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# CONTROLLABILITY OF SEMILINEAR STOCHASTIC EVOLUTION EQUATIONS IN HILBERT SPACE

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Controllability of semilinear stochastic evolution equations is studied by using stochastic versions of the well-known fixed point theorem and semigroup theory. An application to a stochastic partial differential equation is given.

**Key words:** Controllability, Stochastic Evolution Equation, Banach Fixed Point Theorem, Wiener Process.

**AMS subject classifications:** 93B05.

## 1. Introduction

Among the methods employed for the controllability of nonlinear systems in finite and infinite dimensional Banach spaces, fixed point techniques are widely used. Anichini [3], Dauer [7] and Dauer, et. al [9] studied the controllability of classical nonlinear systems by means of Schaefer's theorem, Fan's theorem, and Leray-Schauder's theorem, respectively. Several authors have extended the classical finite dimensional controllability results to infinite dimensional controllability results represented by the evolution equations with bounded and unbounded operators in Banach spaces using semigroup theorem (see [5, 8]).

The semigroup theory gives a unified treatment of a wide class of stochastic parabolic, hyperbolic and functional differential equations, and much effort has been devoted to the study of the controllability results of such evolution equations (see [20]). Stochastic control theory is a stochastic generalization of classical control

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theory. Controllability of nonlinear stochastic systems has been one of the well-known problems discussed in the literature [4, 22, 23]. If the nonlinear terms does not depend on the probability distribution $\mu(t)$ of the process at time $t$, then the process is determined to be a standard Markov process. There are numerous papers in the literature discussing the stability of such stochastic equations in Hilbert spaces (for details see [1, 14, 17]). On the other hand, there are situations where the nonlinear term $f$ depends not only on the state of the process at time $t$ but also on the probability distribution. For example, one may think of an interacting particle system (biological, chemical or physical) in which each particle moves in the space $H$ according to the dynamics described by the equation

$$dx(t) = [Ax(t) + f(x(t), \mu(t))]dt + \sqrt{Q}dw(t),\ t \in J = 0[0,T]$$

$$\mu(t) = \text{probability distribution of } x(t) \tag{1.1}$$

$$x(0) = x_0,$$

with $\mu(t)$ being replaced by the empirical measure

$$\mu_N(t) = \frac{1}{N} \sum_{k=1}^{N} \delta x_k(t) \tag{1.2}$$

of the $N$ particles $x_1(t)$, $x_2(t), \ldots, x_N(t)$ at time $t$. In other words, we have a system of $N$ coupled semilinear stochastic evolution equations:

$$dx_k(t) = [Ax_k(t) + f(x_k(t), \mu_N(t))]dt + \sqrt{Q}dw_k(t), t \in J$$

$$\mu_N(t) = \text{empirical measure given by (1.2)}$$

$$x_k(0) = x_0,\ \ k = 1, 2, \ldots, N.$$

According to the McKean-Vlasov theory (see [2, 18]), under proper conditions, the empirical measure-valued process $\mu_N$ converges in probability as $N$ goes to infinity to a deterministic measure-valued function $\mu$ which corresponds to the probability distribution of the process determined by (1.1). The limiting McKean-Vlasov process has many interesting equilibrium and nonequilibrium asymptotic behaviors (at least in the case $H = R^n$) and therefore has attracted a lot of research attention in recent years (for more information see [10, 11, 13, 16]). For example, a stochastic model for drug distribution in a closed biological system with a simplified heart, one organ or capillary bed, and recirculation of the blood with a constant rate of flow, where the heart is considered as a mixing chamber of constant volume was described in [21]. Drug concentration in the plasma in given areas of the system is assumed to be a random function of time. Assume that for $t \geq 0$, $x_1(s, t; \omega)$ is the concentration in moles per unit volume at points in the capillary at time $t$ and $\omega \in \Omega$, the supporting set of a complete probability measure space $(\Omega, \mathcal{A}, P)$ with $\mathcal{A}$ being the $\sigma$-algebra and $P$ is the probability measure. The heart is considered as a mixing chamber of constant volume given by

$$V = V_e / (\ln(1 + V_e / V_r))$$

where $V_r$ is the residual volume of the heart and $V_e$ is the injection volume. It is assumed that an initial injection is given at the entrance of the heart resulting in a concentration $x(t), 0 \leq t \leq t_1$, of the drug in plasma entering the heart, where $t_1$ is the duration of injection. Let the time required for the blood to flow from the heart exit to the entrance of the organ be $\tau > 0$, and also let $\tau$ be the time required for blood to flow from the exit of the organ to the heart entrance. Drug concentration in the plasma leaving the heart $x(t;\omega)$ satisfies the integral equation (see [6])

