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Research Article

A System of Random Nonlinear Variational Inclusions Involving Random Fuzzy Mappings and $H(\cdot,\cdot)$-Monotone Set-Valued Mappings

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We introduce and study a new system of random nonlinear generalized variational inclusions involving random fuzzy mappings and set-valued mappings with $H(\cdot,\cdot)$-monotonicity in two Hilbert spaces and develop a new algorithm which produces four random iterative sequences. We also discuss the existence of the random solutions to this new kind of system of variational inclusions and the convergence of the random iterative sequences generated by the algorithm.

1. Introduction

The classic variational inequality problem VI(F, K) is to determine a vector $x^* \in K \subset R^n$, such that

$$\left\langle F(x^*)^T, x - x^* \right\rangle \geq 0, \quad \forall x \in K, \tag{1.1}$$

where F is a given continuous function from K to R^n and K is a given closed convex subset of the n-dimensional Euclidean space R^n. This is equivalent to find an $x^* \in K$, such that

$$0 \in F(x^*) + N_{\perp}(x^*), \tag{1.2}$$

where $N_{\perp}$ is normal cone operator.

Due to its enormous applications in solving problems arising from the fields of economics, mechanics, physical equilibrium analysis, optimization and control, transportation equilibrium, and linear or nonlinear programming etcetera, variational inequality and its generalizations have been extensively studied during the past 40 years. For details, we refer readers to [1–7] and the references therein.

It is not a surprise that many practical situations occur by chance and so variational inequalities with random variables/mappings have also been widely studied in the past decade. For instance, some random variational inequalities and random quasivariational inequalities problems have been introduced and studied by Chang [8], Chang and Huang [9, 10], Chang and Zhu [11], Huang [12, 13], Husain et al. [14], Tan et al. [15], Tan [16], and Yuan [7].

It is well known that one of the most important and interesting problems in the theory of variational inequalities is to develop efficient and implementable algorithms for solving variational inequalities and its generalizations. The monotonic properties of associated operators play essential roles in proving the existence of solutions and the convergence of sequences generated by iterative algorithms. In 2001, Huang and Fang [17] were the first to introduce the generalized m-accretive mapping and give the definition of the resolvent operator for generalized m-accretive mappings in Banach spaces. They also showed some properties of the resolvent operator for generalized m-accretive mappings. Recently, Fang and Huang, Verma, and Cho and Lan investigated many generalized operators such as H-monotone, H-accretive, (H,η)-monotone, (H,η)-accretive, and (A,η)-accretive mappings. For details, we refer to [6, 17–22] and the references therein. In 2008, Zou and Huang [23] introduced the $H(\cdot,\cdot)$-accretive operator in Banach spaces which provides a unified framework for the existing H-monotone, (H,η)-monotone, and (A,η)-monotone operators in Hilbert spaces and H-accretive, (H,η)-accretive, and (A,η)-accretive operators in Banach spaces.

In 1965, Zadeh [24] introduced the concept of fuzzy sets, which became a cornerstone of modern fuzzy mathematics. To explore connections among VIs, fuzzy mapping and random mappings, in 1997, Huang [25] introduced the concept of random fuzzy mappings and studied the random nonlinear quasicomplementarity problem for random fuzzy mappings. Later, Huang [26] studied the random generalized nonlinear variational inclusions for random fuzzy mappings. In 2005, Ahmad and Bazán [27] studied a class of random generalized nonlinear mixed variational inclusions for random fuzzy mappings and constructed an iterative algorithm for solving such random problems. For related work in this hot area, we refer to Ahmad and Farajzadeh [28], Ansari and Yao [29], Chang and Huang [9, 10], Cho and Huang [30], Cho and Lan [31], Huang [25, 26, 32], Huang et al. [33], and the references therein.

Motivated and inspired by recent research work mentioned above in this field, in this paper, we try to inject some new energy into this interesting field by studying on a new kind of random nonlinear variational inclusions in two Hilbert spaces. We will prove the existence of random solutions to the system of inclusions and propose an algorithm which produces a convergent iterative sequence. For a suitable choice of some mappings, we can obtain several known results [10, 11, 21, 23, 31, 34] as special cases of the main results of this paper.

2. Preliminaries

Throughout this paper, let $(\Omega,\mathcal{A})$ be a measurable space, where Ω is a set and $\mathcal{A}$ is a σ-algebra over Ω. Let X_1 be a separable real Hilbert space endowed with a norm $\|\cdot\|_{X_1}$ and an inner product $\langle\cdot,\cdot\rangle_{X_1}$. Let X_2 be a separable real Hilbert space endowed with a norm $\|\cdot\|_{X_2}$ and an inner product $\langle\cdot,\cdot\rangle_{X_2}$.

We denote by $D(\cdot,\cdot)$ the Hausdorff metric between two nonempty closed bounded subsets, where the Hausdorff metric between A and B is defined by

$$D(A,B) = \max\left\{\sup_{a\in A}\inf_{b\in B} d(a,b), \sup_{b\in B}\inf_{a\in A} d(a,b)\right\}. \tag{2.1}$$

We denote by $\mathcal{B}(X_1)$, 2^{X_1}, and $CB(X_1)$ the class of Borel σ-fields in X_1, and the family of all nonempty subsets of X_1, the family of all nonempty closed bounded subsets of X_1.

In this paper, to make it self-contained, we start with the following basic definitions and similar definitions can also be found in [26, 32, 34].

Definition 2.1. A mapping $x_1 : \Omega \to X_1$ is said to be measurable if for any $B \in \mathcal{B}(X_1)$,

$$\{t \in \Omega : x(t) \in B\} \in \mathcal{A}. \tag{2.2}$$

Definition 2.2. A mapping $T_1 : \Omega \times X_1 \to X_1$ is called a random mapping if for any $x \in X_1$, $z_1(t) = T_1(t,x)$ is measurable.

Definition 2.3. A random mapping $T_1 : \Omega \times X_1 \to X_1$ is said to be continuous if for any $t \in \Omega$, $T_1(t,\cdot) : X_1 \to X_1$ is continuous.

Definition 2.4. A set-valued mapping $V_1 : \Omega \to 2^{X_1}$ is said to be measurable if for any $B \in \mathcal{B}(X_1)$,

$$V_1^{-1}(B) = \{v \in \Omega : V_1(v) \cap B \neq \emptyset\} \in \mathcal{A}. \tag{2.3}$$

Definition 2.5. A mapping $u : \Omega \to X_1$ is called a measurable selection of a set-valued measurable mapping $U : \Omega \to 2^{X_1}$ if u is measurable and for any $t \in \Omega$, $u(t) \in U(t)$.

