MAGNETOHYDRODYNAMIC BOUNDARY LAYER FLOW PAST A STRETCHING PLATE AND HEAT TRANSFER

M. EZZAT, M. ZAKARIA, AND M. MOURSY

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The present work is concerned with unsteady two-dimensional laminar flow of an incompressible, viscous, perfectly electrically conducting fluid past a nonisothermal stretching sheet in the presence of a transverse magnetic field acting perpendicularly to the direction of fluid. By means of the successive approximation method, the governing equations for momentum and energy have been solved. The effects of surface mass transfer f_ω, Alfven velocity α, Prandtl number P, and relaxation time parameter τ_0 on the velocity and temperature have been discussed. Numerical results are given and illustrated graphically for the problem considered.

1. Introduction

Important aspects of biophysics have been derived from physiology, especially in studies involving the conduction of nerve impulses. It is known that the extracellular fluid has a high concentration of positively charged sodium ions (Na^+) outside the neuron cell, and a high concentration of negatively charged chloride (Cl^-) as well as a lower concentration of positively charged potassium (K^+) inside. A peculiar characteristic of all living cells is that there is always an electric potential difference between the outer and inner surfaces of the cell surrounding membrane. A potential called *resting potential*, which usually measures −75 mV occurs, the minus sign indicating a negative charge inside. The stimulation of the cell by any physical effect (heat, electric current, light, etc.) causes a nerve impulse; subsequently, sodium ions are pumped into the cell, potassium ions are pumped out, and the cell membrane reaches a depolarization stage at which the electric signals are transmitted from one cell to another when the action potential is conducted at speeds that range from 1 to 100 m/s, so that the impulse moves along the fiber [7]. (The Nobel Prize for physiology or medicine was awarded in 1963 for formulating these ionic mechanisms involved in nerve cell activity.) The extracellular fluid can be considered a perfect conducting fluid [4].

Many metallic materials are manufactured after they have been refined sufficiently in the molten state. Therefore, it is a central problem in metallurgical chemistry to study the

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heat transfer on liquid metal which is a perfect electric conductor. For instance, liquid sodium Na (100° C) and liquid potassium K (100° C) exhibit very small electrical receptivity, (ρ_L (exp) $= 9.6\times 10^{-6}\,\Omega$ cm) and (ρ_L (exp) $= 12.97\times 10^{-6}\,\Omega$ cm), respectively.

The classical heat conduction equation has the property that the heat pluses propagate at infinite speed. Much attention was recently paid to the modification of the classical heat conduction equation, so that the heat pluses propagate at finite speed. Mathematically speaking, this modification changes the governing partial differential equation from parabolic to hyperbolic type, and thereby eliminating the unrealistic result that thermal disturbance is realized instantaneously everywhere within a fluid. Cattaneo [1] was the first to offer an explicit mathematical correction of the propagation speed defect inherent in Fourier's heat conduction law. Puri and Kythe [8] investigated the effects of using the Maxwell-Cattaneo model in Stokes' second problem for a viscous fluid and they note that the nondimensional thermal relaxation time τ_0 defined as $\tau_0 = CP$, where C and P are, respectively, the Cattaneo and Prandtl numbers, respectively, is of order $(10)^{-2}$.

Continuous surfaces are surfaces such as those of polymer sheets or filaments continuously drawn from a die. Boundary layer flow on continuous surfaces is an important type of flow occurring in a number of technical processes. Sakiadis [9] introduced the continuous surface concept. Crane [2] considered a moving strip, the velocity of which is proportional to the local distance. The heat and mass transfer on a stretching sheet with suction or blowing was investigated by P. S. Gupta and A. S. Gupta [6]. Dutta et al. [3] studied the temperature distribution on the flow over a stretching sheet. The Newtonian fluid flow behavior was assumed by these authors (see [2, 3, 6]).

Our aim in this paper is to study the heat transfer to a viscous, perfectly electrically conducting fluid from a nonisothermal stretching sheet with suction or injection in the presence of a transverse magnetic field when the medium is taken as a perfect conductor.