$$x(t;\omega) = G(t) + \int_0^t K(s, x(s;\omega);\omega)ds$$

where

$$G(t) = \int_0^{T(t)} (C/V)x(s)ds, \quad T(t) = \begin{cases} t, & 0 \leq t \leq t_1 \\ t_1, & t \geq t_1 \end{cases}$$

$$K(s, x(s;\omega);\omega) = (-C/V)\{x(s;\omega) - x_1(1, s-\tau;\omega)\},$$

and $x_1(1, s;\omega) = 0$ if $s < 0$, where $C$ is the constant volume flow rate of plasma in the capillary bed and $x_1(1, s;\omega)$ is the drug concentration in the plasma leaving the organ at time $s$. The mild solutions are in the form of stochastic integral equations. The main objective of this paper is to derive the controllability conditions of semilinear stochastic evolution equations (1.1) in Hilbert space having the probability measure $\mu(t)$. The Banach fixed point theorem is employed to get the suitable controllability conditions. The considered system is an abstract formulation of stochastic partial differential equations (see [12]).

## 2. Preliminaries

Consider the stochastic evolution equation

$$\frac{dx(t)}{dt} = Ax(t) + (Bu)(t) + f(x(t), \mu(t)) + \sqrt{Q}\frac{dw(t)}{dt}, \quad t \in J = [0, T]$$
$$x(0) = x_0, \tag{2.1}$$

where $A$ is the infinitesimal generator of a strongly continuous semigroup $\{S(t), t \geq 0\}$ of bounded linear operators in a Hilbert space $H$. The state $x(t)$ takes the values in the Hilbert space $H$, the control function $u$ is given in $L^2(J;U)$, and a Hilbert space of admissible control functions with $U$ as a Hilbert space. $B$ is a bounded linear operator from $U$ into $H$. The function $f$ is an appropriate $H$-valued function defined on $H \times M_{\lambda^2}(H)$, $M_{\lambda^2}(H)$ denotes a proper subset of probability measures on $H$; $\mu(t)$ is a probability distribution of $x(t)$; $Q$ is a positive, symmetric, bounded operator on $H$ and $w$ is a given $H$-valued cylindrical Wiener process. Let $(\Omega, \mathfrak{F}, \{\mathfrak{F}_t\}, P)$ be a complete probability space equipped with a family of nondecreasing sub-sigma algebras. $H$ is a real separable Hilbert space with scalar product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. $\mathfrak{B}(H)$ denotes the Borel sigma algebra of subsets of $H$ and $M(H)$ is the space of probability measures on $\mathfrak{B}(H)$ carrying the usual topology of weak convergence Further, let $C(H)$, $C_b(H)$ and $C_b^k(H)$ denote the space of Borel measurable continuous, bounded continuous, and bounded continuous, respectively, up to and including the $k^{th}$ Frechet derivative functions on $H$. The notation $\langle\mu,\varphi\rangle$ means $\int_H \varphi(x)\mu(dx)$ whenever this integral makes sense. Throughout this paper, let $\lambda(x) = 1 + \| x \|$, $x \in H$, and define the Banach space

$$C_\rho(H) = \left\{ \varphi \in C(H) \colon |\varphi|_{C_\rho(H)} = \sup_{x \in H} \frac{\| \varphi(x) \|}{\lambda^2(x)} + \sup_{x \neq y} \frac{\| \varphi(x) - \varphi(y) \|}{\| x - y \|} < \infty \right\}.$$

For $p \geq 1$, let $M^s_{\lambda^p}(H)$ be the Banach space of signed measures $m$ on $H$ satisfying

$$|\mu|_{\lambda^p} = \int_H \lambda^p(x) \, | \, |m| \, |(dx) < \infty$$

where $\| m \| = m^+ + m^-$ and $m = m^+ - m^-$ is the Jordan decomposition of $m$. Let $M_{\lambda^2}(H) = M^s_{\lambda^2}(H) \cap M(H)$ be the set of probability measures on $\mathfrak{B}(H)$. We put on $M_{\lambda^2}(H)$ a topology induced by the following metric:

$$\rho(\mu_1, \mu_2) = \sup \left\{ \langle \varphi, \mu_1 - \mu_2 \rangle \colon |\varphi|_\rho = \sup_{x \in H} \frac{\| \varphi(x) \|}{\lambda^2(x)} + \sup_{x \neq y} \frac{\| \varphi(x) - \varphi(y) \|}{\| x - y \|} \leq 1 \right\}.$$