Definition 2.6. A set-valued mapping $W_1 : \Omega \times X_1 \to 2^{X_1}$ is called random set-valued if for any $x_1 \in X_1$, $W_1(\cdot, x_1) : \Omega \to 2^{X_1}$ is a measurable set valued mapping.

Definition 2.7. A random set-valued mapping $W_1 : \Omega \times X_1 \to CB(X_1)$ is said to be $\xi_E(t)$-D-continuous if there exists a measurable function $\xi_E : \Omega \to (0,+\infty)$, such that

$$D(W_1(t,x_1(t)), W_1(t,x_2(t))) \leq \xi_E(t)\|x_1(t) - x_2(t)\|_{X_1}, \tag{2.4}$$

for all $t \in \Omega$ and $x_1(t), x_2(t) \in X_1$.

Definition 2.8. A set-valued mapping $A : X_1 \to 2^{X_1}$ is said to be monotone if for all $x_1, y_1 \in X_1$ and $u_1 \in A(x_1)$, $v_1 \in A(y_1)$,

$$\langle u_1 - v_1, x_1 - y_1\rangle_{X_1} \geq 0. \tag{2.5}$$

Definition 2.9. Let $f_1, g_1 : X_1 \to X_1$ and $H_1 : X_1 \times X_1 \to X_1$ be three single-valued mappings and $A : X_1 \to 2^{X_1}$ be a set-valued mapping. A is said to be $H_1(\cdot,\cdot)$-monotone with respect to operators f_1 and g_1 if A is monotone and $(H_1(f_1,g_1) + \lambda A)(X_1) = X_1$, for every $\lambda > 0$.

Definition 2.10. The inverses of $A : X_1 \to 2^{X_1}$ and $B : X_2 \to 2^{X_2}$ are defined as follows, respectively,

$$A^{-1}(y) = \{x \in X_1 : y \in A(x)\}, \quad \forall y \in X_1,$$
$$B^{-1}(y) = \{x \in X_2 : y \in B(x)\}, \quad \forall y \in X_2. \tag{2.6}$$

Definition 2.11. $p : \Omega \times X_1 \to X_1$ is said to be

(1) monotone if

$$\langle p(t, x_1(t)) - p(t, x_2(t)), x_1(t) - x_2(t) \rangle_{X_1} \geq 0, \quad \forall t \in \Omega,\ \forall x_1(t), \quad x_2(t) \in X_1, \tag{2.7}$$

(2) strictly monotone if p is monotone and

$$\langle p(t, x_1(t)) - p(t, x_2(t)), x_1(t) - x_2(t) \rangle_{X_1} = 0 \Longleftrightarrow x_1(t) = x_2(t), \quad \forall t \in \Omega,\ \forall x_1(t),\ x_2(t) \in X_1, \tag{2.8}$$

(3) $\delta_p(t)$-strongly monotone if there exists some measurable function $\delta_p : \Omega \to (0, +\infty)$, such that

$$\langle p(t, x_1(t)) - p(t, x_2(t)), x_1(t) - x_2(t) \rangle_{X_1} \geq \delta_p(t) \|x_1(t) - x_2(t)\|_{x_1}^2, \quad \forall t \in \Omega,\ \forall x_1(t),\ x_2(t) \in X_1, \tag{2.9}$$

(4) $\sigma_p(t)$-Lipschitz continuous if there exists some measurable function $\sigma_p : \Omega \to (0, +\infty)$, such that

$$\|p(t, x_1(t)) - p(t, x_2(t))\|_{X_1} \leq \sigma_p(t) \|x_1(t) - x_2(t)\|_{X_1}, \quad \forall t \in \Omega, \quad \forall x_1(t),\ x_2(t) \in X_1. \tag{2.10}$$

Definition 2.12. A single-valued mapping $M : X_1 \times X_1 \times X_2 \to X_1$ is said to be

(1) $\zeta_A(t)$-strongly monotone with respect to the random single-valued mapping $s_M : \Omega \times X_1 \to X_1$ in the first argument if there exists some measurable function $\zeta_A : \Omega \to (0, +\infty)$, such that

$$\langle M(s_M(t, u_1(t)), \cdot, \cdot) - M(s_M(t, u_2(t)), \cdot, \cdot), u_1(t) - u_2(t) \rangle_{X_1} \geq \zeta_A(t) \|u_1(t) - u_2(t)\|_{X_1}^2, \tag{2.11}$$

for all $t \in \Omega$ and $u_1(t), u_2(t) \in X_1$,

(2) $\xi_M(t)$-Lipschitz continuous with respect to the random single-valued mapping $s_M : \Omega \times X_1 \to X_1$ in its first argument if there exists some measurable function $\xi_M : \Omega \to (0, +\infty)$, such that

$$\|M(s_M(t, u_1(t)), \cdot, \cdot) - M(s_M(t, u_2(t)), \cdot, \cdot)\|_{X_1} \leq \xi_M(t) \|u_1(t) - u_2(t)\|_{X_1}, \tag{2.12}$$

for all $t \in \Omega$ and $u_1(t), u_2(t) \in X_1$,

(3) $\beta_M(t)$-Lipschitz continuous with respect to its second argument if there exists some measurable function $\beta_M : \Omega \to (0, +\infty)$, such that

$$\|M(\cdot, x_1(t), \cdot) - M(\cdot, x_2(t), \cdot)\|_{X_1} \leq \beta_M(t)\|x_1(t) - x_2(t)\|_{X_1}, \tag{2.13}$$

for all $t \in \Omega$ and $x_1(t), x_2(t) \in X_1$,

(4) $\eta_M(t)$-Lipschitz continuous with respect to its third argument if there exists some measurable function $\eta_M : \Omega \to (0, +\infty)$ such that

$$\|M(\cdot, \cdot, y_1(t)) - M(\cdot, \cdot, y_2(t))\|_{X_1} \leq \eta_M(t)\|y_1(t) - y_2(t)\|_{X_2}, \tag{2.14}$$

for all $t \in \Omega$ and $y_1(t), y_2(t) \in X_2$;

Definition 2.13. Assume that $p : \Omega \times X_1 \to X_1$ is a random single-valued mapping, $f_1 : X_1 \to X_1$, $g_1 : X_1 \to X_1$, and $H_1(f_1, g_1) : X_1 \to X_1$ are three single-valued mappings, $H_1(f_1, g_1)$ is said to be

