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Research Article

Wiener-Hopf Equations Technique for General Variational Inequalities Involving Relaxed Monotone Mappings and Nonexpansive Mappings

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We show that the general variational inequalities are equivalent to the general Wiener-Hopf equations and use this alterative equivalence to suggest and analyze a new iterative method for finding the common element of the set of fixed points of a nonexpansive mapping and the set of solutions of the general variational inequality involving multivalued relaxed monotone operators. Our results improve and extend recent ones announced by many others.

1. Introduction

Variational inequalities introduced by Stampacchia [1] in the early sixties have witnessed explosive growth in theoretical advances, algorithmic development, and applications across all disciplines of pure and applied sciences (see [1, 2] and the references therein). It combines novel theoretical and algorithmic advances with new domain of applications. Analysis of these problems requires a blend of techniques from convex analysis, functional analysis, and numerical analysis. In recent years, variational inequality theory has been extended and generalized in several directions, using new and powerful methods, to study a wide class of unrelated problems in a unified and general framework. In 1988, Noor [3] introduced and studied a new class of variational inequalities involving two operators, which is known as general variational inequality. We remark that the general variational inequalities are also called Noor variational inequalities. It turned out that oddorder, nonsymmetric obstacle, free, unilateral, nonlinear equilibrium, and moving boundary problems arising in various branches of pure and applied sciences can be studied via Noor variational inequalities (see [3–5]). On the other hand, in 1997, Verma considered the solvability of a new class of variational inequalities involving multivalued relaxed monotone operators (see [6]). Relaxed monotone operators have applications to constrained hemivariational inequalities. Since in the study of constrained problems in reflexive Banach spaces E the set of all admissible elements is nonconvex but star-shaped, corresponding variational formulations are no longer variational inequalities. Using hemivariational inequalities, one can prove the existence of solutions to the following type of nonconvex constrained problems (P): find u in C such that

$$\langle Au - g, v\rangle \geq 0, \quad \forall v \in T_C(u), \tag{1.1}$$

where the admissible set $C \subset E$ is a star-shaped set with respect to a certain ball $B_E(u_0,\rho)$, and $T_C(u)$ denotes Clarke's tangent cone of C at u in C. It is easily seen that when C is convex, (1.1) reduces to the variational inequality of finding u in C such that

$$\langle Au - g, v - u\rangle \geq 0, \quad \forall v \in C. \tag{1.2}$$

Example 1.1 [7]. Let $A : E \to E^*$ be a maximal monotone operator from a reflexive Banach space E into E^* with strong monotonicity and let $C \subset E$ be star-shaped with respect to a ball $B_E(u_0,\rho)$. Suppose that $Au_0 - g \neq 0$ and that distance function d_C satisfies the condition of relaxed monotonicity

$$\langle u^* - v^*, u - v\rangle \geq -c\|u - v\|^2, \quad \forall u, v \in E, \tag{1.3}$$

and for any $u^* \in \partial d_C(u)$ and $v^* \in \partial d_C(v)$ with c satisfying $0 < c < 4a^2\rho/\|Au_0 - g\|^2$, where a is the constant for strong monotonicity of A. Here, ∂d_C is a relaxed monotone operator. Then, the problem (P) has at least one solution.

As a result of interaction between different branches of mathematical and engineering sciences, we now have a variety of techniques to suggest and analyze various numerical methods including projection technique and its variant forms, auxiliary principle and Wiener-Hopf equations for solving variational inequalities and related optimization problems. In this paper, using essentially the projection technique, we show that the general variational inequalities are equivalent to the general Wiener-Hopf equations, whose origin can be traced back to Shi [8]. It has been shown [4, 8–10] that the Wiener-Hopf equations are more flexible and general than the projection methods. Noor [4, 9] has used the Wiener-Hopf equations technique to study the sensitivity analysis and the dynamical systems as well as to suggest and analyze several iterative methods for solving variational inequalities.