Then $F = (M_{\lambda^2}(H), \rho)$ forms a complete metric space, and denote $C(J,F)$ the complete metric space of continuous functions from $J$ to $F$ with the metric

$$D_T(\mu_1, \mu_2) = \sup_{t \in J} \rho(\mu_1(t), \mu_2(t)), \quad \mu_1, \mu_2 \in C(J,F).$$

Let $C(J, L^2(\Omega, \mathfrak{F}, \boldsymbol{P}; H))$ be the Banach space of continuous maps from $J$ into $L^2(\Omega, \mathfrak{F}, \boldsymbol{P}; H)$ satisfying the condition $\sup_{t \in J} E \| x(t) \|^2 < \infty$. Let $K$ be the closed subspace of $C(J, L^2(\Omega, \mathfrak{F}, \boldsymbol{P}; H))$ consisting of measurable and $\mathfrak{F}_t$-adapted processes $x = \{x(t) \colon t \in J\}$. Then $K$ is a Banach space with the norm topology given by

$$\| x \|_K = \left( \sup_{t \in J} E \| x(t) \|^2 \right)^{1/2}.$$

For the existence of solution of (2.1) assume the following hypotheses:

**(H1)** (*i*) $A$ is the infinitesimal generator of a $C_0$-semigroup $\{S(t) \colon t \geq 0\}$ of bounded linear operators on $H$ of negative type

$$\| S(t) \| \leq C_1 e^{-\omega t}, \quad t \geq 0$$

for some positive constants $C_1 > 0$ and $\omega > 0$;

(*ii*) $Q$ is a positive, symmetric, bounded operator in $H$ such that the operator $Q_t$ defined by

$$Q_t = \int_0^t S(r) Q S^*(r) dr$$

is nuclear for all $t \geq 0$ and $\sup_{t \in J} \mathrm{tr} Q_t < \infty$.

(*iii*) $w$ is an $H$-valued Cylindrical Wiener process defined on $(\Omega, \mathfrak{F}, \boldsymbol{P})$ with covariance operator $I$.

**(H2)** $f: H \times F \to H$ satisfies the conditions

$$\| f(x, \mu_1) - f(y, \mu_2) \| \leq C_2( \| x - y \| + \rho(\mu_1, \mu_2))$$

and

$$\| f(x, \mu) \| \leq C_3(1 + \| x \| + | \mu |_\lambda)$$

where $C_2, C_3$ are positive constants.

**(H3)** The linear operator $W$ from $L^2(J;U)$ into $H$ defined by

$$Wu = \int_0^T S(T-s)Bu(s)ds$$

has an invertible operator $W^{-1}$ defined on $H \backslash \text{Ker} W$ (see [15]) and there exist positive constants $C_4$ and $C_5$ such that

$$\| B \|^2 \leq C_4 \text{ and } \| W^{-1} \|^2 \leq C_5.$$

Suppose the hypotheses (H1) and (H2) are satisfied. Then the stochastic process

$$x(t) = S(t)x_0 + \int_0^t S(t-s)[(Bu)(s) + f(x(s), \mu(s))]ds$$
$$+ \int_0^t S(t-s)\sqrt{Q}dw(s) \tag{2.2}$$

for $t \in J$ defined on the probability space $(\Omega, \mathfrak{F}, \boldsymbol{P})$ is said to be a mild solution in $K$ of equation (2.1) for a given initial data $x_0$ (see [1]).

**Definition 2.1:** The stochastic evolution equation (2.1) is said to be *controllable* on $J$, if, for every $x(0) = x_0 \in H$, there exists a control $u \in L^2(J;U)$ such that the solution $x(\cdot)$ of (2.1) satisfies $x(T) = x_1$ where $x_1$ and $T$ are preassigned terminal state and time, respectively. If the system is controllable for all $x_0$ at $t = 0$ and for all $x_1$ at $t = T$, it is called *completely controllable* on $J$.