(1) $\mu_A(t)$-strongly monotone with respect to the mapping p if there exists some measurable function $\mu_A : \Omega \to (0, +\infty)$ such that

$$\begin{aligned}&\langle H_1(f_1(p(t, x_1(t))), g_1(p(t, x_1(t)))) - H_1(f_1(p(t, y_1(t))), g_1(p(t, y_1(t)))), x_1(t) - y_1(t)\rangle_{X_1}\\&\geq \mu_A(t)\|x_1(t) - y_1(t)\|_{X_1}^2,\end{aligned} \tag{2.15}$$

for all $t \in \Omega$ and $x_1(t), y_1(t) \in X_1$,

(2) $a_A(t)$-Lipschitz continuous with respect to the mapping p if there exists some measurable function $a_A : \Omega \to (0, +\infty)$ such that

$$\begin{aligned}&\|H_1(f_1(p(t, x_1(t))), g_1(p(t, x_1(t)))) - H_1(f_1(p(t, y_1(t))), g_1(p(t, y_1(t))))\|_{X_1}\\&\leq a_A(t)\|x_1(t) - y_1(t)\|_{X_1},\end{aligned} \tag{2.16}$$

for all $t \in \Omega$ and $x_1(t), y_1(t) \in X_1$.

(3) α_A-strongly monotone with respect to f_1 in the first argument if there exists a positive constant α_A, such that

$$\langle H_1(f_1(x_1), u_1) - H_1(f_1(y_1), u_1), x_1 - y_1\rangle_{X_1} \geq \alpha_A\|x_1 - y_1\|_{X_1}^2, \tag{2.17}$$

for all $x_1, y_1, u_1 \in X_1$,

(4) β_A-relaxed monotone with respect to g_1 in the second argument if there exists a positive constant β_A, such that

$$\langle H_1(u_1, g_1(x_1)) - H_1(u_1, g_1(y_1)), x_1 - y_1 \rangle_{X_1} \geq -\beta_A \|x_1 - y_1\|_{X_1}^2, \tag{2.18}$$

for all $x_1, y_1, u_1 \in X_1$.

Let $\mathcal{F}(X_1)$ be a collection of all fuzzy sets over X_1. A mapping F from Ω into $\mathcal{F}(X_1)$ is called a fuzzy mapping. If F is a fuzzy mapping on X_1, then for any given $t \in \Omega$, $F(t)$(denote it by F_t in the sequel) is a fuzzy set on X_1 and $F_t(y)$ is the membership function of y in F_t.

Let $A \in \mathcal{F}(X_1)$, $\alpha \in [0,1]$, then the set

$$(A)_\alpha = \{x \in X_1 : A(x) \geq \alpha\} \tag{2.19}$$

is called an α-cut set of fuzzy set A.

Definition 2.14. A random fuzzy mapping $F : \Omega \to \mathcal{F}(X_1)$ is said to be measurable if for any given $\alpha \in (0,1]$, $(F(\cdot))_\alpha : \Omega \to 2^{X_1}$ is a measurable set-valued mapping.

Definition 2.15. A fuzzy mapping $E : \Omega \times X_1 \to \mathcal{F}(X_1)$ is called a random fuzzy mapping if for any given $x_1 \in X_1$, $E(\cdot, x_1) : \Omega \to \mathcal{F}(X_1)$ is a measurable fuzzy mapping.

Remark 2.16. The above is mainly about some definitions in X_1. There are similar definitions and notations for operators in X_2.

Let $E : \Omega \times X_1 \to \mathcal{F}(X_1)$ and $F : \Omega \times X_2 \to \mathcal{F}(X_2)$ be two random fuzzy mappings satisfying the following condition $(**)$:

$(**)$ there exist two mappings $\alpha : X_1 \to (0,1]$ and $\beta : X_2 \to (0,1]$, such that

$$\begin{aligned} &(E_{t,x_1})_{\alpha(x_1)} \in CB(X_1), \quad \forall (t, x_1) \in \Omega \times X_1, \\ &(F_{t,x_2})_{\beta(x_2)} \in CB(X_2), \quad \forall (t, x_2) \in \Omega \times X_2. \end{aligned} \tag{2.20}$$

By using the random fuzzy mappings E and F, we can define the two set-valued mappings E^* and F^* as follows, respectively,

$$\begin{aligned} &E^* : \Omega \times X_1 \longrightarrow CB(X_1), \quad (t, x_1) \longrightarrow (E_{t,x_1})_{\alpha(x_1)}, \quad \forall (t, x_1) \in \Omega \times X_1, \\ &F^* : \Omega \times X_2 \longrightarrow CB(X_2), \quad (t, x_2) \longrightarrow (E_{t,x_2})_{\alpha(x_2)}, \quad \forall (t, x_2) \in \Omega \times X_2. \end{aligned} \tag{2.21}$$

It follows that

$$\begin{aligned} &E^*(t, x_1) = (E_{t,x_1})_{\alpha(x_1)} = \{z_1 \in X_1 : (E_{t,x_1})(z_1) \geq \alpha(x_1)\}, \\ &F^*(t, x_2) = (F_{t,x_2})_{\beta(x_2)} = \{z_2 \in X_2 : (F_{t,x_2})(z_2) \geq \beta(x_2)\}. \end{aligned} \tag{2.22}$$

It is easy to see that E^* and F^* are two random set-valued mappings. We call E^* and F^* the random set-valued mappings induced by the fuzzy mappings E and F, respectively.

Problem 1. Let $f_1, g_1 : X_1 \to X_1$ be two single-valued mappings and $s_M, p : \Omega \times X_1 \to X_1$ be two random single-valued mappings. Let $f_2, g_2 : X_2 \to X_2$ be two single-valued mappings and $s_N, q : \Omega \times X_2 \to X_2$ be two random single-valued mappings. Let $H_1 : X_1 \times X_1 \to X_1$, $H_2 : X_2 \times X_2 \to X_2$, $M : X_1 \times X_1 \times X_2 \to X_1$ and $N : X_2 \times X_1 \times X_2 \to X_2$ be four single-valued mappings. Suppose that $A : X_1 \to 2^{X_1}$ is an $H_1(\cdot,\cdot)$-monotone mapping with respect to f_1 and g_1 and $B : X_2 \to 2^{X_2}$ is an $H_2(\cdot,\cdot)$-monotone mapping with respect to f_2 and g_2. $E : \Omega \times X_1 \to \mathcal{F}(X_1)$ and $F : \Omega \times X_2 \to \mathcal{F}(X_2)$ are two random fuzzy mappings, α, β, E^*, and F^* are the same as the above. Assume that $p(t, u(t)) \cap \operatorname{dom}(A) \neq \emptyset$ and $q(t, v(t)) \cap \operatorname{dom}(B) \neq \emptyset$ for all $t \in \Omega$. We consider the following problem.

