Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 479783, 18 pages
doi:10.1155/2012/479783

Research Article

On Generalised Interval-Valued Fuzzy Soft Sets

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Received 10 August 2011; Revised 18 November 2011; Accepted 22 November 2011

Academic Editor: Jong Hae Kim

Soft set theory, initiated by Molodtsov, can be used as a new mathematical tool for dealing with imprecise, vague, and uncertain problems. In this paper, the concepts of two types of generalised interval-valued fuzzy soft set are proposed and their basic properties are studied. The lattice structures of generalised interval-valued fuzzy soft set are also discussed. Furthermore, an application of the new approach in decision making based on generalised interval-valued fuzzy soft set is developed.

1. Introduction

Most of our real-life problems in social science, economics, medical science, engineering, environmental science, and many other fields have various uncertainties. To deal with these uncertainties, many kinds of theories have been proposed such as theory of probability [1], fuzzy set theory [2], rough set theory [3], intuitionistic fuzzy set theory [4], and interval mathematics [5–7]. Unfortunately, each of these theories has its inherent difficulties, which was pointed out by Molodtsov in [8]. To overcome these difficulties, Molodtsov [8] proposed the soft set theory, which has become a new completely generic mathematical tool for modeling uncertainties.

Recently, the soft set theory has been widely focused in theory and application after Molodtsov's work. Maji and Biswas [9] first introduced the concepts of soft subset, soft superset, soft equality, null soft set, and absolute soft set. They also gave some operations on soft set and verified De Morgan's laws. Ali et al. [10] corrected some errors of former studies and defined some new operations on soft sets. Afterwards, Ali et al. [11] further studied some important properties associated with the new operations and investigated some algebraic structures of soft sets. Sezgin and Atagün [12] extended the theoretical aspect of operations on soft sets. Soft mappings, soft equality, kernels and closures of soft set relations, and soft set relation mappings were presented in [13–15]. On the other hand, soft set theory has a rich potential for application in many fields. Especially, it has been successfully applied to soft decision making [16–18] and some algebra structures such as groups [19, 20], ordered semigroups [21], rings [22], semirings [23], BCK/BCI-algebras [24–26], d-algebras [27], and BL-algebras [28].

Clearly, all of these works mentioned above are based on the classical soft set theory. To improve the capability of soft set theory in dealing with more complex real-life problems, some fuzzy extensions of soft set theory have been studied by many scholars [29–36]. Particularly, Maji et al. [29] firstly proposed the concept of the fuzzy soft set. Roy and Maji [30] presented an application of fuzzy soft set in decision making. Yang et al. [31] defined the interval-valued fuzzy soft set which is based on a combination of the interval-valued fuzzy set and soft set. Majumdar and Samanta [32] generalized the concept of fuzzy soft sets; that is, a degree of which is attached with the parameterization of fuzzy sets while defining a fuzzy soft set.

However, in many practical applications, specially in fuzzy decision-making problems, the membership functions of objects and parameters are very individual, which are dependent on evaluation of experts in general and thus cannot be lightly confirmed. For example, concerning the fuzzy concept "capability", there are three experts who give their evaluations to that of someone as 0.6, 0.76, and 0.8, respectively. Clearly, it is more practical and reasonable to evaluate someone's capability by an interval-valued data [0.6, 0.8] than a certain single value. In this case, therefore, we can make use of interval-valued fuzzy sets which assign to each object or parameter an interval that approximates the "real" (but unknown) membership degree. This paper aims to further generalize the concept of generalised fuzzy soft sets by combining the generalised fuzzy soft sets [32] and interval-valued fuzzy sets [7] and obtain a new soft set model named generalised interval-valued fuzzy soft set. It can be viewed as an interval-valued fuzzy extension of the generalised fuzzy soft set theory [32] or a generalization of the interval-valued fuzzy soft set theory [31].

The rest of this paper is organized as follows. In Section 2, the notions of soft set, fuzzy soft set, generalised fuzzy soft set, and interval-valued fuzzy soft set are recalled. In Section 3, the concept and operations of generalised interval-valued fuzzy soft sets are proposed and some of their properties are investigated. Section 4 studies the lattice structures of generalised interval-valued fuzzy soft set. Section 5 introduces the concept of generalised comparison table, which is applied to decision making based on generalised interval-valued fuzzy soft set. Some illustrative examples are also employed to show that the method presented here is not only reasonable but also more efficient in practical applications. Finally, Section 6 presents the conclusion.

2. Preliminary

In this section, we briefly review the concepts of soft sets, fuzzy soft sets, generalised fuzzy soft sets, interval-valued fuzzy soft set, and so on. Further details could be found in [7, 8, 29, 31, 32, 37]. Throughout this paper, unless otherwise stated, U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and α, β, γ are fuzzy subset of $A, B, C \subseteq E$, respectively.

Definition 2.1 (see [8]). A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F, A) or as the set of ε-approximate elements of the soft set.

Definition 2.2 (see [29]). Let $\mathcal{P}(U)$ denote the set of all fuzzy subsets of U. Then a pair $(\widetilde{F}, A)$ is called a fuzzy soft set over U, where $\widetilde{F}$ is a mapping from A to $\mathcal{P}(U)$.

From the definition, it is clear that $\widetilde{F}(e)$ is a fuzzy set on U for any $e \in A$. The modified definition of fuzzy soft set by Majumdar and Samanta is as follows.

Definition 2.3 (see [32]). Let U be an initial universal set, E a set of parameters, and the pair (U, E) a soft universe. Let $F : E \to \mathcal{P}(U)$ and μ be a fuzzy subset of E; that is, $\mu : E \to [0,1]$. Let $F_\mu : E \to \mathcal{P}(U) \times [0,1]$ be a function defined as follows: $F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in \mathcal{P}(U)$. Then F_μ is called a generalised fuzzy soft set over (U, E).

Definition 2.4 (see [7]). An interval-valued fuzzy set X on a universe U is a mapping $X : U \to \operatorname{Int}([0,1])$, where $\operatorname{Int}([0,1])$ stands for the set of all closed subintervals of $[0,1]$.

The set of all interval-valued fuzzy sets on U is denoted by $\mathcal{F}(U)$. Suppose that $X \in \mathcal{F}(U)$, for all $h \in U$, $\mu_X(h) = [\mu_X^-(h), \mu_X^+(h)]$ is called the degree of membership of an element h to X. And $\mu_X^-(h)$ and $\mu_X^+(h)$ are referred to as the lower and upper degrees of membership of h to X, where $0 \le \mu_X^-(h) \le \mu_X^+(h) \le 1$.

Definition 2.5 (see [7]). Let X and Y be two interval-valued fuzzy sets on universe U. Then the union, intersection, and complement of vague sets are defined as follows:

$$
\begin{aligned}
X \cup Y &= \left\{ \frac{h}{[\mu_X^-(h) \vee \mu_Y^-(h), \mu_X^+(h) \wedge \mu_Y^+(h)]} \mid h \in U \right\}, \\
X \cap Y &= \left\{ \frac{h}{[\mu_X^-(h) \wedge \mu_Y^-(h), \mu_X^+(h) \vee \mu_Y^+(h)]} \mid h \in U \right\}, \\
X^c &= \left\{ \frac{h}{[1 - \mu_X^+(h), 1 - \mu_X^-(h)]} \mid h \in U \right\}.
\end{aligned}
\tag{2.1}
$$

Definition 2.6 (see [31]). Let U be an initial universe, let E be a set of parameters, and let $A \subseteq E$. $\mathcal{F}(U)$ denotes the set of all interval-valued fuzzy sets of U. A pair (F, A) is an interval-valued fuzzy soft set over U, where F is a mapping given by $F : A \to \mathcal{F}(U)$.

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U. For each parameter $e \in A$, $F(e)$ is actually an interval-valued fuzzy set of U, and it can be written as $F(e) = \{(h/\mu_{F(e)}(h)) : h \in U\}$, where $\mu_{F(e)}(h)$ is the interval-valued fuzzy membership degree that object h holds on parameter e.

Definition 2.7 (see [37]). A t-norm is an increasing, associative, and commutative mapping $T : [0,1] \times [0,1] \to [0,1]$ that satisfies the boundary condition: $T(a, 1) = a$ for all $a \in [0,1]$.

The commonly used continuous t-norms are $T(a,b) = \min(a,b)$, $T(a,b) = \max\{0, a + b - 1\}$, and $T(a,b) = a \cdot b$.

Definition 2.8 (see [37]). A t-conorm is an increasing, associative, and commutative mapping $S : [0,1] \times [0,1] \to [0,1]$ that satisfies the boundary condition: $S(a,0) = a$ for all $a \in [0,1]$.

The commonly used continuous t-conorms are $S(a,b) = \max(a,b)$, $S(a,b) = a+b-a\cdot b$, and $S(a,b) = \min\{1, a+b\}$.