## 3. Main Result

**Theorem 3.1:** *Suppose the hypotheses* ($H1$)-($H3$) *are satisfied; then the system* (2.1) *is completely controllable on j.*

**Proof:** Using the hypothesis (H3), define the control

$$u(t) = -W^{-1}[S(T)x_0 + \int_0^T S(T-s)f(x(s), \mu(s))ds + \int_0^T S(T-s)\sqrt{Q}dw(s)](t).$$

Now it is shown that, when using this control, the operator defined by

$$(\Phi x)(t) = S(t)x_0 - \int_0^t S(t-\eta)BW^{-1}[S(T)x_0$$

$$+ \int_0^T S(T-s)f(x(s),\mu(s))ds + \int_0^T S(T-s)\sqrt{Q}dw(s)](\eta)d\eta \tag{3.1}$$

$$+ \int_0^t S(t-s)f(x(s),u(s))ds + \int_0^t S(t-s)\sqrt{Q}dw(s)$$

has a fixed point. This fixed point is a solution of equation (2.1). Clearly $(\Phi x)(0) = x_0$, which means that the control $u$ steers the semilinear evolution equation from the initial state $x_0$ to $x_1$ in time $T$, provided we can obtain a fixed point of the nonlinear operator $\Phi$.

First, we show that $\Phi$ maps $K$ into $K$, for a fixed measure-valued function $\mu \in C(J,F)$. Since $|a+b+c|^2 \leq 9(|a|^2 + |b|^2 + |c|^2)$ for any real numbers $a, b$ and $c$, we have

$$\|(\Phi x)(t)\|^2 \leq 9\{E(\|S(t)x_0\|^2) + E\| \int_0^T S(t-\eta)BW^{-1}[S(T)x_0$$

$$+ \int_0^T S(T-s)f(x(s),\mu(s))ds + \int_0^T S(T-s)\sqrt{Q}dw(s)](\eta)d\eta\|^2\}$$

$$9\| \int_0^t S(t-s)f(x(s),\mu(s))ds\|^2 + 9\| \int_0^t S(t-s)\sqrt{Q}dw(s)\|^2$$

$$\leq 9C_1^2\{E\|x_0\|^2 + TC_4C_5[C_1^2E\|x_0\|^2 + TC_1^2E\int_0^t C_3^2(1 + \|x\|^2 + |\mu(s)|_\lambda^2)ds$$

$$+ trQ_T] + TE\int_0^t C_3^2(1 + \|x\|^2 + |\mu(s)|_\lambda^2)ds\} + 9\text{tr}Q_t$$

$$\leq 9C_1^2\{E\|x_0\|^2 + TC_4C_t[C_1^2E\|x_0\|^2 + T^2C_1^2C_3^2(1 + \|x\|^2 + |\mu(s)|_\lambda^2) + \text{tr}Q_T]$$

$$+ T^2C_3^2(1 + \|x\|^2 + |\mu(s)|_\lambda^2)\} + 9\text{tr}Q_T$$

$$\leq k_1 + k_2\|x\|_K^2$$

where

$$k_1 = 9(1 + TC_1^2C_4C_5)\{C_1^2E \| x_0 \|^2 + T^2C_1^2C_3^2(1 + \sup_{t \in J} | \mu(s) |_\lambda^2) + \text{tr}Q_T\}$$
$$k_2 = 9T^2C_1^2C_3^2(1 + TC_1^2C_4C_5)$$

are two positive constants.

Hence $\| (\Phi x)(t) \|_K^2 < \infty$ for $x \in K$ and it is easy to see that $(\Phi x)(t)$ is $\mathfrak{F}_t$-measurable whenever $x(t)$ is $\mathfrak{F}_t$-measurable and so $\Phi$ maps $K$ into $K$.

To complete the proof, it remains to show that $\Phi x \in C(J, F)$. Let $\Phi x = \{(\Phi x)(t) : t \in J\}$, then it is enough to show that $t \to (\Phi x)(t)$ is continuous, since $x \in K$ and $(\Phi x)(t) \in M_{\lambda^2}(H)$ for any $t \in J$. Let $z(t) = \int_0^t S(t-s)\sqrt{Q}dw(s)$ and applying semigroup property of $S(t)$ for $0 \leq s \leq t \leq T$, we have

$$E \| (\Phi x)(t) - (\Phi x)(s) \|^2 \leq 9E( \| S(s)[S(t-s) - I]x_0 \|^2)$$

$$+ 18E \| \int_s^t S(t-\eta)BW^{-1}[S(T)x_0 + \int_0^T S(T-s)f(x(s), \mu(s))ds + z(T)](\eta)d\eta \|^2$$

$$+ 18E \| \int_0^s [S(t-s) - I]S(t-\eta)BW^{-1}[S(T)x_0$$

$$+ \int_0^T S(T-s)f(x(s), \mu(s))ds + z(T)](\eta)d\eta \|^2$$

$$+ 18E \| \int_s^t S(t-\tau)f(x(\tau), \mu(\tau))d\tau \|^2$$

$$+ 18E \| \int_0^s [S(t-s) - I]S(t-\tau)f(x(\tau), \mu(\tau))d\tau \|^2 + 9 \| z(t) - z(s) \|^2$$