2. Formulation of the problem

The basic equations that govern unsteady two-dimensional flow of viscous fluid in rectangular Cartesian coordinates xyz with the velocity vector $\mathbf{V} = [u(x,y,t), v(x,y,t), 0]$ in the presence of an external magnetic field are

(i) continuity equation

$$\nabla\cdot\mathbf{V} = 0; \tag{2.1}$$

(ii) momentum equation

$$\rho\frac{D\mathbf{V}}{Dt} = -\nabla p + \mu\nabla^2\mathbf{V} + \mathbf{J}\wedge\mathbf{B}; \tag{2.2}$$

(iii) generalized equation of heat conduction

$$\rho C_p\frac{D}{Dt}\left(T + \tau_0\frac{\partial T}{\partial t}\right) = \lambda\nabla^2 T + \mu\left(\Phi + \tau_0\frac{\partial\Phi}{\partial t}\right), \tag{2.3}$$

where T is the temperature, p the pressure, ρ the density, μ the dynamic viscosity, C_P the specific heat at constant pressure, λ the thermal conductivity, $\mathbf{J}$ the current density,

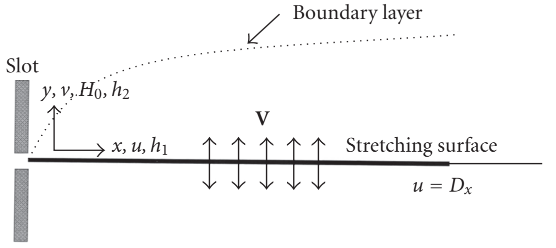

Figure 2.1. Coordinate system for the physical model of the stretching sheet.

$\mathbf{B} = \mu_0 \mathbf{H_0}$, being the electromagnetic induction, $\mathbf{H_0}$ the magnetic field, μ_0 the magnetic permeability, τ_0 a constant with time dimension referred to as the relaxation time, Φ the viscous dissipation function given by

$$\Phi = 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2, \tag{2.4}$$

and the operator D/Dt is defined as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{V} \cdot \nabla). \tag{2.5}$$

Let a constant magnetic field of strength $\mathbf{H_0}$ act in the direction of the y-axis. This produces an induced magnetic field $\mathbf{h}$ and an induced electric field $\mathbf{E}$ that satisfy the linearized equations of electromagnetic, valid for slowly moving media of a perfect conductor [4],

$$\operatorname{curl}\mathbf{h} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}, \tag{2.6}$$

$$\operatorname{curl}\mathbf{E} = -\mu_0 \frac{\partial \mathbf{h}}{\partial t}, \tag{2.7}$$

$$\mathbf{E} = -\mu_0 (\mathbf{V} \wedge \mathbf{H_0}), \tag{2.8}$$

$$\operatorname{div}\mathbf{h} = 0, \tag{2.9}$$

where ε_0 is the electric permeability.

As mentioned above, the applied magnetic field $\mathbf{H_0}$ has components (i.e., $\mathbf{H_0} = (0, H_0, 0)$). It can be easily seen from the above equations that the induced magnetic field has components (i.e., $\mathbf{h} = (h_1, h_2, 0)$), and the vectors $\mathbf{E}$ and $\mathbf{J}$ will have nonvanishing components only in the z-direction, that is,

$$\mathbf{E} = (0,0,E), \qquad \mathbf{J} = (0,0,J). \tag{2.10}$$

We consider the flow past a wall coinciding with plane $y = 0$, and the flow is confined to $y > 0$. Keeping the origin fixed, the wall is stretched by introducing two equal and opposite forces along the x-axis (see Figure 2.1). With the usual boundary layer assumption, (2.1), (2.2), (2.3) and (2.6), (2.7), (2.8), (2.9) reduce to the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.11}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\alpha^2}{H_0}\left(\frac{\partial h_1}{\partial y}-\frac{\partial h_2}{\partial x}-\mu_0\varepsilon_0 H_0\frac{\partial u}{\partial t}\right), \tag{2.12}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\lambda}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{C_p}\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}\right)^2-\tau_0\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right), \tag{2.13}$$

$$\frac{\partial h_1}{\partial t}=H_0\frac{\partial u}{\partial y}, \tag{2.14}$$

$$\frac{\partial h_2}{\partial t}=-H_0\frac{\partial u}{\partial x}, \tag{2.15}$$

where ν is the kinematics viscosity and α is the Alfven velocity given by $\alpha^2=\mu_0 H_0^2/\rho$.