Related to the variational inequalities, we have the problem of finding the fixed points of the nonexpansive mappings, which is the subject of current interest in functional analysis. It is natural to consider a unified approach to these two different problems.

Motivated and inspired by the research going on in this direction, we first introduce a new class of the general Wiener-Hopf equations involving. Using the projection technique, we show that the general Wiener-Hopf equations are equivalent to the general variational inequalities. We use this alterative equivalence from the numerical and approximation viewpoints to suggest and analyze an new iterative scheme for finding the common element of the set of fixed points of nonexpansive mappings and the set of solutions of the general variational inequalities.

2. Preliminaries

Let K be a nonempty closed convex subset of a real Hilbert space H, whose inner product and norm are denoted by $\langle \cdot,\cdot \rangle$ and $\|\cdot\|$, respectively. Let $T,g : H\to H$ be two nonlinear operators, $A : H\to 2^H$ a multivalued relaxed monotone operator, and S_1, S_2 two nonexpansive self-mappings of K. Let P_K be the projection of H into the convex set K.

We now consider the problem of finding $u \in H : g(u) \in K$ such that

$$\langle Tu+w, g(v)-g(u)\rangle \geq 0, \quad \forall v \in H : g(v) \in K,\ w \in Au. \tag{2.1}$$

Note what follows.

(1) If $g \equiv I$, the identity operator, then problem (2.1) is equivalent to finding $u \in K$ such that

$$\langle Tu+w, v-u\rangle \geq 0, \quad \forall v \in K,\ w \in Au, \tag{2.2}$$

which is considered as the Verma general variational inequality introduced and studied by Verma [6] in 1997. Next, we will denote the set of solutions of the general variational inequality (2.2) by $GVI(K,T,A)$.

(2) If $w \equiv 0$, then problem (2.1) reduces to finding $u \in H : g(u) \in K$ such that

$$\langle Tu, g(v)-g(u)\rangle \geq 0, \quad \forall v \in H : g(v) \in K, \tag{2.3}$$

which is known as the general variational inequality introduced and studied by Noor [3] in 1988.

(3) If $w \equiv 0$ and $g \equiv I$, the identity operator, then problem (2.1) collapses to finding $u \in K$ such that

$$\langle Tu, v-u\rangle \geq 0, \quad \forall v \in K, \tag{2.4}$$

which is known as the variational inequality problem, originally introduced and studied by Stampacchia [1] in 1964. Next, we will denote the set of solutions of the variational inequality (2.4) by $VI(K,T)$.

Related to the variational inequalities, we have the problems of solving the Wiener-Hopf equations. To be more precise, Let $Q_K = I - SP_K$, where P_K is the projection of H onto the closed convex set K, I is the identity operator, and S is a nonexpansive self-mapping of K. If g^{-1} exists, then we consider the problem of finding $z \in H$ such that

$$Tg^{-1}SP_Kz + w + \rho^{-1}Q_Kz = 0, \quad \forall w \in Ag^{-1}SP_Kz, \tag{2.5}$$

where $\rho > 0$ is a constant, which is called the general Wiener-Hopf equation involving nonexpansive mappings and multivalued relaxed monotone operators. Next, we denote by $GWHE(H,T,g,S,A)$ the set of solutions of the general Wiener-Hopf equation (2.5).

If $w \equiv 0$, then (2.5) reduces to

$$Tg^{-1}SP_K z + \rho^{-1}Q_K z = 0, \tag{2.6}$$

which is called the general Wiener-Hopf equation involving nonexpansive mappings.

If $w \equiv 0$ and $S \equiv I$, the identity operator, then (2.5) is equivalent to

$$Tg^{-1}P_K z + \rho^{-1}Q_K z = 0, \tag{2.7}$$

where $Q_K = I - P_K$. Equation (2.7) is considered as the classical general Wiener-Hopf equation (see [4]).