Find four measurable mappings $u, x : \Omega \to X_1$ and $v, y : \Omega \to X_2$, such that

$$\begin{gathered}
E_{t,u(t)}(x(t)) \geq \alpha(u(t)), \\
F_{t,v(t)}(y(t)) \geq \beta(v(t)), \\
0 \in M(s_M(t, u(t)), x(t), y(t)) + A(p(t, u(t))), \\
0 \in N(s_N(t, v(t)), x(t), y(t)) + B(q(t, v(t))),
\end{gathered} \tag{2.23}$$

for all $t \in \Omega$.

Problem 1 is called a system of generalized random nonlinear variational inclusions involving random fuzzy mappings and set-valued mappings with $H(\cdot,\cdot)$-monotonicity in two Hilbert spaces. A set of the four measurable mappings x, y, u, and v is called one solution of Problem 1.

3. Random Iterative Algorithm

In order to prove the main results, we need the following lemmas.

Lemma 3.1 (see [23]). *Let H_1, f_1, g_1, and A be defined as in Problem 1. Let $H_1(f_1, g_1)$ be α_A-strongly monotone with respect to f_1, β_A-relaxed monotone with respect to g_1, where $\alpha_A > \beta_A$. Suppose that $A : X_1 \to 2^{X_1}$ is an $H_1(\cdot,\cdot)$-monotone set-valued mapping with respect to f_1 and g_1, then the resolvent operator $R_{A,\lambda}^{H_1(\cdot,\cdot)} = (H(f, g) + \lambda A)^{-1}$ is a single-valued mapping.*

Lemma 3.2 (see [23]). *Let H_1, f_1, g_1, A be defined as in Problem 1. Let $H_1(f_1, g_1)$ be α_A-strongly monotone with respect to f_1, β_A-relaxed monotone with respect to g_1, where $\alpha_A > \beta_A$. Suppose that $A : X_1 \to 2^{X_1}$ is an $H_1(\cdot,\cdot)$-monotone set-valued mapping with respect to f_1 and g_1. Then, the resolvent operator $R_{A,\lambda}^{H_1(\cdot,\cdot)}$ is $1/(\alpha_A - \beta_A)$-Lipschitz continuous.*

Remark 3.3. Some interesting examples concerned with the $H_1(\cdot,\cdot)$-monotone mapping and the resolvent operator $R_{A,\lambda}^{H_1(\cdot,\cdot)}$ can be found in [23].

Lemma 3.4 (see Chang [8]). *Let $V : \Omega \times X_1 \to CB(X_1)$ be a D-continuous random set-valued mapping. Then for any given measurable mapping $u : \Omega \to X_1$, the set-valued mapping $V(\cdot, u(\cdot)) : \Omega \to CB(X_1)$ is measurable.*

Lemma 3.5 (see Chang [8]). *Let $V, W : \Omega \to CB(X_1)$ be two measurable set-valued mappings, and let $\varepsilon > 0$ be a constant and $u : \Omega \to X_1$ a measurable selection of V. Then there exists a measurable selection $v : \Omega \to X_1$ of W, such that for all $t \in \Omega$,*

$$\|u(t) - v(t)\| \leq (1+\varepsilon) D(V(t), W(t)). \tag{3.1}$$

Lemma 3.6. *The four measurable mappings $x, u : \Omega \to X_1$ and $y, v : \Omega \to X_2$ are solution of Problem 1 if and only if, for all $t \in \Omega$,*

$$\begin{gathered} x(t) \in E^*(t, u(t)), \\ x(t) \in F^*(t, v(t)), \\ p(t,u(t)) = R_{A,\lambda}^{H_1(\cdot,\cdot)}\left[H_1\left(f_1(p(t,u(t))), g_1(p(t,u(t)))\right) - \lambda M\left(s_M(t,u(t)), x(t), y(t)\right)\right], \\ q(t,u(t)) = R_{B,\rho}^{H_2(\cdot,\cdot)}\left[H_2\left(f_2(q(t,v(t))), g_2(q(t,v(t)))\right) - \rho N\left(s_N(t,v(t)), x(t), y(t)\right)\right], \end{gathered} \tag{3.2}$$

where $R_{A,\lambda}^{H_1(\cdot,\cdot)} = (H_1(f_1,g_1) + \lambda A)^{-1}$ and $R_{B,\rho}^{H_2(\cdot,\cdot)} = (H_2(f_2,g_2) + \rho B)^{-1}$ are two resolvent operators.

Proof. From the definitions of $R_{A,\lambda}^{H_1(\cdot,\cdot)}$ and $R_{B,\rho}^{H_2(\cdot,\cdot)}$, one has

$$\begin{aligned} &H_1\left(f_1(p(t,u(t))), g_1(p(t,u(t)))\right) - \lambda M\left(s_M(t,u(t)), x(t), y(t)\right) \\ &\qquad \in H_1\left(f_1(p(t,u(t))), g_1(p(t,u(t)))\right) + \lambda A\left(p(t,u(t))\right), \quad \forall t \in \Omega, \\ &H_2\left(f_2(q(t,v(t))), g_2(q(t,v(t)))\right) - \rho N\left(s_N(t,v(t)), x(t), y(t)\right) \\ &\qquad \in H_2\left(f_2(q(t,v(t))), g_2(q(t,v(t)))\right) + \rho B\left(q(t,v(t))\right), \quad \forall t \in \Omega. \end{aligned} \tag{3.3}$$

Hence,

$$\begin{gathered} 0 \in M\left(s_M(t,u(t)), x(t), y(t)\right) + A\left(p(t,u(t))\right), \quad \forall t \in \Omega, \\ 0 \in N\left(s_N(t,v(t)), x(t), y(t)\right) + B\left(q(t,v(t))\right), \quad \forall t \in \Omega. \end{gathered} \tag{3.4}$$

Thus, (x, y, u, v) is a set of solution of Problem 1. This completes the proof. □

Now we use Lemma 3.6 to construct the following algorithm.

Let $u_0 : \Omega \to X_1$ and $v_0 : \Omega \to X_2$ be two measurable mappings, then by Himmelberg [35], there exist $x_0 : \Omega \to X_1$, a measurable selection of $E^*(\cdot, u_0(\cdot)) : \Omega \to CB(X_1)$ and $y_0 : \Omega \to X_2$, a measurable selection of $F^*(\cdot, v_0(\cdot)) : \Omega \to CB(X_2)$. We now propose the following algorithm.