The boundary and initial conditions imposed on (2.5), (2.6), and (2.7) are

$$\begin{aligned}
&y=0: u=Dx, \quad v=V_0, \quad t=0,\\
&y=0: T-T_\infty=T_0x^m, \quad t=0,\\
&y=0: u=Dxe^{\omega t}, \quad v=V_0e^{\omega t}, \quad t>0,\\
&y=0: T-T_\infty=T_0x^me^{\omega t}, \quad t>0,\\
&y\longrightarrow\infty, \quad u\longrightarrow 0, \quad T\longrightarrow T_\infty,
\end{aligned} \tag{2.16}$$

where $D>0$ and ω are constants, V_0 is the velocity condition at the surface, T_0 the mean temperature of the surface, T_∞ the temperature condition far away from the surface, and m the power law exponent [10].

Eliminating h_1 and h_2 between (2.12), (2.14), and (2.15) and taking into account the boundary layer approximations, equation (2.12) yields

$$(1+\alpha^2\mu_0\varepsilon_0)\frac{\partial^2 u}{\partial t^2}+u\frac{\partial^2 u}{\partial t\partial x}+\frac{\partial u}{\partial t}\frac{\partial u}{\partial x}+v\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial v}{\partial t}\frac{\partial u}{\partial y}=\left(\nu\frac{\partial}{\partial t}+\alpha^2\right)\frac{\partial^2 u}{\partial y^2}. \tag{2.17}$$

We introduce the following nondimensional quantities:

$$\begin{aligned}
&x^*=\sqrt{\frac{D}{\nu}}x, \quad y^*=\sqrt{\frac{D}{\nu}}y, \quad t^*=Dt, \quad h_1^*=\frac{h_1}{H_0},\\
&h_2^*=\frac{h_2}{H_0}, \quad u^*=\frac{u}{\sqrt{D\nu}}, \quad v^*=\frac{v}{\sqrt{C\nu}}, \quad P=\frac{\rho C_p\nu}{\lambda},\\
&T^*=\frac{T-T_\infty}{T_0}, \quad \tau_0^*=D\tau_0, \quad \alpha^*=\frac{\alpha}{\sqrt{D\nu}}, \quad E_c=\frac{\sqrt{D\nu}}{T_0C_P},\\
&f_\omega=\frac{V_0}{\sqrt{D\nu}}, \quad \omega^*=\frac{\omega}{D}, \quad E^*=\frac{1}{H_0\mu_0\sqrt{D\nu}}E,
\end{aligned} \tag{2.18}$$

where f_ω is the mass transfer, P the Prandtl number, and E_c the Eckert number. The mass transfer parameter f_ω is positive for injection and negative for suction.

Invoking the nondimensional quantities above, equations (2.11), (2.13), and (2.17) are reduced to the nondimensional equations, dropping the asterisks for convenience,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.19}$$

$$a_1\frac{\partial^2 u}{\partial t^2}+u\frac{\partial^2 u}{\partial t\partial x}+\frac{\partial u}{\partial t}\frac{\partial u}{\partial x}+v\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial v}{\partial t}\frac{\partial u}{\partial y}=\left(\frac{\partial}{\partial t}+\alpha^2\right)\frac{\partial^2 u}{\partial y^2}, \tag{2.20}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{P}\frac{\partial^2 T}{\partial y^2}+E_c\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}\right)^2-\tau_0\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right), \tag{2.21}$$

where $a_1=1+\alpha^2/c^2$ and c is the speed of light given by $c^2=1/\varepsilon_0\mu_0$.