3. Generalised Interval-Valued Fuzzy Soft Set

Obviously, by combining generalised soft set and the interval-valued fuzzy set, it is natural to define the generalised interval-valued fuzzy soft set model. We first define two types of generalised interval-valued fuzzy soft set as follows.

Definition 3.1. Let U be an initial universe and E a set of parameters, $A \subseteq E$, $\widetilde{F} : A \to \mathcal{F}(U)$, and let α be a fuzzy sets of A, that is, $\alpha : A \to [0,1]$. Define a function $\widetilde{F}_\alpha : A \to \mathcal{F}(U) \times [0,1]$ as $\widetilde{F}_\alpha(e) = (\widetilde{F}(e) = \{h/\mu_{\widetilde{F}(e)}(h)\}, \alpha(e))$, where $\mu_{\widetilde{F}(e)}(h) = [\mu^-_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{F}(e)}(h)]$ is an interval value is called the degree of membership an element h *to* $\widetilde{F}(e)$, and $\alpha(e)$ is called the degree of possibility of such belongness. Then $\widetilde{F}_\alpha$ is called type 1 generalised interval-valued fuzzy soft set over the soft universe (U, E).

Here for each parameter e, $\widetilde{F}_\alpha(e)$ indicates not only the degree of belongingness of elements of U in $\widetilde{F}(e)$ but also the degree of preference of such belongingness which is represented by $\alpha(e)$.

Definition 3.2. Let U be an initial universe and E a set of parameters, $A \subseteq E$, $\widetilde{F} : A \to \mathcal{F}(U)$, and let α be an interval-valued fuzzy sets of A; that is, $\alpha : A \to \mathrm{Int}([0,1])$, where $\mathrm{Int}([0,1])$ stands for the set of all closed subintervals of $[0,1]$. Define a function $\widetilde{F}_\alpha : A \to \mathcal{F}(U) \times \mathrm{Int}([0,1])$ as $\widetilde{F}_\alpha(e) = (\widetilde{F}(e) = \{h/\mu_{\widetilde{F}(e)}(h)\}, \alpha(e))$, where $\mu_{\widetilde{F}(e)}(h) = [\mu^-_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{F}(e)}(h)]$ and $\alpha(e) = [\alpha^-(e), \alpha^+(e)]$ are interval values. Then $\widetilde{F}_\alpha$ is called type 2 generalised interval-valued fuzzy soft set over the soft universe (U, E).

It is clear that if $\alpha^-(e) = \alpha^+(e)$ holds for each $a \in A$, then the type 2 generalised interval-valued fuzzy soft set will degenerate to the type 1 generalised interval-valued fuzzy soft set. And if $\mu^-_{\widetilde{F}(e)}(h) = \mu^+_{\widetilde{F}(e)}(h)$ also holds for each $a \in A$, then type 1 generalised interval-valued fuzzy soft set will degenerate to generalised fuzzy soft set [32].

In this paper, the type 2 generalised interval-valued fuzzy soft set is denoted by GIVFS set in short. To illustrate this idea, let us consider the following example.

Example 3.3. Let $U = \{h_1, h_2, h_3\}$ be a set of mobile telephones and $A = \{e_1, e_2, e_3\} \in E$ a set of parameters. The e_i $(i = 1,2,3)$ stand for the parameters "expensive", "beautiful", and "multifunctional", respectively. Let $\widetilde{F}_\alpha : A \to \mathcal{P}(U) \times \mathrm{Int}([0,1])$ be a function given as follows:

$$\begin{aligned}
\widetilde{F}_\alpha(e_1) &= \left(\left\{\frac{h_1}{[0.8,0.9]}, \frac{h_2}{[0.6,0.7]}, \frac{h_3}{[0.5,0.6]}\right\}, [0.7,0.8]\right), \\
\widetilde{F}_\alpha(e_2) &= \left(\left\{\frac{h_1}{[0.7,0.8]}, \frac{h_2}{[0.3,0.4]}, \frac{h_3}{[0.5,0.7]}\right\}, [0.6,0.7]\right), \\
\widetilde{F}_\alpha(e_3) &= \left(\left\{\frac{h_1}{[0.5,0.6]}, \frac{h_2}{[0.5,0.7]}, \frac{h_3}{[0.7,0.8]}\right\}, [0.8,0.9]\right).
\end{aligned} \tag{3.1}$$

Then $\widetilde{F}_\alpha$ is a GIVFS set.

Definition 3.4. Let $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ be GIVFS sets over (U, E). Then $\widetilde{F}_\alpha$ is called a GIVFS subset of $\widetilde{G}_\beta$ if

(1) $A \subseteq B$;

(2) $\widetilde{F}(e)$ is an interval-valued fuzzy subset of $\widetilde{G}(e)$ for any $e \in A$; that is, $\mu^-_{\widetilde{F}(e)}(h) \leq \mu^-_{\widetilde{G}(e)}(h)$ and $\mu^+_{\widetilde{F}(e)}(h) \leq \mu^+_{\widetilde{G}(e)}(h)$ for any $h \in U$ and $e \in A$;

(3) α is an interval-valued fuzzy subset of β; that is, $\alpha^-(e) \leq \beta^-(e)$ and $\alpha^+(e) \leq \beta^+(e)$ for any $e \in A$.

In this case, the above relationship is denoted by $\widetilde{F}_\alpha \Subset \widetilde{G}_\beta$. And $\widetilde{G}_\beta$ is said to be a GIVFS superset of $\widetilde{F}_\alpha$.

Definition 3.5. Let $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ be GIVFS sets over (U, E). Then $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ are said to be GIVFS equal if and only if $\widetilde{F}_\alpha \Subset \widetilde{G}_\beta$ and $\widetilde{G}_\beta \Subset \widetilde{F}_\alpha$.

Definition 3.6. The relative complement of a GIVFS set $\widetilde{F}_\alpha$ is denoted by $\widetilde{F}^r_\alpha$ and is defined by $\widetilde{F}^r_\alpha = (\widetilde{F}^r, \alpha^r)$, where $\widetilde{F}^r : A \rightarrow \mathcal{F}(U)$ is a mapping given by $\widetilde{F}^r(e) = \{h/\mu_{\widetilde{F}^r(e)}(h)\}$ and $\alpha^r : A \rightarrow \text{Int}([0,1])$ *is* a mapping given by $\alpha^r(e)$ for all $h \in U, e \in A$, where $\mu_{\widetilde{F}^r(e)}(h) = [\mu^-_{\widetilde{F}^r(e)}(h), \mu^+_{\widetilde{F}^r(e)}(h)] = [1 - \mu^+_{\widetilde{F}(e)}(h), 1 - \mu^-_{\widetilde{F}(e)}(h)]$, $\alpha^r(e) = [\alpha^{r-}(e), \alpha^{r+}(e)] = [1 - \alpha^+(e), 1 - \alpha^-(e)]$.

Example 3.7. We consider the GIVFS set $\widetilde{F}_\alpha$ given in Example 3.3 and define a GIVFS set $\widetilde{G}_\beta$ as follows:

$$\begin{aligned}
\widetilde{G}_\beta(e_1) &= \left(\left\{\frac{h_1}{[0.7, 0.8]}, \frac{h_2}{[0.4, 0.5]}, \frac{h_3}{[0.4, 0.6]}\right\}, [0.5, 0.6]\right), \\
\widetilde{G}_\beta(e_2) &= \left(\left\{\frac{h_1}{[0.5, 0.6]}, \frac{h_2}{[0.2, 0.4]}, \frac{h_3}{[0.5, 0.6]}\right\}, [0.3, 0.4]\right).
\end{aligned} \tag{3.2}$$

Then $\widetilde{G}_\beta$ is a GIVFS subset of $\widetilde{F}_\alpha$, and the relative complement of a GIVFS set $\widetilde{G}_\beta$ is

$$\begin{aligned}
\widetilde{G}^r_\beta(e_1) &= \left(\left\{\frac{h_1}{[0.2, 0.3]}, \frac{h_2}{[0.5, 0.6]}, \frac{h_3}{[0.4, 0.6]}\right\}, [0.4, 0.5]\right), \\
\widetilde{G}^r_\beta(e_2) &= \left(\left\{\frac{h_1}{[0.4, 0.5]}, \frac{h_2}{[0.6, 0.8]}, \frac{h_3}{[0.4, 0.5]}\right\}, [0.6, 0.7]\right).
\end{aligned} \tag{3.3}$$

Definition 3.8. Let $\overline{1} = [1, 1]$. A GIVFS set $\widetilde{F}_\alpha$ over (U, E) is said to be relative absolute GIVFS set denoted by $\widetilde{\Omega}_A$, if $\mu_{\widetilde{F}(e)}(h) = \overline{1}$ and $\alpha(e) = \overline{1}$ for all $h \in U$ and $e \in A$.