If $w \equiv 0$ and $S \equiv g \equiv I$, the identity operator, then (2.5) collapses to

$$TP_K z + \rho^{-1}Q_K z = 0, \tag{2.8}$$

which is known as the original Wiener-Hopf equation, introduced by Shi [8]. It is well known that the variational inequalities and Wiener-Hopf equations are equivalent. This equivalence has played a fundamental and basic role in developing some efficient and robust methods for solving variational inequalities and related optimization problems.

We now recall some well-known concepts and results.

Definition 2.1. A mapping $T : K \to H$ is said to be relaxed (γ, r)-coercive if there exist two constants $\gamma, r > 0$ such that

$$\langle Tx - Ty, x - y\rangle \geq (-\gamma)\|Tx - Ty\|^2 + r\|x - y\|^2, \quad \forall x, y \in K. \tag{2.9}$$

Definition 2.2. A mapping $A : H \to 2^H$ is called t-relaxed monotone if there exists a constant $t > 0$ such that

$$\langle w_1 - w_2, u - v\rangle \geq -t\|u - v\|^2, \quad \forall w_1 \in Au,\ w_2 \in Av. \tag{2.10}$$

Definition 2.3. A multivalued mapping $A : H \to 2^H$ is said to be μ-Lipschitzian if there exists a constant $\mu > 0$ such that

$$||w_1 - w_2|| \leq \mu\|u - v\|, \quad \forall w_1 \in Au,\ w_2 \in Av. \tag{2.11}$$

Lemma 2.4 (Reich [11]). *Suppose that $\{\delta_k\}_{k=0}^{\infty}$ is a nonnegative sequence satisfying the following inequality:*

$$\delta_{k+1} \leq (1 - \lambda_k)\delta_k + \sigma_k, \quad k \geq 0 \tag{2.12}$$

with $\lambda_k \in [0,1]$, $\sum_{k=0}^{\infty}\lambda_k = \infty$, and $\sigma_k = \circ(\lambda_k)$. Then, $\lim_{k\to\infty}\delta_k = 0$.

Lemma 2.5. *For a given $z \in H$, $u \in K$ satisfies the inequality*

$$\langle u - z, v - u \rangle \geq 0, \quad \forall v \in K \tag{2.13}$$

if and only if $u = P_K z$, where P_K is the projection of H into K.

It is well-known that the projection operator P_K is nonexpansive.

Lemma 2.6. *The function $u \in H : g(u) \in K$ satisfies the general variational inequality (2.1) if and only if $u \in H$ satisfies the relation*

$$g(u) = P_K[g(u) - \rho(Tu + w)], \quad \forall w \in Au, \tag{2.14}$$

where $\rho > 0$ is a constant and P_K is the metric projection of H onto K.

Proof. The proof follows from Lemma 2.5. □

Remark 2.7. If $u \in GVI(K,T,g,A)$ such that $g(u) \in F(S_1) \subset K$, where S_1 is nonexpansive self-mapping of K, one can easily see that

$$g(u) = S_1 g(u) = P_K[g(u) - \rho(Tu + w)] = S_1 P_K[g(u) - \rho(Tu + w)], \tag{2.15}$$

where $\rho > 0$ is a constant. If further, assume, $u \in F(S_2)$, where S_2 is also a nonexpansive self-mapping of K, then we obtain

$$u = (1 - a_n)u + a_n S_2 u, \tag{2.16}$$

where the sequence $\{a_n\} \subset [0,1]$ for all $n \geq 0$. If $u \in H$ such that $g(u) \in F(S_1)$ is a common element of $F(S_2)$ and $GVI(K,T,g,A)$, then combining (2.15) with (2.16), we have

$$u = (1 - a_n)u + a_n S_2\{u - g(u) + S_1 P_K[g(u) - \rho(Tu + w)]\}, \tag{2.17}$$

where $\rho > 0$ is a constant and the sequence $\{a_n\} \subset [0,1]$ for all $n > 0$.

3. Main results

In this section, we use the general Wiener-Hopf equation (2.5) to suggest and analyze a new iterative method for finding the common element of the set of fixed points of a nonexpansive mapping and the set of solutions of the general variational inequality (2.1). For this purpose, we need the following result.