From (2.16), the reduced boundary conditions are

$$\begin{aligned}
&y=0: u=Dx, \quad v=f_\omega, \quad t=0,\\
&y=0: T=x^m, \quad t=0,\\
&y=0: u=Dxe^{\omega t}, \quad v=f_\omega e^{\omega t}, \quad t>0,\\
&y=0: T=x^m e^{\omega t}, \quad t>0,\\
&y\longrightarrow\infty, \quad u\longrightarrow 0, \quad T\longrightarrow 0.
\end{aligned} \tag{2.22}$$

3. The method of successive approximations

A process of successive approximations [10] will integrate the unsteady boundary layer equations (2.19), (2.20), and (2.21). Selecting a system of coordinates, which is at rest with respect to the plate and the magnetohydrodynamic flow of a perfectly conducting fluid that moves with respect to the plane surface, we can assume that the velocities u, v, angular velocity ω, and the temperature T possess a series solution of the form

$$\begin{aligned}
u(x,y,t)&=\sum_{i=0}^{\infty}u_i(x,y,t),\\
v(x,y,t)&=\sum_{i=0}^{\infty}v_i(x,y,t),\\
T(x,y,t)&=\sum_{i=1}^{\infty}T_i(x,y,t),
\end{aligned} \tag{3.1}$$

where $u_i=0(\varepsilon^i)$ is an i-integer and ε is a small number.

Each term in the series (3.1) must satisfy the continuity equation (2.19):

$$\frac{\partial u_i}{\partial x}+\frac{\partial v_i}{\partial y}=0 \quad (i=0,1,2,\ldots). \tag{3.2}$$

Substituting the series (3.1) into equations (2.20) and (2.21) and setting equal to zero terms of the same order, one obtains equations for finding components of the series (3.1):

$$\left(\frac{\partial}{\partial t}+\alpha^2\right)\frac{\partial^2 u_0}{\partial y^2}-a_1\frac{\partial^2 u_0}{\partial t^2}=0, \tag{3.3}$$

$$\left(\frac{\partial}{\partial t}+\alpha^2\right)\frac{\partial^2 u_1}{\partial y^2}-\frac{\partial^4 u_1}{\partial t^2\partial y^2}-a_1\frac{\partial^2 u_1}{\partial t^2}=u_0\frac{\partial^2 u_0}{\partial t\partial x}+\frac{\partial u_0}{\partial t}\frac{\partial u_0}{\partial x}+v_0\frac{\partial^2 u_0}{\partial t\partial y}+\frac{\partial v_0}{\partial t}\frac{\partial u_0}{\partial y}, \tag{3.4}$$

$$\frac{\partial^2 T_0}{\partial y^2}-P\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T_0}{\partial t}=0, \tag{3.5}$$

$$\frac{\partial^2 T_1}{\partial y^2}-P\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial T_1}{\partial t}=P\left(1+\tau_0\frac{\partial}{\partial t}\right)\left\{u_0\frac{\partial T_0}{\partial x}+v_0\frac{\partial T_0}{\partial y}\right\}-E_c\left(1+\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial u_0}{\partial y}\right)^2. \tag{3.6}$$

Combining (3.1) and (2.22), we have the corresponding boundary conditions

$$\begin{gathered}
y=0: u_0=xe^{\omega t},\quad u_i=0,\quad i=1,2,\ldots,\quad t>0,\\
y=0: v_0=f_\omega e^{\omega t},\quad v_i=0,\quad i=1,2,\ldots,\quad t>0,\\
y=0: T_0=x^m e^{\omega t},\quad T_i=0,\quad i=1,2,\ldots,\quad t>0,\\
y\longrightarrow\infty,\quad u_i\longrightarrow 0,\quad T_i\longrightarrow 0,\quad i=0,1,2,\ldots.
\end{gathered} \tag{3.7}$$

In the following analysis, the first two terms in the series solution (3.1) will be retained. It is a known fact that such solution is satisfactory in the phases of the nonperiodic motion after it has been started from rest (till the moment when the first separation of boundary layer occurs) and in the case of periodic motion when the amplitude of oscillation is small. Higher-order approximations u_3 can be easily obtained in principle. However, the complexity of the method of successive approximations increases rapidly as higher approximations are considered. It is also known that the third- and higher-terms series solutions give small changes in the results, compared with the two-terms series solutions.