Definition 3.9. Let $\overline{0} = [0, 0]$. A GIVFS set $\widetilde{F}_\alpha$ over (U, E) is said to be relative null GIVFS set, denoted by $\widetilde{\Phi}_A$, if $\mu_{\widetilde{F}(e)}(h) = \overline{0}$ and $\alpha(e) = \overline{0}$ for all $h \in U$ and $e \in A$.

Definition 3.10. The union of two GIVFS sets $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ over (U, E) denoted by $\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta$ is a GIVFS set $\widetilde{H}_\gamma$ and defined as $\widetilde{H}_\gamma : A \cup B \to \mathcal{F}(U) \times \mathrm{Int}([0,1])$ such that, for all $h \in U$ and $e \in A \cup B$,

$$\widetilde{H}_\gamma(e) = \begin{cases} \left(\left\{\dfrac{h}{\mu_{\widetilde{F}(e)}(h)}\right\}, \alpha(e)\right), & \text{if } e \in A - B, \\ \left(\left\{\dfrac{h}{\mu_{\widetilde{G}(e)}(h)}\right\}, \beta(e)\right), & \text{if } e \in B - A, \\ \left(\left\{\dfrac{h}{\mu_{\widetilde{H}(e)}(h)}\right\}, \gamma(e)\right), & \text{if } e \in A \cap B, \end{cases} \tag{3.4}$$

where $\mu_{\widetilde{H}(e)}(h) = S(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)) = [S(\mu^-_{\widetilde{F}(e)}(h), \mu^-_{\widetilde{G}(e)}(h)), S(\mu^+_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{G}(e)}(h))]$ and $\gamma(e) = S(\alpha(e), \beta(e)) = [S(\alpha^-(e), \beta^-(e)), S(\alpha^+(e), \beta^+(e))]$.

Definition 3.11. The intersection of two GIVFS sets $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ over (U, E) denoted by $\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta$ is a GIVFS set $\widetilde{H}_\gamma$ and defined as $\widetilde{H}_\gamma : A \cap B \to \mathcal{F}(U) \times \mathrm{Int}([0,1])$ such that, for all $h \in U$ and $e \in A \cap B$, $\widetilde{H}_\gamma(e) = (\{h/\mu_{\widetilde{H}(e)}(h)\}, \gamma(e))$, where $\mu_{\widetilde{H}(e)}(h) = T(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)) = [T(\mu^-_{\widetilde{F}(e)}(h), \mu^-_{\widetilde{G}(e)}(h)), T(\mu^+_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{G}(e)}(h))]$ and $\gamma(e) = T(\alpha(e), \beta(e)) = [T(\alpha^-(e), \beta^-(e)), T(\alpha^+(e), \beta^+(e))]$.

Example 3.12. We consider the GIVFS sets $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ given in Examples 3.3 and 3.7, respectively, and consider $S(x, y) = \max\{x, y\}$ and $T(x, y) = \min\{x, y\}$. Then

$$\begin{aligned} \left(\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta\right)(e_1) &= \left(\left\{\frac{h_1}{[0.8, 0.9]}, \frac{h_2}{[0.6, 0.7]}, \frac{h_3}{[0.5, 0.6]}\right\}, [0.7, 0.8]\right), \\ \left(\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta\right)(e_2) &= \left(\left\{\frac{h_1}{[0.7, 0.8]}, \frac{h_2}{[0.3, 0.4]}, \frac{h_3}{[0.5, 0.7]}\right\}, [0.6, 0.7]\right), \\ \left(\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta\right)(e_3) &= \left(\left\{\frac{h_1}{[0.5, 0.6]}, \frac{h_2}{[0.5, 0.7]}, \frac{h_3}{[0.7, 0.8]}\right\}, [0.8, 0.9]\right), \\ \left(\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta\right)(e_1) &= \left(\left\{\frac{h_1}{[0.7, 0.8]}, \frac{h_2}{[0.4, 0.5]}, \frac{h_3}{[0.4, 0.6]}\right\}, [0.5, 0.6]\right), \\ \left(\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta\right)(e_2) &= \left(\left\{\frac{h_1}{[0.5, 0.6]}, \frac{h_2}{[0.2, 0.4]}, \frac{h_3}{[0.5, 0.6]}\right\}, [0.3, 0.4]\right). \end{aligned} \tag{3.5}$$

Proposition 3.13. *Let $\widetilde{F}_\alpha$ be a GIVFS set over (U, E). Then the following holds*

(1) $\widetilde{F}_\alpha \tilde{\cap} \widetilde{\Omega}_A = \widetilde{F}_\alpha$,

(2) $\widetilde{F}_\alpha \tilde{\cup} \widetilde{\Omega}_A = \widetilde{\Omega}_A$,

(3) $\widetilde{F}_\alpha \tilde{\cap} \widetilde{\Phi}_A = \widetilde{\Phi}_A$,

(4) $\widetilde{F}_\alpha \tilde{\cup} \widetilde{\Phi}_A = \widetilde{F}_\alpha$.

Proof. It is easily obtained from Definitions 3.8–3.11. □

Theorem 3.14. *Let $\widetilde{F}_\alpha$, $\widetilde{G}_\beta$, and $\widetilde{H}_\gamma$ be GIVFS sets over (U, E). Then the following holds*

(1) $\widetilde{F}_\alpha \Cap \widetilde{G}_\beta = \widetilde{G}_\beta \Cap \widetilde{F}_\alpha$,

(2) $\widetilde{F}_\alpha \Cap (\widetilde{G}_\beta \Cap \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \Cap \widetilde{G}_\beta) \Cap \widetilde{H}_\gamma$,

(3) $\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta = \widetilde{G}_\beta \tilde{\cup} \widetilde{F}_\alpha$,

(4) $\widetilde{F}_\alpha \tilde{\cup} (\widetilde{G}_\beta \tilde{\cup} \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta) \tilde{\cup} \widetilde{H}_\gamma$.

Proof. It is easily obtained from Definitions 3.10 and 3.11. □

Definition 3.15. The restricted union of two GIVFS sets $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ over (U, E) denoted by $\widetilde{F}_\alpha \Cup \widetilde{G}_\beta$ is a GIVFS set $\widetilde{H}_\gamma$ and defined as $\widetilde{H}_\gamma : A \cap B \to \digamma(U) \times \mathrm{Int}([0,1])$ such that, for all $h \in U$ and $e \in A \cap B$, $\widetilde{H}_\gamma(e) = (\{h/\mu_{\widetilde{H}(e)}(h)\}, \gamma(e))$, where $\mu_{\widetilde{H}(e)}(h) = S(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)) = [S(\mu^-_{\widetilde{F}(e)}(h), \mu^-_{\widetilde{G}(e)}(h)), S(\mu^+_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{G}(e)}(h))]$ and $\gamma(e) = S(\alpha(e), \beta(e)) = [S(\alpha^-(e), \beta^-(e)), S(\alpha^+(e), \beta^+(e))]$.

Definition 3.16. The extended intersection of two GVS sets $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ over (U, E), denoted by $\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta$, is a GVS set $\widetilde{H}_\gamma : A \cup B \to \digamma(U) \times \mathrm{Int}([0,1])$ which is defined as, for all $h \in U, e \in A \cup B$,

$$\widetilde{H}_\gamma(e) = \begin{cases} \left(\left\{\dfrac{h}{\mu_{\widetilde{F}(e)}(h)}\right\}, \alpha(e)\right), & \text{if } e \in A - B, \\ \left(\left\{\dfrac{h}{\mu_{\widetilde{G}(e)}(h)}\right\}, \beta(e)\right), & \text{if } e \in B - A, \\ \left(\left\{\dfrac{h}{\mu_{\widetilde{H}(e)}(h)}\right\}, \gamma(e)\right), & \text{if } e \in A \cap B, \end{cases} \quad (3.6)$$

where $\mu_{\widetilde{H}(e)}(h) = T(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)) = [T(\mu^-_{\widetilde{F}(e)}(h), \mu^-_{\widetilde{G}(e)}(h)), T(\mu^+_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{G}(e)}(h))]$ and $\gamma(e) = T(\alpha(e), \beta(e)) = [T(\alpha^-(e), \beta^-(e)), T(\alpha^+(e), \beta^+(e))]$.

Theorem 3.17. *Let $\widetilde{F}_\alpha$, $\widetilde{G}_\beta$, and $\widetilde{H}_\gamma$ be three GIVFS sets over (U, E). Then the following holds:*

(1) $\widetilde{F}_\alpha \Cup \widetilde{G}_\beta = \widetilde{G}_\beta \Cup \widetilde{F}_\alpha$,

(2) $\widetilde{F}_\alpha \Cup (\widetilde{G}_\beta \Cup \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \Cup \widetilde{G}_\beta) \Cup \widetilde{H}_\gamma$,

(3) $\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta = \widetilde{G}_\beta \tilde{\cap} \widetilde{F}_\alpha$,

(4) $\widetilde{F}_\alpha \tilde{\cap} (\widetilde{G}_\beta \tilde{\cap} \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta) \tilde{\cap} \widetilde{H}_\gamma$.