Algorithm 3.7. For any given measurable mappings $u_0 : \Omega \to X_1$ and $v_0 : \Omega \to X_2$, iterative sequences that attempt to solve Problem 1 are defined as follows:

$$\begin{aligned} u_{n+1}(t) &= u_n(t) - p(t, u_n(t)) \\ &\quad + R_{A,\lambda}^{H_1(\cdot,\cdot)}\left[H_1\left(f_1(p(t,u_n(t))), g_1(p(t,u_n(t)))\right) - \lambda M\left(s_M(t,u_n(t)), x_n(t), y_n(t)\right)\right], \\ v_{n+1}(t) &= v_n(t) - q(t, v_n(t)) \\ &\quad + R_{B,\rho}^{H_2(\cdot,\cdot)}\left[H_2\left(f_2(q(t,v_n(t))), g_2(q(t,v_n(t)))\right) - \rho N\left(s_N(t,v_n(t)), x_n(t), y_n(t)\right)\right]. \end{aligned} \tag{3.5}$$

Choose $x_{n+1}(t) \in E^*(t, u_{n+1}(t))$ and $y_{n+1}(t) \in F^*(t, v_{n+1}(t))$, such that

$$\begin{aligned} \|x_{n+1}(t) - x_n(t)\|_{X_1} &\le (1+\varepsilon_{n+1}) D(E^*(t,u_{n+1}(t)), E^*(t,u_n(t))), \\ \left\|y_{n+1}(t) - y_n(t)\right\|_{X_2} &\le (1+\varepsilon_{n+1}) D(F^*(t,v_{n+1}(t)), F^*(t,v_n(t))), \end{aligned} \tag{3.6}$$

for any $t \in \Omega$ and $n = 0, 1, 2, 3, \ldots$.

Remark 3.8. The existence of x_n and y_n is guaranteed by Lemmas 3.4 and 3.5.

4. Existence and Convergence

Theorem 4.1. *Let X_1 and X_2 be two separable real Hilbert spaces. Suppose that $s_M, p : \Omega \times X_1 \to X_1$ and $s_N, q : \Omega \times X_2 \to X_2$ are four random mappings. Suppose that $f_1, g_1 : X_1 \to X_1$, $H_1 : X_1 \times X_1 \to X_1$, $f_2, g_2 : X_2 \to X_2$, $H_2 : X_2 \times X_2 \to X_2$ are six single-valued mappings. Assume that*

(1) $A : X_1 \to 2^{X_1}$ *is an $H_1(\cdot,\cdot)$-monotone with respect to operators f_1 and g_1,*

(2) $B : X_2 \to 2^{X_2}$ *is an $H_2(\cdot,\cdot)$-monotone with respect to operators f_2 and g_2,*

(3) $p(t,u(t)) \cap \mathrm{dom}(A) \ne \emptyset$ *and* $q(t,v(t)) \cap \mathrm{dom}(B) \ne \emptyset$ *for all* $t \in \Omega$,

(4) $M : X_1 \times X_1 \times X_2 \to X_1$ *is $\zeta_A(t)$-monotone with respect to the mapping s_M in the first argument, $\xi_M(t)$-Lipschitz continuous with respect to mapping s_M in the first argument, $\beta_M(t)$-Lipschitz continuous with respect to the second argument and $\eta_M(t)$-Lipschitz continuous with respect to the third argument,*

(5) $N : X_2 \times X_1 \times X_2 \to X_2$ *is $\zeta_B(t)$-monotone with respect to the mapping s_N in the first argument, $\xi_N(t)$-Lipschitz continuous with respect to mapping s_N in the first argument, $\beta_N(t)$-Lipschitz continuous with respect to the second argument and $\eta_N(t)$-Lipschitz continuous with respect to the third argument,*

(6) *Let $E : \Omega \times X_1 \to \mathcal{F}(X_1)$ and $F : \Omega \times X_2 \to \mathcal{F}(X_2)$ be two random fuzzy mappings satisfying the condition $(**)$, α, β, E^* and F^* are four mappings induced by E and F. E^* and F^* are $\xi_E(t)$-D-Lipschitz and $\xi_F(t)$-D-Lipschitz continuous, respectively;*

(7) *p is $\delta_p(t)$-strongly monotone with respect to its second argument, $\sigma_p(t)$-Lipschitz continuous with respect to its second argument, $H_1(f_1, g_1)$ is $\mu_A(t)$-strongly monotone*

with respect to the mapping p *and* $a_A(t)$*-Lipschitz continuous with respect to the mapping* p*,*

(8) $H_1(f_1, g_1)$ *is* α_A*-strongly monotone with respect to* f_1*, and* β_A*-relaxed monotone with respect to* g_1*, where* $\alpha_A > \beta_A$*,*

(9) q *is* $\delta_q(t)$*-strongly monotone with respect to its second argument and* $\sigma_q(t)$*-Lipschitz continuous with respect to its second argument,* $H_2(f_2, g_2)$ *is* $\mu_B(t)$*-strongly monotone with respect to the mapping* q*, and* $a_B(t)$*-Lipschitz continuous with respect to the mapping* q*,*

(10) $H_2(f_2, g_2)$ *is* α_B*-strongly monotone with respect to* f_2 *and* β_B*-relaxed monotone with respect to* g_2*, where* $\alpha_B > \beta_B$*,*

If

$$\begin{aligned}
A(t) &= \frac{\lambda}{\alpha_A - \beta_A}\beta_M(t)\xi_E(t) + \sqrt[2]{1 - 2\delta_p(t) + \left[\sigma_p(t)\right]^2} \\
&\quad + \frac{1}{\alpha_A - \beta_A}\sqrt[2]{1 - 2\mu_A(t) + [a_A(t)]^2} \\
&\quad + \frac{1}{\alpha_A - \beta_A}\sqrt[2]{1 - 2\lambda\zeta_A(t) + \lambda^2[\xi_M(t)]^2}, \\
B(t) &= \frac{\lambda}{\alpha_A - \beta_A}\eta_M(t)\xi_F(t), \\
C(t) &= \frac{\rho}{\alpha_B - \beta_B}\beta_N(t)\xi_E(t), \\
D(t) &= \frac{\rho}{\alpha_B - \beta_B}\eta_N(t)\xi_F(t) + \sqrt[2]{1 - 2\delta_q(t) + [\sigma_q(t))]^2} \\
&\quad + \frac{1}{\alpha_B - \beta_B}\sqrt[2]{1 - 2\mu_B(t) + [a_B(t)]^2} \\
&\quad + \frac{1}{\alpha_B - \beta_B}\sqrt[2]{1 - 2\rho\zeta_B(t) + \rho^2[\xi_N(t)]^2}, \\
&0 < A(t) + C(t) < 1, \quad \forall t \in \Omega, \\
&0 < B(t) + D(t) < 1, \quad \forall t \in \Omega,
\end{aligned} \tag{4.1}$$

then there exist four measurable mappings $x, u : \Omega \to X_1$ *and* $y, v : \Omega \to X_2$*, such that* (x, y, u, v) *is a set of solution of Problem 1. Moreover,*

$$\lim_{n\to\infty} x_n(t) = x(t), \quad \lim_{n\to\infty} y_n(t) = y(t), \quad \lim_{n\to\infty} u_n(t) = u(t), \quad \lim_{n\to\infty} v_n(t) = v(t), \tag{4.2}$$

where $x_n(t)$*,* $y_n(t)$*,* $u_n(t)$*, and* $v_n(t)$ *are defined as in Algorithm 3.7.*