$$\leq 9E( \| S(s)[S(t-s) - I]x_0 \|^2)$$

$$+ 18C_1^2C_4C_5(t-s)^2E \| S(T)x_0 + \int_0^T S(T-s)f(x(s), \mu(s))ds + z(T) \|^2$$

$$+ 18E \| \int_0^s [S(t-s) - I]S(t-\eta)BW^{-1}[S(T)x_0 \int_0^T S(T-s)f(x(s), \mu(s))ds$$

$$+ z(T)](\eta)d\eta \|^2 + 18C_1^2C_3(t-s)E \int_0^t (1 + \| x(\tau) \|^2 + | \mu(\tau) |_\lambda^2)d\tau$$

$$18E \,\|\, \int_0^s [S(t-s)-I]S(t-\tau)f(x(\tau),\mu(\tau))d\tau \,\|^2 + 9 \,\|\, z(t)-z(s) \,\|^2. \qquad (3.2)$$

Since $S(t)$ is strong continuous (see Pazy [19]), $[S(t-s)-I]h$ converges to 0 as $t \longrightarrow s$ for any $h \in H$. One can easily derive by Lebesgue's dominated convergence theorem that the first, third and fifth terms on the right-hand side of equation (3.2) tends to 0 as $t \longrightarrow s$. Further, since $z(t)$ is a continuous process, $z(t)-z(s)$ converges to 0 as $t \longrightarrow s$ with probability 1. Lebesgue's dominated convergence theorem and the fact $\mathrm{tr} Q_t < \infty$ can be used to claim that

$$\lim_{t \longrightarrow s} E \,\|\, z(t)-z(s) \,\|^2 = 0.$$

Thus, $E \,\|\, (\Phi x)(t) - (\Phi x)(s) \,\|^2$ converges to 0 as $t \longrightarrow s$. For any $\varphi \in C_{\lambda^2}(H)$, by definition of the metric $\rho$, we have

$$\begin{aligned} \| \langle \varphi, ((\Phi x)(t) - (\Phi x)(s)) \rangle \| &= \| E[(\varphi x)(t) - (\varphi x)(s)] \| \\ &\leq |\varphi|_\rho E \,\|\, x(t) - X(s) \,\| \end{aligned}$$

and therefore,

$$\lim_{t \longrightarrow s} \rho((\Phi x)(t), (\Phi x)(s)) = 0,$$

hence $(\Phi x)(t) \in C(J,F)$. Now we prove that $\Phi$ is a contraction map on $C(J,F)$ and therefore has a unique fixed point. Indeed, for $x, y \in K$ satisfying $x(0) = y(0)$ we have

$$\begin{aligned} & E \,\|\, (\Phi x)(t) - (\Phi y)(t) \,\|^2 \\ \leq\ & E \,\|\, \int_0^t S(t-\eta)BW^{-1}[S(T-s)[f(x(s),\mu(s)) - f(y(s),\mu(s))]ds](\eta)d\eta \,\|^2 \\ & + E \,\|\, \int_0^t S(t-s)[f(x(s),\mu(s)) - f(y(s),\mu(s))]ds \,\|^2 \\ & \leq (1 + TC_1^2C_4C_5)TC_1^2C_2 \int_0^t E \,\|\, x(s) - y(s) \,\|^2 ds \\ & = C_6 \int_0^t E \,\|\, x(s) - y(s) \,\|^2 ds, \end{aligned}$$

where $C_6 = (1 + TC_1^2C_4C_5)TC_1^2C_2$. For any integer $n \geq 1$, by iterations, it follows that

$$\| (\Phi^n x)(t) - (\Phi^n y)(t) \|_K^2 \leq \frac{C_6^n T^n}{n!} \| x(s) - y(s) \|^2.$$

Since for sufficiently large $n$, $\frac{C_6^n T^n}{n!} < 1$, $\Phi^n$ is a contraction map on $K$ and therefore $\Phi$ itself has a unique fixed point $x$ in $K$. Any fixed point of $\Phi$ is a solution of (2.1) on $J$ satisfying $(\Phi x)(t) = x(t) \in H$ for all $x_0$ and $T$. Thus, the system (2.1) is completely controllable on $J$.