Proof. It is easily obtained from Definitions 3.15 and 3.16. □

Theorem 3.18. *Let $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ be two GIVFS sets over (U, E). Then the following holds:*

(1) $(\widetilde{F}_\alpha \Cap \widetilde{G}_\beta)^r = (\widetilde{F}_\alpha)^r \Cup (\widetilde{G}_\beta)^r$,

(2) $(\widetilde{F}_\alpha \Cup \widetilde{G}_\beta)^r = (\widetilde{F}_\alpha)^r \Cap (\widetilde{G}_\beta)^r$.

Proof. (1) Suppose that $\widetilde{F}_\alpha \Cap \widetilde{G}_\beta = \widetilde{H}_\gamma$, then $C = A \cap B$, and, for all $e \in C,\ h \in U$,

$$\begin{aligned}
\mu_{\widetilde{H}(e)}(h) &= T\left(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)\right) \\
&= \left[T\left(\mu^-_{\widetilde{F}(e)}(h), \mu^-_{\widetilde{G}(e)}(h)\right), T\left(\mu^+_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{G}(e)}(h)\right)\right], \\
\gamma(e) &= T(\alpha(e), \beta(e)) \\
&= \left[T(\alpha^-(e), \beta^-(e)), T(\alpha^+(e), \beta^+(e))\right].
\end{aligned} \tag{3.7}$$

Moreover, we have $(\widetilde{F}_\alpha \Cap \widetilde{G}_\beta)^r = \widetilde{H}^r_\gamma$, $C = A \cap B$, and for all $e \in C,\ h \in U$,

$$\begin{aligned}
\mu_{\widetilde{H}^r(e)}(h) &= \left[1 - T\left(\mu^+_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{G}(e)}(h)\right), 1 - T\left(\mu^-_{\widetilde{F}(e)}(h), \mu^-_{\widetilde{G}(e)}(h)\right)\right], \\
\gamma^r(e) &= \left[1 - T(\alpha^+(e), \beta^+(e)), 1 - T(\alpha^-(e), \beta^-(e))\right].
\end{aligned} \tag{3.8}$$

Assume that the parameters set of a GIVFS set $\widetilde{J}_\delta$ is denoted D, and $\widetilde{F}^r_\alpha \Cup \widetilde{G}^r_\beta = \widetilde{J}_\delta$. Then $D = A \cap B$. Since

$$\begin{aligned}
\mu_{\widetilde{F}^r(e)}(h) &= \left[1 - \mu^+_{\widetilde{F}(e)}(h), 1 - \mu^-_{\widetilde{F}(e)}(h)\right], \qquad \alpha^r(e) = \left[1 - \alpha^+(e), 1 - \alpha^-(e)\right], \\
\mu_{\widetilde{G}^r(e)}(h) &= \left[1 - \mu^+_{\widetilde{G}(e)}(h), 1 - \mu^-_{\widetilde{G}(e)}(h)\right], \qquad \beta^r(e) = \left[1 - \beta^+(e), 1 - \beta^-(e)\right].
\end{aligned} \tag{3.9}$$

Then, for each $e \in D,\ h \in U$,

$$\begin{aligned}
\mu_{\widetilde{J}(e)}(h) &= S\left(\mu_{\widetilde{F}^r(e)}(h), \mu_{\widetilde{G}^r(e)}(h)\right) \\
&= \left[S\left(1 - \mu^+_{\widetilde{F}(e)}(h), 1 - \mu^+_{\widetilde{G}(e)}(h)\right), S\left(1 - \mu^-_{\widetilde{F}(e)}(h), 1 - \mu^-_{\widetilde{G}(e)}(h)\right)\right] \\
&= \left[1 - T\left(\mu^+_{\widetilde{F}(e)}(h), \mu^+_{\widetilde{G}(e)}(h)\right), 1 - T\left(\mu^-_{\widetilde{F}(e)}(h), \mu^-_{\widetilde{G}(e)}(h)\right)\right] \\
&= \mu_{\widetilde{H}^r(e)}(h), \\
\delta(e) &= S(\alpha^r(e), \beta^r(e)) \\
&= \left[S(1 - \alpha^+(e), 1 - \beta^+(e)), S(1 - \alpha^-(e), 1 - \beta^-(e))\right] \\
&= \left[1 - T(\alpha^+(e), \beta^+(e)), 1 - T(\alpha^-(e), \beta^-(e))\right] \\
&= \gamma^r(e).
\end{aligned} \tag{3.10}$$

Therefore, $\widetilde{H}^r_\gamma$ and $\widetilde{J}_\delta$ are the same GIVFS sets. Thus, $(\widetilde{F}_\alpha \Cap \widetilde{G}_\beta)^r = (\widetilde{F}_\alpha)^r \Cup (\widetilde{G}_\beta)^r$.

(2) The proof is similar to that of (1). □

Definition 3.19. The "AND" of two GIVFS sets $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ over (U, E), denoted by $\widetilde{F}_\alpha \barwedge \widetilde{G}_\beta$, is defined as $\widetilde{H}_\gamma : A \times B \to \mathcal{F}(U) \times \mathrm{Int}([0,1])$ such that for all $h \in U$ and $(a,b) \in A \times B$, $\widetilde{H}_\gamma(a,b) =$

$(\{h/\mu_{\widetilde{H}(a,b)}(h)\}, \gamma(a,b))$, where $\mu_{\widetilde{H}(a,b)}(h) = T(\mu_{\widetilde{F}(a)}(h), \mu_{\widetilde{G}(b)}(h)) = [T(\mu^-_{\widetilde{F}(a)}(h), \mu^-_{\widetilde{G}(b)}(h)), T(\mu^+_{\widetilde{F}(a)}(h), \mu^+_{\widetilde{G}(b)}(h))]$ and $\gamma(a,b) = T(\alpha(a), \beta(b)) = [T(\alpha^-(a), \beta^-(b)), T(\alpha^+(a), \beta^+(b))]$.

Definition 3.20. The "OR" of two GIVFS sets $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ over (U, E), denoted by $\widetilde{F}_\alpha \veebar \widetilde{G}_\beta$, is defined as $\widetilde{H}_\gamma : A \times B \to \mathcal{F}(U) \times \text{Int}([0,1])$ such that for all $h \in U$ and $(a,b) \in A \times B$, $\widetilde{H}_\gamma(a,b) = (\{h/\mu_{\widetilde{H}(a,b)}(h)\}, \gamma(a,b))$, where $\mu_{\widetilde{H}(a,b)}(h) = S(\mu_{\widetilde{F}(a)}(h), \mu_{\widetilde{G}(b)}(h)) = [S(\mu^-_{\widetilde{F}(a)}(h), \mu^-_{\widetilde{G}(b)}(h)), S(\mu^+_{\widetilde{F}(a)}(h), \mu^+_{\widetilde{G}(b)}(h))]$ and $\gamma(a,b) = S(\alpha(a), \beta(b)) = [S(\alpha^-(a), \beta^-(b)), S(\alpha^+(a), \beta^+(b))]$.

Theorem 3.21. *Let $\widetilde{F}_\alpha$, $\widetilde{G}_\beta$, and $\widetilde{H}_\gamma$ be three GIVFS sets over (U, E). Then the following holds*

(1) $\widetilde{F}_\alpha \veebar (\widetilde{G}_\beta \veebar \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \veebar \widetilde{G}_\beta) \veebar \widetilde{H}_\gamma$,

(2) $\widetilde{F}_\alpha \barwedge (\widetilde{G}_\beta \barwedge \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \barwedge \widetilde{G}_\beta) \barwedge \widetilde{H}_\gamma$.

Proof. It is easily obtained from Definitions 3.19 and 3.20. □

Theorem 3.22. *Let $\widetilde{F}_\alpha$ and $\widetilde{G}_\beta$ be two GIVFS sets over (U, E). Then the following holds*

(1) $(\widetilde{F}_\alpha \veebar \widetilde{G}_\beta)^r = (\widetilde{F}_\alpha)^r \barwedge (\widetilde{G}_\beta)^r$,

(2) $(\widetilde{F}_\alpha \barwedge \widetilde{G}_\beta)^r = (\widetilde{F}_\alpha)^r \veebar (\widetilde{G}_\beta)^r$.