4. Solution of the problem

We suppose that the exact solutions of the differential equations (3.3) and (3.5) are of the form

$$u_0(x,y,t)=xe^{\omega t}f_1'(y), \tag{4.1}$$

$$T_0(x,y,t)=x^m e^{\omega t}\psi_1(y), \tag{4.2}$$

using (3.2), and

$$v_0(x,y,t)=-e^{\omega t}f_1(y). \tag{4.3}$$

Then from (3.3) and (3.5) and using (4.1) and (4.2), one obtains the differential equations of the unknown functions $f_1(y)$, $\psi_1(y)$ and the corresponding boundary conditions

$$\begin{gathered} f_1''' - k_1^2 f_1' = 0, \\ \psi_1'' - P_1 \psi_1 = 0, \\ y = 0 : f_1 = -f_\omega, \quad f_1' = 1, \\ y = 0 : \psi_1 = 1, \\ y \longrightarrow \infty, \qquad f_1' \longrightarrow 0, \qquad \psi_1 \longrightarrow 0, \end{gathered} \tag{4.4}$$

where $k_1^2 = a_1 \omega^2/(\alpha^2 + \omega)$ and $P_1 = \omega P(1 + \omega \tau_0)$.

The solutions of system (4.4) are of the form

$$f_1(y) = \frac{1}{k_1}\left(1 - e^{-k_1 y}\right) - f_\omega, \tag{4.5}$$

$$\psi_1(y) = e^{-\sqrt{P_1} y}. \tag{4.6}$$

Assuming the solutions of the differential equation (3.4) is of the form

$$u_1(x, y, t) = x e^{2\omega t} f_2'(y), \tag{4.7}$$

we can obtain an exact solution of (3.6) if we consider the case $m = 2$:

$$T_1(x, y, t) = x^2 e^{2at} \psi_2(y), \tag{4.8}$$

and using (4.1), (4.2), (4.3), (4.7), and (4.8), one obtains from (3.4), (3.6), and (3.7) the differential equations for $f_2(y)$ and $\psi_2(y)$ and the corresponding boundary conditions

$$\begin{gathered} f_2''' - k_2^2 f_2' = \frac{k_2^2}{2\omega a_1}\left[f_1'^2 - f_1 f_1''\right], \\ \psi_2'' - P_2 \psi_2 = \frac{P_2}{2a}\left[2\psi_1 f_1' - \psi_1' f_1\right] - E_c' f_1''^2, \\ y = 0 : f_2 = 0, \quad f_2' = 0, \\ y = 0 : \psi_2 = 0, \\ y \longrightarrow \infty, \qquad \psi_2 \longrightarrow 0, \qquad f_2' \longrightarrow 0, \end{gathered} \tag{4.9}$$

where

$$k_2^2 = \frac{4a_1\omega^2}{\left[2\omega + \alpha^2\right]}, \qquad E_c' = E_c(1 + 2\tau_0 \omega), \qquad P_2 = 2\omega P(1 + 2\omega\tau_0). \tag{4.10}$$

Using (4.5) and (4.6), one obtains the solutions of system (4.9):

$$f_2(y) = A_1 + A_2 e^{-k_1 y} + A_3 e^{-k_2 y}, \tag{4.11}$$

$$\psi_2(y) = B_1 e^{-\sqrt{P_1} y} + B_2 e^{-2k_1 y} + B_3 e^{-(k_1 + \sqrt{P_1}) y} + B_4 e^{-\sqrt{P_2} y}, \tag{4.12}$$

where

$$A_1 = -A_2 - A_3, \qquad A_3 = -\frac{k_1}{k_2}A_2, \qquad A_2 = \frac{k_1[1 - k_1 f_\omega]}{2a_1\omega(k_2^2 - k_1^2)},$$
$$B_1 = \frac{P_2\sqrt{P_1}(1 - k_1 f_\omega)}{2ak_1(P_1 - P_2)}, \qquad B_2 = -\frac{E_c' k_1^2}{P_1 - P_2}, \tag{4.13}$$
$$B_3 = \frac{(2k_1 - \sqrt{P_1})P_2}{2ak_1(P_1 - P_2)}, \qquad B_4 = -(B_1 + B_2 + B_3).$$