Proof. To simplify calculations, for any $n \in N$, we let

$$\begin{aligned} s_n(t) &= H_1(f_1(p(t,u_n(t))), g_1(p(t,u_n(t)))) - \lambda M(s_M(t,u_n(t)), x_n(t), y_n(t)), \\ t_n(t) &= H_2(f_2(q(t,v_n(t))), g_2(q(t,v_n(t)))) - \rho N(s_N(t,v_n(t)), x_n(t), y_n(t)), \end{aligned} \tag{4.3}$$

$$S_n(t) = -p(t,u_n(t)) + R_{A,\lambda}^{H_1(\cdot,\cdot)}(s_n(t)), \quad T_n(t) = -q(t,v_n(t)) + R_{B,\rho}^{H_2(\cdot,\cdot)}(t_n(t)). \tag{4.4}$$

We use $\|\cdot\|_1$ to replace $\|\cdot\|_{X_1}$ and $\|\cdot\|_2$ to replace $\|\cdot\|_{X_2}$.

Thus,

$$u_{n+1}(t) = u_n(t) + S_n(t), \tag{4.5}$$

$$v_{n+1}(t) = v_n(t) + T_n(t). \tag{4.6}$$

By the definition of $S_n(t)$, $T_n(t)$, s_n, and t_n, we have the following

$$\begin{aligned} \|S_n(t)\|_1 &= \|S_{n-1}(t) + S_n(t) - S_{n-1}(t)\|_1 \\ &= \|[u_n(t) - u_{n-1}(t)] + S_n(t) - S_{n-1}(t)\|_1 \\ &\le \left\|[u_n(t) - u_{n-1}(t)] - [p(t,\mathrm{u}_n(t)) - p(t,u_{n-1}(t))]\right\|_1 \\ &\quad + \left\|R_{A,\lambda}^{H_1(\cdot,\cdot)}(s_n(t)) - R_{A,\lambda}^{H_1(\cdot,\cdot)}(s_{n-1}(t))\right\|_1, \end{aligned} \tag{4.7}$$

$$\begin{aligned} \|T_n(t)\|_2 &= \|T_{n-1}(t) + T_n(t) - T_{n-1}(t)\|_2 \\ &= \|[v_n(t) - v_{n-1}(t)] + T_n(t) - T_{n-1}(t)\|_2 \\ &\le \left\|[v_n(t) - v_{n-1}(t)] - [q(t,v_n(t)) - q(t,v_{n-1}(t))]\right\|_2 \\ &\quad + \left\|R_{B,\rho}^{H_2(\cdot,\cdot)}(t_n(t)) - R_{B,\rho}^{H_2(\cdot,\cdot)}(t_{n-1}(t))\right\|_2. \end{aligned} \tag{4.8}$$

We first prove that for the two sequences $u_n(t)$ and $v_n(t)$, there exist two sequences $\{A_n(t)\}$ and $\{B_n(t)\}$ in $[0,1]$, such that

$$\|u_{n+1}(t) - u_n(t)\|_1 \le A_n(t)\|u_n(t) - u_{n-1}(t)\|_1 + B_n(t)\|v_n(t) - v_{n-1}(t)\|_2. \tag{4.9}$$

In fact, for the first term in (4.7), from the $\delta_p(t)$-strongly monotonicity and $\sigma_p(t)$-Lipschitz continuity of the function p, we have the following:

$$\begin{aligned} &\left\|[u_n(t) - u_{n-1}(t)] - [p(t,u_n(t)) - p(t,u_{n-1}(t))]\right\|_1^2 \\ &\quad = \|u_n(t) - \mathrm{u}_{n-1}(t)\|_1^2 - 2\langle p(t,u_n(t)) - p(t,u_{n-1}(t)), u_n(t) - u_{n-1}(t)\rangle_1 \\ &\qquad + \left\|p(t,u_n(t)) - p(t,u_{n-1}(t))\right\|_1^2 \\ &\quad \le \left\{1 - 2\delta_p(t) + \left[\sigma_p(t)\right]^2\right\}\|u_n(t) - u_{n-1}(t)\|_1^2. \end{aligned} \tag{4.10}$$

For the second term in (4.7), it follows from Lemma 3.2 that

$$
\begin{aligned}
&\left\| R_{A,\lambda}^{H_1(\cdot,\cdot)}(s_n(t)) - R_{A,\lambda}^{H_1(\cdot,\cdot)}(s_{n-1}(t)) \right\|_1 \\
&\quad \le \frac{1}{\alpha_A - \beta_A} \| s_n(t) - s_{n-1}(t) \|_1 \\
&\quad \le \frac{1}{\alpha_A - \beta_A} \big\| H_1\big(f_1(p(t,u_n(t))), g_1(p(t,u_n(t)))\big) - \lambda M\big(s_M(t,u_n(t)), x_n(t), y_n(t)\big) \\
&\qquad\qquad - \big[H_1\big(f_1(p(t,u_{n-1}(t))), g_1(p(t,u_{n-1}(t)))\big) - \lambda M\big(s_M(t,u_{n-1}(t)), x_{n-1}(t), y_{n-1}(t)\big)\big] \big\|_1 \\
&\quad \le \frac{1}{\alpha_A - \beta_A} \\
&\qquad \times \Big\{ \big\| u_n(t) - u_{n-1}(t) - \big[H_1\big(f_1(p(t,u_n(t))), g_1(p(t,u_n(t)))\big) \\
&\qquad\quad - H_1\big(f_1(p(t,u_{n-1}(t))), g_1(p(t,u_{n-1}(t)))\big)\big] \big\|_1 \\
&\qquad\quad + \big\| -u_n(t) - u_{n-1}(t) - \lambda \big[M\big(s_M(t,u_n(t)), x_n(t), y_n(t)\big) - M\big(s_M(t,u_{n-1}(t)), x_n(t), y_n(t)\big)\big] \big\|_1 \\
&\qquad\quad + \lambda \big\| M(s_M(t,u_{n-1}(t)), x_n(t), y_n(t)) - M\big(s_M(t,u_{n-1}(t)), x_{n-1}(t), y_n(t)\big) \big\|_1 \\
&\qquad\quad + \lambda \big\| M\big(s_M(t,u_{n-1}(t)), x_{n-1}(t), y_n(t)\big) - M\big(s_M(t,u_{n-1}(t)), x_{n-1}(t), y_{n-1}(t)\big) \big\|_1 \Big\}.
\end{aligned}
\tag{4.11}
$$