Proof. (1) Suppose that $\widetilde{F}_\alpha \veebar \widetilde{G}_\beta = \widetilde{H}_\gamma$, then $C = A \times B$, and, for all $(a,b) \in C$, $h \in U$,

$$\begin{aligned} \mu_{\widetilde{H}(a,b)}(h) &= S\left(\mu_{\widetilde{F}(a)}(h), \mu_{\widetilde{G}(b)}(h)\right) \\ &= \left[S\left(\mu^-_{\widetilde{F}(a)}(h), \mu^-_{\widetilde{G}(b)}(h)\right), S\left(\mu^+_{\widetilde{F}(a)}(h), \mu^+_{\widetilde{G}(b)}(h)\right)\right], \\ \gamma(a,b) &= S(\alpha(a), \beta(b)) \\ &= \left[S(\alpha^-(a), \beta^-(b)), S(\alpha^+(a), \beta^+(b))\right]. \end{aligned} \tag{3.11}$$

Moreover, we have $(\widetilde{F}_\alpha \veebar \widetilde{G}_\beta)^r = \widetilde{H}^r_\gamma$, $C = A \times B$, and for all $(a,b) \in C, h \in U$,

$$\begin{aligned} \mu_{\widetilde{H}^r(a,b)}(h) &= \left[1 - S\left(\mu^+_{\widetilde{F}(a)}(h), \mu^+_{\widetilde{G}(b)}(h)\right), 1 - S\left(\mu^-_{\widetilde{F}(a)}(h), \mu^-_{\widetilde{G}(b)}(h)\right)\right], \\ \gamma^r(a,b) &= \left[1 - S(\alpha^+(a), \beta^+(b)), 1 - S(\alpha^-(a), \beta^-(b))\right]. \end{aligned} \tag{3.12}$$

Assume that the parameters set of a GIVFS set $\widetilde{J}_\delta$ is denoted D, and $\widetilde{F}^r_\alpha \barwedge \widetilde{G}^r_\beta = \widetilde{J}_\delta$. Then $D = A \times B$. Since for all $a \in A$, $b \in B$, $h \in U$,

$$\begin{aligned} \mu_{\widetilde{F}^r(a)}(h) &= \left[1 - \mu^+_{\widetilde{F}(a)}(h), 1 - \mu^+_{\widetilde{F}(a)}(h)\right], \qquad \alpha^r(a) = \left[1 - \alpha^+(a), 1 - \alpha^-(a)\right], \\ \mu_{\widetilde{G}^r(b)}(h) &= \left[1 - \mu^+_{\widetilde{G}(b)}(h), 1 - \mu^-_{\widetilde{G}(b)}(h)\right], \qquad \beta^r(b) = \left[1 - \beta^+(b), 1 - \beta^-(b)\right], \end{aligned} \tag{3.13}$$

then, for each $(a,b) \in D,\ h \in U$,

$$\begin{aligned}
\mu_{\widetilde{J}(a,b)}(h) &= T\left(\mu_{\widetilde{F}^r(a)}(h), \mu_{\widetilde{G}^r(b)}(h)\right) \\
&= \left[T\left(1-\mu^+_{\widetilde{F}(a)}(h), 1-\mu^+_{\widetilde{G}(b)}(h)\right), T\left(1-\mu^-_{\widetilde{F}(a)}(h), 1-\mu^-_{\widetilde{G}(b)}(h)\right)\right] \\
&= \left[1-S\left(\mu^+_{\widetilde{F}(a)}(h), \mu^+_{\widetilde{G}(b)}(h)\right), 1-S\left(\mu^-_{\widetilde{F}(a)}(h), \mu^-_{\widetilde{G}(b)}(h)\right)\right] \\
&= \mu_{\widetilde{H}^r(a,b)}(h), \\
\delta(a,b) &= T(\alpha^r(a), \beta^r(b)) \\
&= [T(1-\alpha^+(a), 1-\beta^+(b)), T(1-\alpha^-(a), 1-\beta^-(b))] \\
&= [1-S(\alpha^+(a), \beta^+(b)), 1-S(\alpha^-(a), \beta^-(b))] \\
&= \gamma^r(a,b).
\end{aligned} \tag{3.14}$$

Therefore, $\widetilde{H}^r_\gamma$ and $\widetilde{J}_\delta$ are the same GIVFS sets. Thus, $(\widetilde{F}_\alpha \veebar \widetilde{G}_\beta)^r = (\widetilde{F}_\alpha)^r \barwedge (\widetilde{G}_\beta)^r$.

(2) The proof is similar to that of (1). □

4. The Lattice Structures of GIVFS Sets

The lattice structures of soft sets have been studied by Qin and Hong in [14]. In this section, we will discuss the lattice structures of GIVFS sets. The following proposition shows the idempotent law with respect to operations $\widetilde{\cup}$ and $\Cap$ does not hold in general.

Proposition 4.1. *Let $\widetilde{F}_\alpha$ be a GIVFS sets over (U,E). Then the following holds*

(1) $\widetilde{F}_\alpha \Subset (\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha)$,

(2) $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha) \Subset \widetilde{F}_\alpha$.

To illuminate the above proposition, we give an example as follows.

Example 4.2. We consider the GIVFS set $\widetilde{F}_\alpha$ given in Example 3.3. We have that the following

(1) If $S(a,b) = a+b-a\cdot b$, then $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha)(e_1) = (\{h_1/[0.96,0.99], h_2/[0.84,0.91], h_3/[0.75,0.84]\}, [0.91,0.96]) \supseteq \widetilde{F}_\alpha(e_1)$, $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha)(e_2) \supseteq \widetilde{F}_\alpha(e_2)$, and $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha)(e_3) \supseteq \widetilde{F}_\alpha(e_3)$; that is, $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha) \supseteq \widetilde{F}_\alpha$.

(2) If $S(a,b) = \min(1, a+b)$, then $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha)(e_1) = (\{h_1/[1.0,1.0], h_2/[1.0,1.0], h_3/[1.0,1.0]\}, [1.0,1.0]) \supseteq \widetilde{F}_\alpha(e_1)$, $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha)(e_2) \supseteq \widetilde{F}_\alpha(e_2)$, and $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha)(e_3) \supseteq \widetilde{F}_\alpha(e_3)$; that is, $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha) \supseteq \widetilde{F}_\alpha$.

(3) if $T(a,b) = a\cdot b$, then $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha)(e_1) = (\{h_1/[0.64,0.81], h_2/[0.36,0.49], h_3/[0.25,0.36]\}, [0.49,0.64]) \subseteq \widetilde{F}_\alpha(e_1)$, $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha)(e_2) \subseteq \widetilde{F}_\alpha(e_2)$ and $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha)(e_3) \subseteq \widetilde{F}_\alpha(e_3)$, that is, $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha) \Subset \widetilde{F}_\alpha$;

(4) If $T(a,b) = \max(0, a+b-1)$, then $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha)(e_1) = (\{h_1/[0.6,0.8], h_2/[0.2,0.4], h_3/[0.0,0.2]\}, [0.4,0.6]) \subseteq \widetilde{F}_\alpha(e_1)$, $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha)(e_2) \subseteq \widetilde{F}_\alpha(e_2)$ and $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha)(e_3) \subseteq \widetilde{F}_\alpha(e_3)$; that is, $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha) \Subset \widetilde{F}_\alpha$.

For convenience, let $\widetilde{\mathfrak{S}}(U,E)$ denote the set of all GIVFS sets over (U,E); that is, $\widetilde{\mathfrak{S}}(U,E) = \{\widetilde{F}_\alpha \mid A \subseteq E,\ F : A \to \mathcal{F}(U),\ \alpha : A \to \text{Int}([0,1])\}$.

From Proposition 4.1, we can see that $(\widetilde{\mathfrak{S}}(U,E), \Cap, \widetilde{\cup})$ is not a lattice in general. However, if $T(a,b) = \min(a,b)$ and $S(a,b) = \max(a,b)$, then the idempotent law and absorption law with respect to operations $\widetilde{\cup}$ and $\Cap$ hold. In the remainder of this section, we always consider $T(a,b) = \min(a,b)$ and $S(a,b) = \max(a,b)$.

Theorem 4.3. *Let $A, B \subseteq E$, $\widetilde{F}_\alpha$, and $\widetilde{G}_\beta$ be GIVFS sets over (U,E). Then the following hold:*

(1) $(\widetilde{F}_\alpha \Cap \widetilde{F}_\alpha) = \widetilde{F}_\alpha$,

(2) $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{F}_\alpha) = \widetilde{F}_\alpha$,

(3) $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{G}_\beta) \Cap \widetilde{F}_\alpha = \widetilde{F}_\alpha$,

(4) $(\widetilde{F}_\alpha \widetilde{\Cap} \widetilde{G}_\beta) \widetilde{\cup} \widetilde{F}_\alpha = \widetilde{F}_\alpha$.

Proof. (1) and (2) are trivial to prove. We prove only (3) since (4) can be proved similarly.

