^+ < kz + (E - \Pi)^{-1} D, \quad \text{for } t \ge t_0. \tag{4.8}$$

From (4.8), there exists a constant vector $\sigma \ge 0$, such that

$$\overline{\lim_{t\to+\infty}} [x(t)]^+ = \sigma \le kz + [E - \Pi]^{-1} D. \tag{4.9}$$

Next we will show that $\sigma \in S$. From $\lim_{t\to+\infty}\Phi(t, t_0) = 0$, $w_{ij}, \phi_{ij} \in UC_t$, and $\int_0^{+\infty} \zeta_{ij}(s)ds < +\infty$, for any $\varepsilon > 0$ and $e = (1, 1, \ldots, 1)^T \in \mathbb{R}_+^{n\times 1}$, there exist a positive number A and a positive constant matrix $R \in \mathbb{R}_+^{n\times n}$ such that for all $t \ge t_0 + A$

$$[\Phi(t, t_0)]^+ [\varphi(t_0)]^+ < \frac{\varepsilon e}{4}, \qquad \int_{t_0}^{t} [\Phi(t, s)]^+ ds \le R, \tag{4.10}$$

$$\int_{-\infty}^{t} \zeta(t - v)\left[kz + (E - \Pi)^{-1} D\right] dv \le Re, \qquad \int_{t_0}^{t-A} [\Phi(t, s)]^+ ds \le \frac{\varepsilon}{4} R^{-1}, \tag{4.11}$$

$$\int_{t_0}^{t-A} [\Phi(t, s)]^+ B(s)\left[kz + (E - \Pi)^{-1} D\right] ds \le \frac{\varepsilon e}{4}\ , \tag{4.12}$$

$$\int_{A}^{+\infty} \zeta(u)\left[kz + (E - \Pi)^{-1} D\right] du \le \frac{\varepsilon R^{-1} e}{4}. \tag{4.13}$$

According to the definition of superior limit and $\lim_{t\to+\infty}(t-\tau(t)) = +\infty$, there exists sufficient large $t_2 \geq t_0 + 2A$, such that for any $t \geq t_2$,

$$[x(t)]^+_{r(t)} < \sigma + \varepsilon e, \quad \text{where } r(t) = 2A + \sup_{t-2A<s<t} \tau(s). \tag{4.14}$$

So, from $(\mathbb{A}_1)$, (4.5), and (4.10)-(4.14), when $t \geq t_2$, we obtain

$$\begin{aligned}
[x(t)]^+ &\leq [\Phi(t,t_0)]^+\left[\varphi(t_0)\right]^+ + \int_{t_0}^{t}[\Phi(t,s)]^+B(s)[x(s)]^+_{\tau(s)}ds \\
&\quad + \int_{t_0}^{t}[\Phi(t,s)]^+\int_{-\infty}^{s}\zeta(s-v)[x(v)]^+_{\tau(v)}dv\,ds + D \\
&\leq \frac{\varepsilon e}{4} + \left[\int_{t_0}^{t-A} + \int_{t-A}^{t}\right][\Phi(t,s)]^+B(s)[x(s)]^+_{\tau(s)}ds \\
&\quad + \int_{t_0}^{t-A}[\Phi(t,s)]^+\int_{-\infty}^{s}\zeta(s-v)[x(v)]^+_{\tau(v)}dv\,ds \\
&\quad + \int_{t-A}^{t}[\Phi(t,s)]^+\int_{-\infty}^{s-A}\zeta(s-v)[x(v)]^+_{\tau(v)}dv\,ds \\
&\quad + \int_{t-A}^{t}[\Phi(t,s)]^+\int_{s-A}^{s}\zeta(s-v)[x(v)]^+_{\tau(v)}dv\,ds + D \\
&\leq \frac{\varepsilon e}{4} + \frac{\varepsilon e}{4} + \int_{t-A}^{t}[\Phi(t,s)]^+B(s)[x(s)]^+_{\tau(s)}ds + \frac{\varepsilon}{4}R^{-1}Re \\
&\quad + \int_{t-A}^{t}[\Phi(t,s)]^+\int_{A}^{+\infty}\zeta(u)\left[kz + (E-\Pi)^{-1}D\right]du\,ds \\
&\quad + \int_{t-A}^{t}[\Phi(t,s)]^+\int_{s-A}^{s}\zeta(s-v)dv\,ds[x(t)]^+_{r(t)} + D \\
&\leq \frac{3\varepsilon e}{4} + \int_{t-A}^{t}[\Phi(t,s)]^+B(s)ds[x(t)]^+_{r(t)} + \int_{t_0}^{t}[\Phi(t,s)]^+ds\frac{\varepsilon R^{-1}e}{4} \\
&\quad + \int_{t-A}^{t}[\Phi(t,s)]^+\int_{s-A}^{s}\zeta(s-v)dv\,ds(\sigma+\varepsilon e) + D \\
&\leq \frac{3\varepsilon e}{4} + \int_{t_0}^{t}[\Phi(t,s)]^+B(s)ds(\sigma+\varepsilon e) + \frac{\varepsilon}{4}RR^{-1}e \\
&\quad + \int_{t_0}^{t}[\Phi(t,s)]^+\int_{-\infty}^{s}\zeta(s-v)dv\,ds(\sigma+\varepsilon e) + D \\
&\leq \varepsilon e + \Pi(\sigma+\varepsilon e) + D.
\end{aligned} \tag{4.15}$$

Due to (4.9) and the definition of superior limit, there exists $t_3 \geq t_2$, such that $[x(t_3)]^+ > \sigma - \varepsilon e$. So,

$$\sigma - \varepsilon e < \varepsilon e + \Pi(\sigma + \varepsilon e) + D, \quad t \geq t_3. \tag{4.16}$$

Letting $\varepsilon \to 0$, we have $\sigma \leq (E - \Pi)^{-1} D$, that is $\sigma \in S$, and the proof is completed. □

Corollary 4.3. *If $x(t) = 0$ is an equilibrium point of system* (4.1)*, suppose that the conditions of Theorem 4.2. hold and $J(t) = 0$, then the equilibrium $x(t) = 0$ is globally asymptotically stable.*

Remark 4.4. In Corollary 4.3., if $B(t) = B\lambda$, $\zeta(t-u) = K(t-u)\lambda$, $B = (b_{ij})_{n\times n} > 0$, $\lambda = \operatorname{diag}(\lambda_1, \ldots, \lambda_n) > 0$, $K(t-u) = K(t,u) \in C[R \times R, R_+^{n\times n}]$, $K(t-s) = (k_{ij}(t-s))_{n\times n}$, $\int_0^{+\infty} k_{ij}(s)ds < a_{ij}$, $A(t) = \operatorname{diag}(-\alpha_1, \ldots, -\alpha_n)$, $\alpha_i > 0$ $(i = 1, 2, \ldots, n)$, then we can get Theorems 3 and 4 in [16]. In fact $\Phi(t, t_0) = \operatorname{diag}(e^{-\alpha_1(t-t_0)}, \ldots, e^{-\alpha_n(t-t_0)}) \in UC_t$, Theorems 3 and 4 in [16] satisfy all conditions of Corollary 4.3.

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