Proposition 3.1. *The general variational inequality (2.1) has a solution $u \in H$ such that $g(u) \in F(S_1)$ if and only if the general Wiener-Hopf equation (2.5) involving a nonexpansive* *self-mapping* S_1 *has a solution* $z \in H$, *where*

$$z = g(u) - \rho(Tu + w), \quad w \in Au,$$
$$g(u) = S_1 P_K z, \tag{3.1}$$

where P_K *is the projection of* H *onto* K *and* $\rho > 0$ *is a constant.*

Proof. Pick $u \in GVI(K,T,g,A)$ such that $g(u) \in F(S_1)$. Observe that (2.15) yields

$$g(u) = S_1 P_K[g(u) - \rho(Tu + w)], \quad \forall w \in Au. \tag{3.2}$$

Let

$$z = g(u) - \rho(Tu + w), \quad \forall w \in Au. \tag{3.3}$$

Combining (3.2) with (3.3), we have

$$g(u) = S_1 P_K z,$$
$$z = g(u) - \rho(Tu + w), \quad \forall w \in Au, \tag{3.4}$$

which yields

$$z = S_1 P_K z - \rho(Tg^{-1} S_1 P_K z + w), \quad \forall w \in Ag^{-1} S_1 P_K z. \tag{3.5}$$

It follows that

$$Tg^{-1} S_1 P_K z + w + \rho^{-1} Q_K z = 0, \quad \forall w \in Ag^{-1} S_1 P_K z, \tag{3.6}$$

where $Q_K = I - S_1 P_K$.

So, $z \in H$ is a solution of the general Wiener-Hopf equation (2.5). This completes the proof. □

Remark 3.2. Observing Proposition 3.1, one can easily see the general variational inequality (2.1) and the general Wiener-Hopf equation (2.5) are equivalent. This equivalence is very useful from the numerical point of view. Using the equivalence and by an appropriate rearrangement, we suggest and analyze a new iterative algorithm for finding the common element of the set of fixed points of a nonexpansive mapping and the set of solutions of the general variational inequality.

Algorithm 3.3. The approximate solution $\{u_n\}$ is generated by the following iterative algorithm: $u_0 \in K$ and

$$z_n = g(u_n) - \rho(Tu_n + w_n),$$
$$u_{n+1} = (1 - a_n)u_n + a_n S_2[u_n - g(u_n) + S_1 P_K z_n], \tag{3.7}$$

where $\{a_n\}$ is a sequence in $[0,1]$ for all $n \geq 0$ and S_1 and S_2 are two nonexpansive self-mappings of K.

If $\{w_n\} \equiv 0$ and $S_1 \equiv I$, the identity operator, Algorithm 3.3 reduces to the following algorithm, which is essentially a one-step iterative method refined from Noor [12].

Algorithm 3.4. The approximate solution $\{u_n\}$ is generated by the following iterative algorithm: $u_0 \in K$ and

$$z_n = g(u_n) - \rho T u_n,$$
$$u_{n+1} = (1-a_n)u_n + a_n S_2[u_n - g(u_n) + P_K z_n], \tag{3.8}$$

where $\{a_n\}$ is a sequence in $[0,1]$ for all $n \geq 0$ and S_2 is a nonexpansive self-mappings of K.

If $\{w_n\} \equiv 0$ and $g \equiv S_1 \equiv I$, the identity operator, Algorithm 3.3 reduces to the following algorithm.

Algorithm 3.5. The approximate solution $\{u_n\}$ is generated by the following iterative algorithm: $u_0 \in K$ and

$$z_n = u_n - \rho T u_n,$$
$$u_{n+1} = (1-a_n)u_n + a_n S_2 P_K z_n, \tag{3.9}$$

where $\{a_n\}$ is a sequence in $[0,1]$ for all $n \geq 0$ and S_2 is a nonexpansive self-mappings of K.

If the mapping T is α-inverse strongly monotone mapping, then Algorithm 3.5 can be viewed as Takahashi and Toyoda's [2].