Suppose that the parameter sets of two GIVFS sets $\widetilde{J}_\delta$ and $\widetilde{K}_\eta$ are denoted by M and N, respectively. Let $\widetilde{F}_\alpha \widetilde{\cup} \widetilde{G}_\beta = \widetilde{J}_\delta$ and $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{G}_\beta) \Cap \widetilde{F}_\alpha = \widetilde{K}_\eta$. Then $M = A \cup B$, $N = (A \cup B) \cap A = A$. And, for each $e \in A$ and $h \in U$,

(i) if $e \notin B$, then $\mu_{\widetilde{K}(e)}(h) = T(\mu_{\widetilde{J}(e)}(h), \mu_{\widetilde{F}(e)}(h)) = \min(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{F}(e)}(h)) = \mu_{\widetilde{F}(e)}(h)$, and $\eta(e) = T(\alpha(e), \alpha(e)) = \min(\alpha(e), \alpha(e)) = \alpha(e)$,

(ii) if $e \in B$, then $\mu_{\widetilde{K}(e)}(h) = \min(\mu_{\widetilde{J}(e)}(h), \mu_{\widetilde{F}(e)}(h)) = \min(\max(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)), \mu_{\widetilde{F}(e)}(h)) = \mu_{\widetilde{F}(e)}(h)$, and $\eta(e) = T(S(\alpha(e), \beta(e)), \alpha(e)) = \min(\max(\alpha(e), \beta(e)), \alpha(e)) = \alpha(e)$.

Thus $\widetilde{K}_\eta = \widetilde{F}_\alpha$; that is, $(\widetilde{F}_\alpha \widetilde{\cup} \widetilde{G}_\beta) \Cap \widetilde{F}_\alpha = \widetilde{F}_\alpha$. □

Theorem 4.4. *Let $A, B, C \subseteq E$, $\widetilde{F}_\alpha$, $\widetilde{G}_\beta$, and $\widetilde{H}_\gamma$ be GIVFS sets over (U,E). Then the following hold:*

(1) $\widetilde{F}_\alpha \Cap (\widetilde{G}_\beta \widetilde{\cup} \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \Cap \widetilde{G}_\beta) \widetilde{\cup} (\widetilde{F}_\alpha \Cap \widetilde{H}_\gamma)$,

(2) $\widetilde{F}_\alpha \widetilde{\cup} (\widetilde{G}_\beta \Cap \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \widetilde{\cup} \widetilde{G}_\beta) \Cap (\widetilde{F}_\alpha \widetilde{\cup} \widetilde{H}_\gamma)$.

Proof. (1) Suppose that the parameter sets of two GIVFS sets $\widetilde{J}_\delta$ and $\widetilde{K}_\eta$ are denoted by M and N, respectively. Let $\widetilde{F}_\alpha \Cap (\widetilde{G}_\beta \widetilde{\cup} \widetilde{H}_\gamma) = \widetilde{J}_\delta$ and $(\widetilde{F}_\alpha \Cap \widetilde{G}_\beta) \widetilde{\cup} (\widetilde{F}_\alpha \Cap \widetilde{H}_\gamma) = \widetilde{K}_\eta$. Then $M = A \cap (B \cup C) = (A \cap B) \cup (A \cap C) = N$. And, for each $e \in M, h \in U$, it follows that $e \in A$ and $e \in B \cup C$,

(i) if $e \in A$, $e \notin B$, $e \in C$, then $\mu_{\widetilde{J}(e)}(h) = T(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{H}(e)}(h)) = \min(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{H}(e)}(h)) = \mu_{\widetilde{K}(e)}(h)$, and $\delta(e) = T(\alpha(e), \gamma(e)) = \min(\alpha(e), \gamma(e)) = \eta(e)$,

(ii) if $e \in A$, $e \in B$, $e \notin C$, then $\mu_{\widetilde{J}(e)}(h) = T(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)) = \min(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)) = \mu_{\widetilde{K}(e)}(h)$, and $\delta(e) = T(\alpha(e), \beta(e)) = \min(\alpha(e), \beta(e)) = \eta(e)$,

(iii) if $e \in A$, $e \in B$, $e \in C$, then $\mu_{\widetilde{J}(e)}(h) = \min(\mu_{\widetilde{F}(e)}(h), \max(\mu_{\widetilde{G}(e)}(h), \mu_{\widetilde{H}(e)}(h))) = \max(\min(\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{G}(e)}(h)), \min((\mu_{\widetilde{F}(e)}(h), \mu_{\widetilde{H}(e)}(h))) = \mu_{\widetilde{K}(e)}(h)$, and $\delta(e) = T(\alpha(e), S(\beta(e), \gamma(e))) = \min(\alpha(e), \max(\beta(e), \gamma(e))) = \max(\min(\alpha(e), \beta(e)), \min(\alpha(e), \gamma(e))) = S(T(\alpha(e), \beta(e)), T(\alpha(e), \gamma(e))) = \eta(e)$.

Thus $\widetilde{J}_\delta = \widetilde{K}_\eta$; that is, $\widetilde{F}_\alpha \Cap (\widetilde{G}_\beta \widetilde{\cup} \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \Cap \widetilde{G}_\beta) \widetilde{\cup} (\widetilde{F}_\alpha \Cap \widetilde{H}_\gamma)$.

(2) The proof is similar to that of (1). □

Theorem 4.5. (1) $(\widetilde{\mathfrak{S}}(U,E),\Cap,\tilde{\cup})$ *is a distributive lattice.*

(2) *Let* $\leq_1$ *be the order relation in* $\widetilde{\mathfrak{S}}(U,E)$ *and* $\widetilde{F}_\alpha,\widetilde{G}_\beta \in \widetilde{\mathfrak{S}}(U,E)$. *One has* $\widetilde{F}_\alpha \leq_1 \widetilde{G}_\beta$ *if and only if* $A \subseteq B$, $\mu_{\widetilde{F}(e)}(h) \leq \mu_{\widetilde{G}(e)}(h)$ *and* $\alpha(e) \leq \beta(e)$ *for all* $e \in A$ *and* $h \in U$.

Proof. (1) The proof is straightforward from Theorems 3.14, 4.3, and 4.4.

(2) Suppose that $\widetilde{F}_\alpha \leq_1 \widetilde{G}_\beta$. Then $\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta = \widetilde{G}_\beta$. So by Definition 3.10, we have $A \cup B = B$, $\max(\mu_{\widetilde{F}(e)}(h),\mu_{\widetilde{G}(e)}(h)) = \mu_{\widetilde{G}(e)}(h)$, and $\max(\alpha(e),\beta(e)) = \beta(e)$ for all $e \in A$ and $h \in U$. It follows that $A \subseteq B$, $\mu_{\widetilde{F}(e)}(h) \leq \mu_{\widetilde{G}(e)}(h)$ and $\alpha(e) \leq \beta(e)$ for all $e \in A$ and $h \in U$. Conversely, suppose that $A \subseteq B$, $\mu_{\widetilde{F}(e)}(h) \leq \mu_{\widetilde{G}(e)}(h)$ and $\alpha(e) \leq \beta(e)$ for all $e \in A$ and $h \in U$. We can easily verify that $\widetilde{F}_\alpha \tilde{\cup} \widetilde{G}_\beta = \widetilde{G}_\beta$. Thus $\widetilde{F}_\alpha \leq_1 \widetilde{G}_\beta$. □

For operators $\Cup$ and $\tilde{\cap}$, we can obtain similar results as follows.

Theorem 4.6. *Let* $\widetilde{F}_\alpha$ *and* $\widetilde{G}_\beta$ *be GIVFS sets over* (U,E). *Then the following hold:*

(1) $(\widetilde{F}_\alpha \Cup \widetilde{F}_\alpha) = \widetilde{F}_\alpha$,

(2) $(\widetilde{F}_\alpha \tilde{\cap} \widetilde{F}_\alpha) = \widetilde{F}_\alpha$,

(3) $(\widetilde{F}_\alpha \Cup \widetilde{G}_\beta) \tilde{\cap} \widetilde{F}_\alpha = \widetilde{F}_\alpha$,

(4) $(\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta) \Cup \widetilde{F}_\alpha = \widetilde{F}_\alpha$.

Theorem 4.7. *Let* $\widetilde{F}_\alpha$, $\widetilde{G}_\beta$ *and* $\widetilde{H}_\gamma$ *be GIVFS sets over (U,E). Then the following hold:*

(1) $\widetilde{F}_\alpha \tilde{\cap} (\widetilde{G}_\beta \Cup \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \tilde{\cap} \widetilde{G}_\beta) \Cup (\widetilde{F}_\alpha \tilde{\cap} \widetilde{H}_\gamma)$,

(2) $\widetilde{F}_\alpha \Cup (\widetilde{G}_\beta \tilde{\cap} \widetilde{H}_\gamma) = (\widetilde{F}_\alpha \Cup \widetilde{G}_\beta) \tilde{\cap} (\widetilde{F}_\alpha \Cup \widetilde{H}_\gamma)$.