There are four terms in (4.11). Since $H_1(f_1, g_1)$ is $\mu_A(t)$-strongly monotone and $a_A(t)$-Lipschitz continuous with respect to the mapping p, then for the first term in (4.11), we can obtain the following

$$
\begin{aligned}
&\big\| u_n(t) - u_{n-1}(t) - H_1\big(f_1(p(t,u_n(t))), g_1(p(t,u_n(t)))\big) + H_1\big(f_1(p(t,u_{n-1}(t))), g_1(p(t,u_{n-1}(t)))\big) \big\|_1^2 \\
&\quad = \| u_n(t) - u_{n-1}(t) \|_1^2 - 2\big\langle H_1\big(f_1(p(t,u_n(t))), g_1(p(t,u_n(t)))\big) \\
&\qquad\qquad - H_1\big(f_1(p(t,u_{n-1}(t))), g_1(p(t,u_{n-1}(t)))\big), u_n(t) - u_{n-1}(t) \big\rangle_1 \\
&\qquad + \big\| H_1\big(f_1(p(t,u_n(t))), g_1(p(t,u_n(t)))\big) - H_1\big(f_1(p(t,u_{n-1}(t))), g_1(p(t,u_{n-1}(t)))\big) \big\|_1^2 \\
&\qquad \le \Big\{ 1 - 2\mu_A(t) + [a_A(t)]^2 \Big\} \| u_n(t) - u_{n-1}(t) \|_1^2.
\end{aligned}
\tag{4.12}
$$

For the second term, Since M is $\zeta_A(t)$-monotone with respect to the mapping s_M in the first argument and $\xi_M(t)$-Lipschitz continuous with respect to mapping s_M in the first argument, so

$$\begin{aligned}
&\left\| u_n(t) - u_{n-1}(t) - \lambda\left[M\left(s_M(t,u_n(t)), x_n(t), y_n(t)\right) - M\left(s_M(t,u_{n-1}(t)), x_n(t), y_n(t)\right)\right]\right\|_1^2 \\
&\quad = \|u_n(t) - u_{n-1}(t)\|_1^2 - 2\lambda\left\langle M\left(s_M(t,u_n(t)), x_n(t), y_n(t)\right)\right. \\
&\qquad\qquad \left. - M\left(s_M(t,u_{n-1}(t)), x_n(t), y_n(t)\right), u_n(t) - u_{n-1}(t)\right\rangle_1 \\
&\quad + \lambda^2\left\| M\left(s_M(t,u_n(t)), x_n(t), y_n(t)\right) - M\left(s_M(t,u_{n-1}(t)), x_n(t), y_n(t)\right)\right\|_1^2 \\
&\le \|u_n(t) - u_{n-1}(t)\|_1^2 - 2\lambda\zeta_A(t)\|u_n(t) - u_{n-1}(t)\|_1^2 + \lambda^2[\xi_M(t)]^2\|u_n(t) - u_{n-1}(t)\|_1^2 \\
&\le \left\{1 - 2\lambda\zeta_A(t) + \lambda^2[\xi_M(t)]^2\right\}\|u_n(t) - u_{n-1}(t)\|_1^2.
\end{aligned} \tag{4.13}$$

For the third term, M is $\beta_M(t)$-Lipschitz continuous with respect to the second argument and E^* is $\xi_E(t)$-D-Lipschitz continuous, therefore, we must have the following:

$$\begin{aligned}
&\left\| M(s_M(t,u_{n-1}(t)), x_n(t), y_n(t)) - M(s_M(t,u_{n-1}(t)), x_{n-1}(t), y_n(t))\right\|_1 \\
&\quad \le \beta_M(t)\|x_n(t) - x_{n-1}(t)\|_1 \\
&\quad \le \beta_M(t)(1+\varepsilon_n)D(E^*(t,u_n(t)), E^*(t,u_{n-1}(t))) \\
&\quad \le \beta_M(t)(1+\varepsilon_n)\xi_E(t)\|u_n(t) - u_{n-1}(t))\|_1.
\end{aligned} \tag{4.14}$$

Similarly, because $\eta_M(t)$-Lipschitz continuous with respect to the third argument and F^* is $\xi_F(t)$-D-Lipschitz continuous, so we can derive

$$\begin{aligned}
&\left\| M\left(s_M(t,u_{n-1}(t)), x_{n-1}(t), y_n(t)\right) - M\left(s_M(t,u_{n-1}(t)), x_{n-1}(t), y_{n-1}(t)\right)\right\|_1 \\
&\quad \le \eta_M(t)\left\| y_n(t) - y_{n-1}(t)\right\|_2 \\
&\quad \le \eta_M(t)(1+\varepsilon_n)D(F^*(t,v_n(t)), F^*(t,v_{n-1}(t))) \\
&\quad \le \eta_M(t)(1+\varepsilon_n)\xi_F(t)\|v_n(t) - v_{n-1}(t))\|_2.
\end{aligned} \tag{4.15}$$

If we let

$$\begin{aligned}
A_n(t) = {} & \frac{\lambda}{\alpha_A - \beta_A}\beta_M(t)(1+\varepsilon_n)\xi_E(t) + \sqrt[2]{1 - 2\delta_p(t) + \left[\sigma_p(t)\right]^2} \\
& + \frac{1}{\alpha_A - \beta_A}\sqrt[2]{1 - 2\mu_A(t) + [a_A(t)]^2} \\
& + \frac{1}{\alpha_A - \beta_A}\sqrt[2]{1 - 2\lambda\zeta_A(t) + \lambda^2[\xi_M(t)]^2}
\end{aligned} \tag{4.16}$$

and

$$B_n(t) = \frac{\lambda}{\alpha_A - \beta_A}\eta_M(t)(1+\varepsilon_n)\xi_F(t), \tag{4.17}$$

then, from (4.5) to (4.17), we can have

$$\|u_{n+1}(t) - u_n(t)\|_1 \le A_n(t)\|u_n(t) - u_{n-1}(t)\|_1 + B_n(t)\|v_n(t) - v_{n-1}(t)\|_2. \tag{4.18}$$