If $\{a_n\} = 1$, $\{w_n\} \equiv 0$, and $g = S_1 = S_2 = I$, the identity operator, Algorithm 3.3 reduces to the following algorithm, which was considered by Noor [4].

Algorithm 3.6. The approximate solution $\{u_n\}$ is generated by the following iterative algorithm: $u_0 \in K$ and

$$z_n = u_n - \rho T u_n,$$
$$u_{n+1} = P_K z_n, \tag{3.10}$$

where $\{a_n\}$ is a sequence in $[0,1]$ for all $n \geq 0$.

If $\{a_n\} = 1$ and $g = S_1 = S_2 = I$, the identity operator, Algorithm 3.3 collapses to the following algorithm, which was studied by Verma [6].

Algorithm 3.7. Given $u_0 \in H$, the approximate solution $\{u_n\}$ is generated by the following iterative algorithm:

$$u_{n+1} = P_K[u_n - \rho(Tu_n + w_n)]. \tag{3.11}$$

THEOREM 3.8. *Let K be a nonempty closed convex subset of a real Hilbert space H. Let $T: K \to H$ be a relaxed (γ_1, r_1)-coercive and μ_1-Lipschitz continuous mapping, $g: K \to H$ a relaxed (γ_2, r_2)-coercive and μ_2-Lipschitz continuous mapping, $A: H \to 2^H$ a t-relaxed monotone and μ_3-Lipschitz continuous mapping, and S_1, S_2 two nonexpansive self-mappings of K such that $F(S_1) \neq \emptyset$, $F(S_2) \cap GVI(K,T,g,A) \neq \emptyset$, and $GWHE(H, T, g, S, A) \neq \emptyset$, respectively. Let $\{z_n\}, \{u_n\}$, and $\{g(u_n)\}$ be sequences generated by Algorithm 3.3, where $\{\alpha_n\}$ is a* *sequence in* $[0,1]$. *Assume that the following conditions are satisfied:*

(C1) $\theta = k_1 + 2k_2 < 1$,

where $k_1 = \sqrt{1 + 2\rho(\gamma_1\mu_1^2 - r_1 + t) + \rho^2(\mu_1 + \mu_3)^2}$ *and* $k_2 = \sqrt{1 + 2\mu_2^2\gamma_2 - 2r_2 + \mu_2^2}$;

(C2) $\sum_{n=0}^{\infty} a_n = \infty$.

Then, the sequences $\{z_n\}$, $\{u_n\}$, *and* $\{g(u_n)\}$ *converge strongly to* $z \in GWHE(H, T, g, S_1, A)$, $u \in F(S_2) \cap GVI(K,T,g,A)$, *and* $g(u) \in F(S_1)$, *respectively.*

Proof. Let $z \in H$ be an element of $GWHE(H,T,g,S_1,A)$ and $u \in F(S_2) \cap GVI(K,T,g,A)$ such that $g(u) \in F(S_1)$. From (2.17) and Proposition 3.1, we have

$$\begin{gathered} z = g(u) - \rho(T_u + w), \\ u = (1 - a_n)u + a_n S_2[u - g(u) + S_1 P_K z]. \end{gathered} \tag{3.12}$$

First, we estimate that $||u_{n+1} - u||$. From (3.7) and (3.12), we obtain

$$\begin{aligned} &||u_{n+1} - u|| \\ &\quad = ||(1 - a_n)u_n + a_n S_2[u_n - g(u_n) + S_1 P_K z_n] - u|| \\ &\quad \le (1 - a_n)||u_n - u|| + a_n||S_2[u_n - g(u_n) + S_1 P_K z_n] - S_2[u - g(u) + S_1 P_K z]|| \\ &\quad \le (1 - a_n)||u_n - u|| + a_n||(u_n - u) - [g(u_n) - g(u)]|| + a_n||z_n - z||. \end{aligned} \tag{3.13}$$