Theorem 4.8. (1) $(\widetilde{\mathfrak{S}}(U,E),\Cup,\tilde{\cap})$ *is a distributive lattice.*

(2) *Let* $\leq_2$ *be the order relation in* $\widetilde{\mathfrak{S}}(U,E)$ *and* $\widetilde{F}_\alpha,\widetilde{G}_\beta \in \widetilde{\mathfrak{S}}(U,E)$. $\widetilde{F}_\alpha \leq_2 \widetilde{G}_\beta$ *if and only if* $B \subseteq A$, $\mu_{\widetilde{F}(e)}(h) \leq \mu_{\widetilde{G}(e)}(h)$ *and* $\alpha(e) \leq \beta(e)$ *for all* $e \in B$.

It is worth noting that $(\widetilde{\mathfrak{S}}(U,E),\Cap,\Cup)$ and $(\widetilde{\mathfrak{S}}(U,E),\tilde{\cap},\tilde{\cup})$ are not lattices, as the absorption laws of them do not hold necessarily. To illustrate this, we give an example as follows.

Example 4.9. *Let* $U = \{h_1,h_2,h_3\}$ *be the universe,* $E = \{e_1,e_2,e_3\}$ *the set of parameters,* $A = \{e_1,e_2\}$, $B = \{e_2,e_3\}$. *The GIVFS sets* $\widetilde{F}_\alpha$ *and* $\widetilde{G}_\beta$ *over* (U,E) *are given as*

$$\begin{aligned}
\widetilde{F}_\alpha(e_1) &= \left(\left\{\frac{h_1}{[0.5,0.7]},\frac{h_2}{[0.3,0.4]},\frac{h_3}{[0.6,0.7]}\right\},[0.8,0.9]\right),\\
\widetilde{F}_\alpha(e_2) &= \left(\left\{\frac{h_1}{[0.6,0.8]},\frac{h_2}{[0.2,0.3]},\frac{h_3}{[0.7,0.9]}\right\},[0.4,0.5]\right),\\
\widetilde{G}_\beta(e_2) &= \left(\left\{\frac{h_1}{[0.1,0.3]},\frac{h_2}{[0.4,0.5]},\frac{h_3}{[0.5,0.6]}\right\},[0.6,0.8]\right),\\
\widetilde{G}_\beta(e_3) &= \left(\left\{\frac{h_1}{[0.3,0.4]},\frac{h_2}{[0.5,0.8]},\frac{h_3}{[0.4,0.6]}\right\},[0.5,0.7]\right).
\end{aligned} \tag{4.1}$$

Suppose that $(\widetilde{F}_\alpha \Cup \widetilde{G}_\beta) \Cap \widetilde{F}_\alpha = \widetilde{H}_\gamma$. Then $C = A \cap B = \{e_2\} \neq A$. So $\widetilde{H}_\gamma \neq \widetilde{F}_\alpha$, that is, $(\widetilde{F}_\alpha \Cup \widetilde{G}_\beta) \Cap \widetilde{F}_\alpha \neq \widetilde{F}_\alpha$.

Again, suppose that the parameters set of a GIVFS set $\widetilde{J}_\delta$ is denoted by D, and $(\widetilde{F}_\alpha \widetilde{\cap} \widetilde{G}_\beta) \widetilde{\cup} \widetilde{F}_\alpha = \widetilde{J}_\delta$. Then $D = A \cup B = \{e_1, e_2, e_3\} \neq A$, Therefore, $\widetilde{J}_\delta \neq \widetilde{F}_\alpha$, that is, $(\widetilde{F}_\alpha \widetilde{\cap} \widetilde{G}_\beta) \widetilde{\cup} \widetilde{F}_\alpha \neq \widetilde{F}_\alpha$.

5. An Application of GIVFS Sets

In this section we present a simple application of GIVFS set in an interval-valued fuzzy decision making problem. We first give the following definition.

Definition 5.1. Let $\widetilde{F}_\alpha$ be a GIVFS set, $h_i, h_j \in U, e_k \in A$. One says membership value of h_j lowerly exceeds or equals to the membership value of h_i with respect to the parameter e_k if $\mu^-_{\widetilde{F}(e_k)}(h_i) \le \mu^-_{\widetilde{F}(e_k)}(h_j)$. The corresponding characteristic function is defined as follows:

$$f^-_{e_k}(h_i, h_j) = \begin{cases} 1, & \text{if } \mu^-_{\widetilde{F}(e_k)}(h_i) \le \mu^-_{\widetilde{F}(e_k)}(h_j), \\ 0, & \text{otherwise.} \end{cases} \tag{5.1}$$

Definition 5.2. Let $\widetilde{F}_\alpha$ be a GIVFS set, $h_i, h_j \in U, e_k \in A$. One says membership value of h_j upperly exceeds or equals to the membership value of h_i with respect to the parameter e_k if $\mu^+_{\widetilde{F}(e_k)}(h_i) \le \mu^+_{\widetilde{F}(e_k)}(h_j)$. The corresponding characteristic function is defined as follows:

$$f^+_{e_k}(h_i, h_j) = \begin{cases} 1, & \text{if } \mu^+_{\widetilde{F}(e_k)}(h_i) \le \mu^+_{\widetilde{F}(e_k)}(h_j); \\ 0, & \text{otherwise.} \end{cases} \tag{5.2}$$

Remark 5.3. Let $\widetilde{F}_\alpha$ be a GIVFS set, $h_i, h_j \in U$, and $e_k \in A$. For convenience, we denote the vectors $(f^-_{e_k}(h_i, h_j), f^+_{e_k}(h_i, h_j))$ and $(\alpha^-(e_k), \alpha^+(e_k))$ as $\overrightarrow{f_{e_k}(h_i, h_j)}$ and $\overrightarrow{\alpha(e_k)}$, respectively.

Now we can define the generalised comparison table about GIVFS set $\widetilde{F}_\alpha$.

Definition 5.4. Let $\widetilde{F}_\alpha$ be a GIVFS set. The generalised comparison table about $\widetilde{F}_\alpha$ is a square table in which the number of rows and number of columns are equal. Both rows and columns are labeled by the object names of the universe such as $h_1, h_2, \ldots, h_n$, and the entries are C_{ij}, given as follows:

$$C_{ij} = \sum_{k=1}^{m} \left(\overrightarrow{f_{e_k}(h_i, h_j)} \cdot \overrightarrow{\alpha(e_k)} \right), \quad i, j = 1, 2, \ldots, n. \tag{5.3}$$

Clearly, for $i, j = 1, \ldots, n$, $k = 1, \ldots, m$, $0 \le C_{ij} \le 2m$, and $C_{ii} = \sum_{k=1}^{m}(\alpha^-(e_k) + \alpha^+(e_k))$, where n and m are the numbers of objects and parameters present in a GIVFS set, respectively.

Remark 5.5. The generalised comparison table is different from the comparison table in [30]. First, the comparison in the generalised comparison table is between two interval values, instead of two single values. Second, the entries C_{ij} of the generalised comparison table are numbers of real interval $[0,1]$ in general, instead of single values 0 and 1. Hence, the generalised comparison table is an extension of the comparison table in [30]. If each interval degenerates to a point and $\alpha(e) = 1$ for each $e \in A$, then the generalised comparison table will be degenerate to the comparison table in [30].

In the generalised comparison table, the row sum and the column sum of an object h_i are denoted by p_i and q_i, respectively, and the score of an object h_i is denoted as S_i which can be given by $S_i = p_i - q_i$. Now we present an algorithm as follows.

Algorithm 5.6. (1) Input the objects set U and the parameter set $A \subseteq E$.

(2) Consider the GIVFS set $\widetilde{F}_\alpha$ in tabular form.

(3) By calculating the entries C_{ij}, construct generalised comparison table.

(4) Compute the score of each h_i using row sum and the column sum.

(5) The optimal decision is to select h_k if the score of h_k is maximum.

(6) If k has more than one value then any one of h_k may be chosen.

To illustrate the basic idea of the above algorithm, let us consider the following example.

Example 5.7. Let us consider a GIVFS set which describes the capability of the candidates who are wanted to fill a position for a company. Suppose that there are six candidates in the universe $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ is the set of decision parameters, where e_i $(i = 1,2,3,4,5,6)$ stands for the parameters "experience", "computer knowledge", "young age", "higher education", "good health", and "over-married", respectively.