From (2.14) and (2.15), by virtue of the transform equation (2.18), we get

$$\frac{\partial h_1}{\partial t} = \frac{\partial u}{\partial y},$$
$$\frac{\partial h_2}{\partial t} = -\frac{\partial u}{\partial x}. \tag{4.14}$$

Now, from (4.1),(4.7), and (4.14), the components of the induced magnetic field are given by

$$h_1(x, y, t) = \frac{xe^{\omega t}}{2\omega}(2f_1''(y) + \varepsilon e^{\omega t} f_2''(y)),$$
$$h_2(x, y, t) = -\frac{e^{\omega t}}{2\omega}(2f_1'(y) + \varepsilon e^{at} f_2'(y)). \tag{4.15}$$

From (2.6), (2.8), (4.1), and (4.7), by virtue of the transform equation (2.18), the electric field and the electric current density are given by

$$E(x, y, t) = -xe^{\omega t}(f_1'(y) + \varepsilon e^{\omega t} f_2'(y)),$$
$$J(x, y, t) = \frac{e^{\omega t}}{2a}[2(f_1' - xf_1''') + \varepsilon e^{at}(f_2' - xf_2''')]. \tag{4.16}$$

From the velocity filed, we can study the wall shear stress τ, as given by

$$\tau(x, t) = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}. \tag{4.17}$$

Form (3.1), (4.1), (4.5), (4.7), and (4.11), we obtain

$$\tau = \mu D\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu D x e^{\omega t}[-k_1 + \varepsilon e^{\omega t}(k_1^2 A_2 + k_2^2 A_3)]. \tag{4.18}$$

The local friction coefficient C_f is then given by

$$C_f = \frac{\tau}{(1/2)\mu D}. \tag{4.19}$$

It follows from (4.19) that

$$C_f = 2xe^{\omega t}\left[-k_1 + \varepsilon e^{\omega t}\left(k_1^2 A_2 + k_2^2 A_3\right)\right]. \tag{4.20}$$

Fourier's law may write the local heat flux q.
Let $q = x^2 e^{at} q_0 + x^2 e^{at} q_1$, where q_0 and q_1 are given by

$$q_0 = -\frac{\lambda}{1+\tau_0 n}\Psi_1'(0), \qquad q_1 = -\frac{\lambda}{1+2\tau_0 n}\Psi_2'(0). \tag{4.21}$$

The local heat transfer coefficient is given by

$$q = -\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{4.22}$$

From (4.3), (4.11), (4.17), (4.21), and (4.22), we obtain

$$\begin{aligned} q &= \lambda\frac{U_0}{\nu}\left(T_0 - T_\infty\right)\left(\frac{\partial T}{\partial y}\right)_{y=0} \\ &= -\lambda x^2 e^{\omega t}\frac{U_0}{\nu}\left(T_0 - T_\infty\right)\left[\sqrt{P_1}B_1 + 2k_1 B_2 + \left(k_1 + \sqrt{P_1}\right)B_3 + \sqrt{P_2}B_4\right]. \end{aligned} \tag{4.23}$$

The local heat transfer coefficient is given by

$$\begin{aligned} h(x,t) &= \frac{q(x,t)}{\left(T_0 - T_\infty\right)} \\ &= -\lambda x^2 e^{\omega t}\frac{U_0}{\nu}\left[\sqrt{P_1}B_1 + 2k_1 B_2 + \left(k_1 + \sqrt{P_1}\right)B_3 + \sqrt{P_2}B_4\right]. \end{aligned} \tag{4.24}$$

The local Nusselt number is given by

$$N(x,t) = \frac{h(x,t)}{\lambda} = -x^2 e^{\omega t}\frac{U_0}{\nu}\left[\sqrt{P_1}B_1 + 2k_1 B_2 + \left(k_1 + \sqrt{P_1}\right)B_3 + \sqrt{P_2}B_4\right]. \tag{4.25}$$