Under the assumptions of this theorem, in a similar way, we can also show that, for the other two sequences $\{v_n(t)\}$ and $\{u_n(t)\}$, there exist two sequences $\{C_n(t)\}$ and $\{D_n(t)\}$ in $[0,1]$, such that

$$\|v_{n+1}(t) - v_n(t)\|_2 \le C_n(t)\|u_n(t) - u_{n-1}(t)\|_1 + D_n(t)\|v_n(t) - v_{n-1}(t)\|_2. \tag{4.19}$$

We now can claim easily that $u_n(t)$, $x_n(t)$ are two Cauchy sequences in X_1 and $v_n(t)$, $y_n(t)$ are two Cauchy sequences in X_2. In fact, from (4.18) and (4.19), we can obtain the following inequalities:

$$\begin{aligned}
&\|u_{n+1}(t) - u_n(t)\|_1 + \|v_{n+1}(t) - v_n(t)\|_2 \\
&\quad \le (A_n(t) + C_n(t))\|u_n(t) - u_{n-1}(t)\|_1 + (B_n(t) + D_n(t))\|v_n(t) - v_{n-1}(t)\|_2 \\
&\quad \le \max(A_n(t) + C_n(t), B_n(t) + D_n(t))(\|u_n(t) - u_{n-1}(t)\|_1 + \|v_n(t) - v_{n-1}(t)\|_2).
\end{aligned} \tag{4.20}$$

If we let

$$\begin{gathered}
\theta_n(t) = \max(A_n(t) + C_n(t), B_n(t) + D_n(t)), \\
\theta(t) = \max(A(t) + C(t), B(t) + D(t)),
\end{gathered} \tag{4.21}$$

then we have the following:

$$\begin{gathered}
\lim_{n\to\infty} A_n(t) = A(t), \quad \lim_{n\to\infty} B_n(t) = B(t), \quad \lim_{n\to\infty} C_n(t) = C(t), \\
\lim_{n\to\infty} D_n(t) = D(t), \quad \lim_{n\to\infty} \theta_n(t) = \theta(t).
\end{gathered} \tag{4.22}$$

It follows from the assumptions of Theorem 4.1 that $0 < \theta(t) < 1$, for all $t \in \Omega$, and so $\{u_n(t)\}$ and $\{v_n(t)\}$ are both Cauchy sequences. For sequences $\{x_n\}$ and $\{y_n\}$, since

$$\begin{gathered}
\|x_{n+1}(t) - x_n(t)\|_1 \le (1 + \varepsilon_{n+1})D(E^*(t, u_{n+1}(t)), E^*(t, u_n(t))) \le 2\xi_E(t)\|u_{n+1}(t) - u_n(t)\|_1, \\
\|y_{n+1}(t) - y_n(t)\|_2 \le (1 + \varepsilon_{n+1})D(F^*(t, v_{n+1}(t)), F^*(t, v_n(t))) \le 2\xi_F(t)\|v_{n+1}(t) - v_n(t)\|_2,
\end{gathered} \tag{4.23}$$

thus, $\{x_n(t)\}$ and $\{y_n(t)\}$ are also Cauchy sequences in Hilbert spaces X_1 and X_2, respectively.

We now show that there exist four measurable mappings $x, u : \Omega \to X_1$ and $y, v : \Omega \to X_2$ such that (x, y, u, v) is a set of solution of Problem 1 and

$$\lim_{n\to\infty} x_n(t) = x(t), \quad \lim_{n\to\infty} y_n(t) = y(t), \quad \lim_{n\to\infty} u_n(t) = u(t), \quad \lim_{n\to\infty} v_n(t) = v(t), \tag{4.24}$$

where $\{x_n(t)\}$, $\{y_n(t)\}$, $\{u_n(t)\}$, and $\{v_n(t)\}$ are four iterative sequences generated by Algorithm 3.7.

Because X_1,X_2 are two Hilbert spaces and $\{x_n(t)\}$,$\{y_n(t)\}$,$\{u_n(t)\}$, and $\{v_n(t)\}$ are four Cauchy sequences, thus, there exist four elements $\{x(t)\}$, $\{y(t)\}$, $\{u(t)\}$, and $\{v(t)\}$ such that

$$\lim_{n\to\infty} x_n(t) = x(t), \quad \lim_{n\to\infty} y_n(t) = y(t), \quad \lim_{n\to\infty} u_n(t) = u(t), \quad \lim_{n\to\infty} v_n(t) = v(t). \tag{4.25}$$

Furthermore,

$$\begin{aligned} d(x(t), E^*(t, u(t))) &= \inf\{\|x(t) - a\| : a \in E^*(t, u(t))\} \\ &\leq \|x(t) - x_n(t)\|_1 + d(x_n(t), E^*(t, u(t))) \\ &\leq \|x(t) - x_n(t)\|_1 + D(E^*(t, u_n(t)), E^*(t, u(t))) \\ &\leq \|x(t) - x_n(t)\|_1 + \xi_E(t)\|u_n(t) - u(t)\|_1. \end{aligned} \tag{4.26}$$

Since $\lim_{n\to\infty} x_n(t) = x(t)$, $\lim_{n\to\infty} u_n(t) = u(t)$, and $E^*(\mathrm{t}, u(t)) \in CB(X_1)$, we have the following:

$$x(t) \in E^*(t, u(t)). \tag{4.27}$$

Similar argument leads to the fact that

$$y(t) \in F^*(t, v(t)). \tag{4.28}$$

By the continuity of p, q, H_1, f_1, g_1, H_2, f_2, g_2, E^*, F^*, $R_{A,\lambda}^{H_1(\cdot,\cdot)}$, and $R_{B,\rho}^{H_2(\cdot,\cdot)}$, we have the following:

$$\begin{aligned} p(t, u(t)) &= R_{A,\lambda}^{H_1(\cdot,\cdot)}\left[H_1\left(f_1\left(p(t, u(t))\right), g_1\left(p(t, u(t))\right)\right) - \lambda M\left(s_M(t, u(t)), x(t), y(t)\right)\right], \\ q(t, u(t)) &= R_{B,\rho}^{H_2(\cdot,\cdot)}\left[H_2\left(f_2\left(q(t, v(t))\right), g_2\left(q(t, v(t))\right)\right) - \rho N\left(s_N(t, v(t)), x(t), y(t)\right)\right]. \end{aligned} \tag{4.29}$$

So, by Lemma 3.6, $(x,\ y,\ u,\ v)$ is a set of solution to Problem 1.

This completes the proof. □

Remark 4.2. By some suitable choices of mappings in Theorem 4.1, the main results in this paper extend many existing ones, for instance, the main results in [10, 11, 21, 23, 26, 31, 34].

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