Here, the degree of possibility of belongingness of the parameter e_i can be interpreted as the degree of importance of the parameter to the position. Our purpose is to find out the best candidate for the company based on her expected parameters. Suppose that the company do not consider the parameter "over-married"; that is, the degree of importance of parameter e_6 is regarded as $\overline{0}$. In this case, let $A = \{e_1, e_2, e_3, e_4, e_5\} \subset E$, and let $\alpha : A \to \mathrm{Int}([0,1])$ be an interval-valued fuzzy subset of A, which is given by the company as follows: $\alpha(e_1) = [0.7, 0.8]$, $\alpha(e_2) = [0.5, 0.6]$, $\alpha(e_3) = [0.8, 0.9]$, $\alpha(e_4) = [0.6, 0.7]$, $\alpha(e_5) = [0.4, 0.5]$. And consider the GIVFS set $\widetilde{F}_\alpha$ as follows:

$$
\begin{aligned}
&\widetilde{F}_\alpha(e_1)\\
&= \left(\left\{\frac{h_1}{[0.70,0.85]}, \frac{h_2}{[0.85,0.90]}, \frac{h_3}{[0.65,0.75]}, \frac{h_4}{[0.80,0.90]}, \frac{h_5}{[0.60,0.70]}, \frac{h_6}{[0.65,0.80]}\right\}, [0.7,0.8]\right),\\
&\widetilde{F}_\alpha(e_2)\\
&= \left(\left\{\frac{h_1}{[0.75,0.80]}, \frac{h_2}{[0.60,0.70]}, \frac{h_3}{[0.60,0.70]}, \frac{h_4}{[0.70,0.75]}, \frac{h_5}{[0.80,0.90]}, \frac{h_6}{[0.70,0.80]}\right\}, [0.5,0.6]\right),\\
&\widetilde{F}_\alpha(e_3)\\
&= \left(\left\{\frac{h_1}{[0.80,0.90]}, \frac{h_2}{[0.55,0.66]}, \frac{h_3}{[0.65,0.80]}, \frac{h_4}{[0.68,0.75]}, \frac{h_5}{[0.70,0.80]}, \frac{h_6}{[0.75,0.85]}\right\}, [0.8,0.9]\right),
\end{aligned}
$$

Table 1: Tabular representation of the GIVFS set $\widetilde{F}_\alpha$.

	e_1	e_2	e_3	e_4	e_5
h_1	[0.70,0.85]	[0.75,0.80]	[0.80,0.90]	[0.70,0.80]	[0.65,0.75]
h_2	[0.85,0.90]	[0.60,0.70]	[0.55,0.66]	[0.65,0.75]	[0.60,0.70]
h_3	[0.65,0.75]	[0.60,0.70]	[0.65,0.80]	[0.70,0.78]	[0.80,0.90]
h_4	[0.80,0.90]	[0.70,0.75]	[0.68,0.75]	[0.62,0.70]	[0.60,0.76]
h_5	[0.60,0.70]	[0.80,0.90]	[0.70,0.80]	[0.72,0.82]	[0.75,0.85]
h_6	[0.65,0.80]	[0.70,0.80]	[0.75,0.85]	[0.80,0.90]	[0.70,0.75]
α	[0.7,0.8]	[0.5,0.6]	[0.8,0.9]	[0.6,0.7]	[0.4,0.5]

Table 2: The generalised comparison table about $\widetilde{F}_\alpha$.

	h_1	h_2	h_3	h_4	h_5	h_6
h_1	6.50	5.00	5.60	4.50	3.20	4.80
h_2	1.50	6.50	2.60	3.20	1.50	1.50
h_3	1.50	5.00	6.50	3.10	3.30	1.60
h_4	2.00	4.50	3.40	6.50	1.50	2.50
h_5	3.30	5.00	4.10	5.00	6.50	2.00
h_6	2.80	5.00	5.60	4.50	4.50	6.50

Table 3: The score of h_i about $\widetilde{F}_\alpha$.

	Row sum (p_i)	Column sum (q_i)	The score (S_i)
h_1	29.60	17.60	12.00
h_2	16.80	31.00	−14.20
h_3	21.00	27.80	−6.80
h_4	20.40	26.80	−6.40
h_5	25.90	20.50	5.40
h_6	28.90	18.90	10.00

$$\begin{aligned}
&\widetilde{F}_\alpha(e_4)\\
&=\left(\left\{\frac{h_1}{[0.70,0.80]},\frac{h_2}{[0.65,0.75]},\frac{h_3}{[0.70,0.78]},\frac{h_4}{[0.62,0.70]},\frac{h_5}{[0.72,0.82]},\frac{h_6}{[0.80,0.90]}\right\},[0.6,0.7]\right),\\
&\widetilde{F}_\alpha(e_5)\\
&=\left(\left\{\frac{h_1}{[0.65,0.75]},\frac{h_2}{[0.60,0.70]},\frac{h_3}{[0.80,0.90]},\frac{h_4}{[0.60,0.76]},\frac{h_5}{[0.75,0.85]},\frac{h_6}{[0.70,0.75]}\right\},[0.4,0.5]\right).
\end{aligned} \tag{5.4}$$

The tabular representation of the GIVFS set $\widetilde{F}_\alpha$ is given in Table 1.

It is easy to calculate the entries C_{ij} by the formula 5.3. For example, let us calculate C_{21}. Firstly, we compute $\overrightarrow{f_{e_k}(h_2,h_1)}$ for each $e_k \in A$, where $\overrightarrow{f_{e_1}(h_2,h_1)} = (0,0)$, $\overrightarrow{f_{e_k}(h_2,h_1)} = (1,1)$, $k = 2,3,4,5$. Secondly, we can obtain $C_{21} = 5.0$ by computing $\sum_{k=1}^{m}(\overrightarrow{f_{e_k}(h_2,h_1)} \cdot \overrightarrow{\alpha(e_k)})$, where $\overrightarrow{\alpha(e_1)} = (0.7, 0.8)$, $\overrightarrow{\alpha(e_2)} = (0.5, 0.6)$, $\overrightarrow{\alpha(e_3)} = (0.8, 0.9)$, $\overrightarrow{\alpha(e_4)} = (0.6, 0.7)$, $\overrightarrow{\alpha(e_5)} = (0.4, 0.5)$. And the generalised comparison table about the GIVFS set $\widetilde{F}_\alpha$ is given in Table 2.

Table 4: Tabular representation of the GIVFS set $\widetilde{G}_\beta$.

	e_1	e_2	e_3	e_4	e_5
h_1	[0.70,0.85]	[0.75,0.80]	[0.80,0.90]	[0.70,0.80]	[0.65,0.75]
h_2	[0.85,0.90]	[0.60,0.70]	[0.55,0.66]	[0.65,0.75]	[0.60,0.70]
h_3	[0.65,0.75]	[0.60,0.70]	[0.65,0.80]	[0.70,0.78]	[0.80,0.90]
h_4	[0.80,0.90]	[0.70,0.75]	[0.68,0.75]	[0.62,0.70]	[0.60,0.76]
h_5	[0.60,0.70]	[0.80,0.90]	[0.70,0.80]	[0.72,0.82]	[0.75,0.85]
h_6	[0.65,0.80]	[0.70,0.80]	[0.75,0.85]	[0.80,0.90]	[0.70,0.75]
β	[0.7,0.8]	[0.5,0.6]	**[0.4,0.5]**	**[0.8,0.9]**	**[0.6,0.7]**

Table 5: The generalised comparison table about $\widetilde{G}_\beta$.

	h_1	h_2	h_3	h_4	h_5	h_6
h_1	6.50	5.00	5.20	4.30	2.40	4.20
h_2	1.50	6.50	2.60	3.80	1.50	1.50
h_3	2.10	5.00	6.50	3.50	3.30	2.00
h_4	2.20	4.10	3.00	6.50	1.50	2.70
h_5	4.10	5.00	3.70	5.00	6.50	2.40
h_6	3.60	5.00	5.20	4.30	4.10	6.50

Table 6: The score of h_i about $\widetilde{G}_\beta$.

	Row sum (p_i)	Column sum (q_i)	The score (S_i)
h_1	27.60	20.00	7.60
h_2	17.40	30.60	−13.20
h_3	22.40	26.20	−3.80
h_4	20.00	27.40	−7.40
h_5	26.70	19.30	7.40
h_6	28.70	19.30	9.40

From Table 2, we can obtain the row sum and column sum and compute the score of each h_i, which are presented in Table 3.

From Table 3, it is clear that the maximum score is $S_1 = 12.00$. So h_1 could be selected as the optimal alternative.

It is worth noting that, unlike [30], the decision result depends not only on $\widetilde{F}(e)$ but also on $\alpha(e)$. For example, consider the GIVFS set $\widetilde{G}_\beta$ with data as in Table 4, where $B = A$ and $\widetilde{G}(e) = \widetilde{F}(e)$, but $\beta(e) \neq \alpha(e)$ for each $e \in B$.

The generalised comparison table and the score of h_i about the GIVFS set $\widetilde{G}_\beta$ can be seen in Tables 5 and 6, respectively.

From Table 6, it is clear that the maximum score is $S_6 = 9.40$. Hence, the optimal alternative is h_6, but not h_1.

6. Conclusion

This paper can be viewed as a continuation of the study of Majumdar and Samanta [32], Yang et al. [31], and Roy and Maji [30]. We extended the generalised fuzzy soft set and defined two types of generalised interval-valued fuzzy soft set and studied some of their properties. We also gave the application of GIVFS sets in dealing with some decision-making problems by defining generalised comparison table.

Acknowledgment

This work is supported by the National Natural Science Foundation of China (no. 11071061) and the National Basic Research Program of China (no. 2011CB311808).

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