## 4. Example

Consider the following nonlinear stochastic partial differential equation of the form

$$\partial_t x(t,\xi) = \frac{\partial^2}{\partial \xi^2} x(t,\xi)dt + f(\xi, \mu(t), \langle x(t,\cdot), h_1\rangle, \ldots, \langle x(t,\cdot), h_n\rangle)dt + (Bu)(t)dt$$

$$+ \sum_{k=1}^{\infty} \lambda_k \sin(k\xi) d\beta_k(t), \xi \in (0,\pi), t > 0$$

$$x(t,0) = x(t,\pi) = 0, t > 0 \tag{4.1}$$

$$x(0,\cdot) = x_0(\cdot) \in X = H = L^2(0,\pi),$$

with the following assumptions given by:

(1) Let $\text{dom}A = H^2(0,\pi) \cap H_0^1(0,\pi)$ and $(A\phi)\xi = \frac{\partial^2}{\partial \xi^2}\phi(\xi)$, $\xi \in (0,\pi)$, $\phi \in \text{dom}A$ and $B$ is a bounded linear operator from the control space $U = L^2(0,\pi)$ into $H$.

(2) Define the function $G: H \to H$ by choosing $h_1, h_2, \ldots, h_n \in H$ and a function $f: [0,\pi] \times F \times R^n \to R$, $(\xi, \mu, y_1, y_2, \ldots, y_n) \to f(\xi, \mu, y_1, y_2, \ldots, y_n)$ and setting

$$(Gg)(\xi) = f(\xi, \mu, \langle g, h_1\rangle, \langle g, h_2\rangle, \ldots, \langle g, h_n\rangle), \xi \in [0,\pi], g \in H,$$

$\langle \cdot, \cdot \rangle$ is the usual scalar product in $H$.

(3) Also for $y_1, y_2, y_n \in J$ and $i = 1, 2, \ldots, n$, assume $f, \frac{\partial f}{\partial \xi}, \frac{\partial f}{\partial y_i}, \frac{\partial^2 f}{\partial \xi \partial y_i}$ are bounded and continuous on $[0,\pi] \times F \times R^n$, such that

$$f(0, \mu, y_1, y_2, \ldots, y_n) = f(\pi, \mu, y_1, y_2, \ldots, y_n) = 0,$$

$$[\partial f / \partial y_i](0, \mu, y_1, y_2, \ldots, y_n) = [\partial f / \partial y_i](\pi, \mu, y_1, y_2, \ldots, y_n) = 0.$$

(4) The functions $e_k(\xi) = \sqrt{2}/\pi \sin k\xi$, $\xi \in (0,\pi)$, for an orthonormal basis of $H$ consisting of eigenvectors of $A$ corresponding to the eigenvalues $\alpha_k = -k^2$, $k = 1, 2, \ldots$ etc., and $\beta_k(t)$ are standard, real independent Wiener processes. Take a sequence of numbers $\{\lambda_k\}$ and define the operator $Q$ by setting $Qe_k = \lambda_k e_k$, $k = 1, 2, \ldots$. Assume that $\lambda_k > 0$, $\sup_k \lambda_k < \infty$, $\sup_k k\lambda_k^{-1/2} e^{-tk^2} < \infty$ for $t > 0$.

Then (4.1) has an abstract formulation of the following nonlinear stochastic equation in a Hilbert space with constant, but possibly degenerate diffusion term

$$\frac{dx(t)}{dt} = Ax(t) + f(x(t), \mu(t)) + (Bu)(t) + Q^{1/2}\frac{dw(t)}{dt}, t \in J. \tag{4.2}$$

$$x(0) = x_0 \in H$$

where the linear operator $A$ is the infinitesimal generator of a strongly continuous semigroup $e^{At}$, $t \geq 0$ in $H$, $Q$ is a continuous linear, self-adjoint nonnegative operator in $H$, and the operators defined by

$$Q_t x = \int_0^t e^{sA} Q e^{sA^*} x_0 ds, x_0 \in H$$

are trace class. Further $f: H \rightarrow H$ is Lipschitz continuous and $w(t)$, $t \geq 0$ is a cylindrical Wiener process in $H$. Then 94.2) has a unique solution as the following (see [12])

$$x(t) = e^{tA} x_0 + \int_0^t e^{(t - sA}[(Bu)(s) + f(x(s), \mu(t))]ds + \int_0^t e^{(t - sA} Q^{1/2} dw(s).$$

Hence by Theorem 3.1, for $S(t) = e^{tA}$, the system (4.1) is completely controllable on $J$.

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