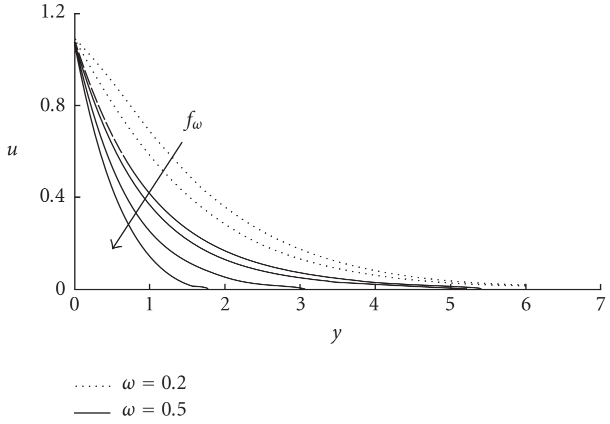

Figure 5.1. Effect of surface mass transfer on velocity distribution, where $f_\omega = 3.0, 1.0, 0.0, -1, -2, -3$.

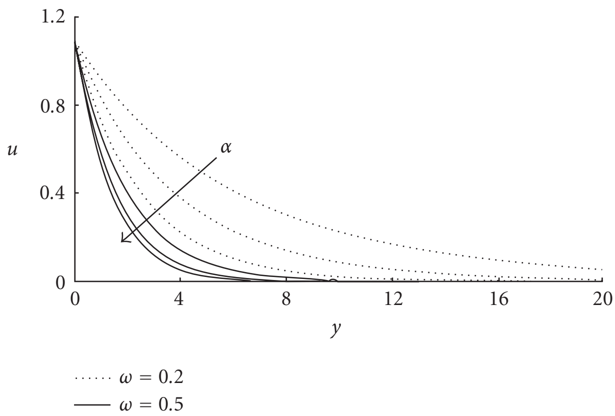

Figure 5.2. Effect of Alfven velocity α on velocity distribution, where $\alpha = 0.6, 0.3, 0.0$.

5. Results and discussion

The velocity profiles for $\omega t = 1.0$, $\alpha = 0.2$, and for different values of f_ω are shown in Figure 5.1. As might be expected, suction ($f_\omega < 0$) broadens the velocity distribution and thickens the boundary-layer thickness, while injection ($f_\omega > 0$) thins it. Also, the wall shear stress would be increased with the application of suction whereas injection tends to decrease the wall shear stress. This can be readily understood from the fact that the wall velocity gradient is increased with the increase of the value of f_ω. The effects of Alfven velocity α on the velocity profiles are presented in Figure 5.2 for $f_\omega = 2$, and $\omega t = 1$. In this figure, the dotted lines represent the solution of this flow, when $\omega = 0.2$, and the solid lines represent the solution of this flow obtained when $\omega = 0.5$. It can be seen from this figure that the velocity field increases with the increase of values of the Alfven velocity parameter α, and an increase in the value of ω leads to a decrease in the velocity.

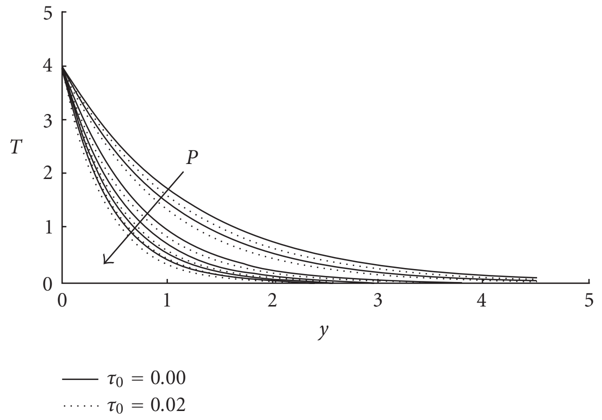

Figure 5.3. Temperature distribution for various values of Prandtl number P, where $P = 0.7, 1.0, 2.0, 3.0, 4.0, 5.0$.