Next, we evaluate $\|(u_n - u) - [g(u_n) - g(u)\|$. By the relaxed (γ_2, r_2)-coercive and μ_2-Lipschitzian definition on g, we have

$$\begin{aligned} &||(u_n - u) - [g(u_n) - g(u)]||^2 \\ &\quad = ||u_n - u||^2 - 2\langle g(u_n) - g(u), u_n - u\rangle + ||g(u_n) - g(u)||^2 \\ &\quad \le ||u_n - u||^2 - 2\Big[-\gamma_2||g(u_n) - g(u)||^2 + r_2||u_n - u||^2\Big] + \mu_2^2||u_n - u||^2 \\ &\quad \le (1 + 2\mu_2^2\gamma_2 - 2r_2 + \mu_2^2)||u_n - u||^2 = k_2^2||u_n - u||^2, \end{aligned} \tag{3.14}$$

where $k_2 = \sqrt{1 + 2\mu_2^2\gamma_2 - 2r_2 + \mu_2^2}$. Next, we evaluate $||z_n - z||$. In a similar way, using the relaxed (γ_1, r_1)-coercive and μ_1-Lipschitzian definition on $T$and the t-relaxed monotone and μ_3-Lipschitzian definition on A, we have

$$\begin{aligned} &||(u_n - u) - \rho[(Tu_n + w_n) - (T_u + w)]||^2 \\ &\quad = ||u_n - u||^2 - 2\rho\langle Tu_n + w_n - (Tu + w), u_n - u\rangle + \rho^2||(Tu_n + w_n) - (Tu + w)||^2 \\ &\quad \le ||u_n - u||^2 - 2\rho(\langle Tu_n - Tu, u_n - u\rangle + \langle w_n - w, u_n - u\rangle) \\ &\qquad + \rho^2(||Tu_n - Tu|| + ||w_n - w||)^2 \\ &\quad \le \Big[1 + 2\rho(\gamma_1\mu_1^2 - r_1 + t) + \rho^2(\mu_1 + \mu_3)^2\Big]||u_n - u||^2 = k_1^2||u_n - u||^2, \end{aligned} \tag{3.15}$$

where $k_1 = \sqrt{1+2\rho(\gamma_1\mu_1^2 - r_1 + t) + \rho^2(\mu_1+\mu_3)^2}$. From (3.7) and (3.12), we have

$$\begin{aligned}||z_n - z|| &= ||g(u_n) - g(u) - \rho[(Tu_n + w_n) - (Tu + w)]|| \\ &\leq ||u_n - u - [g(u_n) - g(u)]|| + ||u_n - u - \rho[(Tu_n + w_n) - (Tu + w)]||.\end{aligned} \tag{3.16}$$

Now, substituting (3.14) and (3.15) into (3.16), we have

$$||z_n - z|| \leq (k_1 + k_2)||u_n - u||. \tag{3.17}$$

Substituting (3.14) and (3.17) into (3.13), we have

$$\begin{aligned}||u_{n+1} - u|| &\leq [1 - (1 - k_1 - 2k_2)a_n]||u_n - u|| \\ &= [1 - (1 - \theta)a_n]||u_n - u||,\end{aligned} \tag{3.18}$$

where $\theta = k_1 + 2k_2 < 1$. Thus, from (C1), (C2) and Lemma 2.4, we have $\lim_{n\to\infty}\|u_n - u\| = 0$. Also from (3.17), we have $\lim_{n\to\infty}\|z_n - z\| = 0$. On the other hand, we have

$$||g(u_n) - g(u)|| \leq \mu_2||u_n - u||. \tag{3.19}$$

It follows that $\lim_{n\to\infty}\|g(u_n) - g(u)\| = 0$. This completes the proof. □

Remark 3.9. In this paper, we show that the general variational inequalities involving three nonlinear operators are equivalent to a new class of general Wiener-Hopf equations. The iterative methods suggested and analyzed in this paper are very convenient and are reasonably easy to use for the computation. It is interesting to use the technique in this paper to develop other new iterative methods for solving the general variational inequalities in different directions.

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