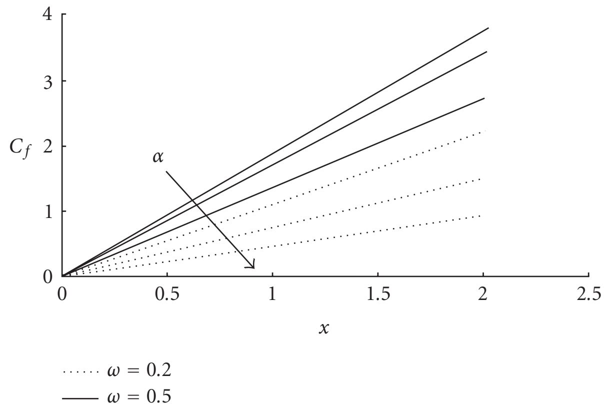

Figure 5.4. Effect of Alfven velocity α on friction coefficient, where $\alpha = 0.0, 0.2, 0.4$.

Results for a typical temperature profile are illustrated in Figure 5.3 for various values of Prandtl number and relaxation time. The thermal boundary layer thickness is more reduced together with a larger wall temperature gradient when the relaxation time $\tau_0 = 0.02$. Also, it is observed that an increase in the value of P leads to a decrease in the temperature field.

The skin friction coefficient C_f is plotted against x in Figure 5.4 for different values of α and two values of ω. The effects of Alfven velocity α are observed from Figure 5.4. An increase in the value of α leads to a decrease in the skin fraction coefficient. Also, the skin fraction coefficient is found to increase when $\omega = 0.5$ as compared to when $\omega = 0.2$.

The effects of Prandtl number is observed from Figure 5.5. An increase in the Prandtl number leads to an increase in the local Nusselt number. Also, it can be seen from this figure that the local Nusselt number increases slowly when τ_0 increases.

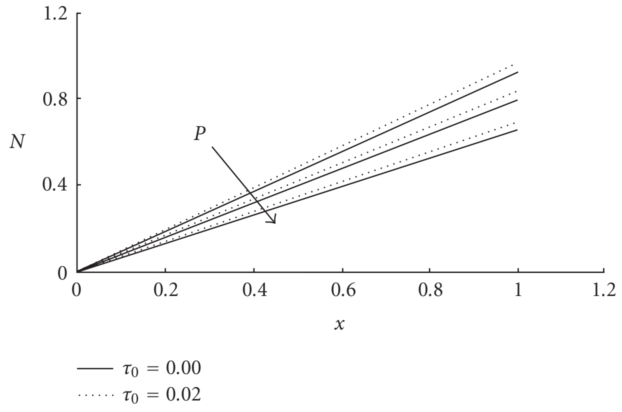

Figure 5.5. Local Nusselt number, where $P = 0.9, 0.7, 0.5$, versus x.

6. Concluding remarks

The electromagnetic flow has many applications in electric heating, mathematical biology, biofluid mechanics, biomedical engineering, and the blood. To study the effect of the electric field on the particles, we must take another term in the governing equation (2.2); it will lead to the discussion of the attraction force among the particles suspended in the fluid (in a forthcoming paper). For liquid metals, the term $\varepsilon_0(\partial E/\partial t)$ is usually negligible.

The generalized thermofluid with one relaxation time based on a modified Fourier law of heat conduction for isotropic media in the absence of heat sources was developed in Section 2. This modification allows for so-called second-sound effects in fluid, hence thermal disturbances propagate with finite wave speeds. This remedies the physically unacceptable situation in classical thermofluid that predicts infinite speed of propagation for such disturbance [5].

In this work, we use a more general model of equations, which includes the relaxation time of heat conduction and the electric permeability of the electromagnetic field. The inclusion of the relaxation time and electric permeability modifies the governing thermal and electromagnetic equations, changing them from parabolic to hyperbolic type, and thereby eliminating the unrealistic result that thermal disturbance is realized instantaneously everywhere within a fluid [10].

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M. Ezzat: Department of Mathematics, Faculty of Education, Alexandria University, El-Shatby 21526, Alexandria, Egypt

E-mail address: m_ezzat2002@yahoo.com

M. Zakaria: Department of Mathematics, Faculty of Education, Alexandria University, El-Shatby 21526, Alexandria, Egypt

E-mail address: zakariandm@yahoo.com

M. Moursy: Department of Mathematics, Faculty of Education, Alexandria University, El-Shatby 21526, Alexandria, Egypt

E-mail address: mfky@yahoo.com