INSTABILITY THROUGH POROUS MEDIUM OF TWO VISCOUS SUPERPOSED CONDUCTING FLUIDS

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ABSTRACT. The stability of the plane interface separating two viscous superposed conducting fluids through porous medium is studied when the whole system is immersed in a uniform horizontal magnetic field. The stability analysis is carried out for two highly viscous fluids of equal kinematic viscosities, for mathematical simplicity. It is found that the stability criterion is independent of the effects of viscosity and porosity of the medium and is dependent on the orientation and magnitude of the magnetic field. The magnetic field is found to stabilize a certain wave number range of the unstable configuration. The behaviour of growth rates with respect to viscosity, porosity and medium permeability are examined analytically.

1. INTRODUCTION.

    The instability of the plane interface between two fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been discussed by Chandrasekhar [1]. Bhatia [2] has studied the influence of viscosity on the stability of the plane interface separating two incompressible superposed conducting fluids of

uniform densities, when the whole system is immersed in a uniform horizontal magne-
tic field. He has carried out the stability analysis for two highly viscous fluids
of equal kinematic viscosities and different uniform densities. When the fluid
slowly percolates through the pores of the rock, the gross effect is represented by
Darcy's law which states that the usual viscous term in the equations of fluid
motion is replaced by the resistance term (μ/k_1) $\vec{q}$, where μ is the viscosity of the
fluid, k_1 the permeability of the medium and $\vec{q}$ the velocity of the fluid. Wooding
[3] has experimentally observed, in the absence of viscous dissipation and consider-
ing only Darcy resistance, that convection sets on as a fairly regular cellular
pattern in the horizontal. This problem in the case of a conducting fluid consider-
ing both Darcy and viscous resistances has been investigated by Prabhamani and
Rudraiah [4]. Saville [5] has studied the stability of motions involving fluid
interfaces in porous media. Various problems of fluid flows through porous medium
have been treated by Saffman and Taylor [6], Chouke et al [7], Scheidegger [8],
Yih [9], Nayfeh [10] and Rudraiah and Prabhamani [11].

The instability of two viscous superposed conducting fluids through porous
medium may find applications in geophysics. It is therefore the motivation of this
study to examine the effects of viscosity and medium permeability on the stability
of the plane interface separating two incompressible superposed conducting fluids of
uniform densities, when the whole system is immersed in a uniform horizontal magnetic
field. We examine the roles of viscosity, medium permeability and magnetic field on
the instability problem. This aspect forms the subject matter of the present study
wherein we have carried out the stability analysis for two highly viscous fluids of
equal kinematic viscosities and different uniform densities.

2.   PERTURBATION EQUATIONS.

Consider the motion of an incompressible, infinitely conducting viscous fluid
(of variable viscosity $\mu_o(z)$) in the presence of a uniform magnetic field $\vec{H}(H_x,H_y,0)$.
Let $\vec{q}(u,v,w)$, $\delta\rho$, δp and $\vec{h}(h_x,h_y,h_z)$ denote the perturbations in velocity, density ρ,
pressure p and magnetic field $\vec{H}$ respectively. Then the linearized perturbation
equations of a fluid flowing through a porous medium when both Darcy as well as viscous
resistances are present are:

$$\frac{\rho}{\varepsilon} \frac{\partial \vec{q}}{\partial t} = -\nabla \delta p + \vec{g}\delta\rho + \frac{1}{4\pi} (\nabla \times \vec{h}) \times \vec{H} - \frac{\rho\nu}{k_1} \vec{q} + \frac{1}{\varepsilon} [\mu\nabla^2\vec{q} + (\nabla\vec{q})\cdot\nabla\mu + (\nabla\mu\cdot\nabla)\vec{q}],$$ (2.1)

$$\nabla\cdot\vec{q} = 0, \quad \nabla\cdot\vec{h} = 0,$$ (2.2)

$$\varepsilon \frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{q}\times\vec{H}),$$ (2.3)

$$\varepsilon \frac{\partial}{\partial t} \delta\rho + (\vec{q}\cdot\nabla)\rho = 0.$$ (2.4)

Equation (4) ensures that the density of every particle remains unchanged as we follow it with its motion. $\nu(=\mu/\rho)$ denotes the kinematic viscosity of the fluid, ε is porosity ($0<\varepsilon<1$) and $\vec{g}$ $(0,0,-g)$ is the acceleration due to gravity. $\varepsilon \to 1$ and $k_1 \to \infty$ correspond to nonporous medium. Analyzing the disturbances into normal modes, we assume that the perturbed quantities have the space (x,y,z) and time (t) dependence of the form

$$f(z)\exp(ik_x x + ik_y y + nt),$$ (2.5)

where k_x, k_y are horizontal wave numbers ($k^2 = k_x^2 + k_y^2$), n is the growth rate of the harmonic disturbance and $f(z)$ is some function of z.

For perturbations of the form (2.5), equations (2.1)-(2.4) give

$$\rho(\frac{n}{\varepsilon} + \frac{\nu}{k_1}) u + \frac{H_y}{4\pi} (ik_x h_y - ik_y h_x) = -ik_x\delta p + \frac{1}{\varepsilon} [\mu(D^2-k^2)u + (D\mu)(ik_y w + Du)],$$ (2.6)

$$\rho(\frac{n}{\varepsilon} + \frac{\nu}{k_1}) v + \frac{H_x}{4\pi} (ik_y h_x - ik_x h_y) = -ik_y\delta p + \frac{1}{\varepsilon} [\mu(D^2-k^2)v + (D\mu)(ik_y w + Dv)],$$ (2.7)

$$\rho(\frac{n}{\varepsilon} + \frac{\nu}{k_1})w + \frac{H_x}{4\pi} (Dh_x - ik_x h_z) + \frac{H_y}{4\pi} (Dh_y - ik_y h_z) = -D\delta p - g\delta\rho + \frac{1}{\varepsilon} [\mu(D^2-k^2)w + 2(D\mu)(Dw)],$$ (2.8)

$$ik_x u + ik_y v + Dw = 0, \quad ik_x h_x + ik_y h_y + Dh_z = 0,$$ (2.9)

$$n\delta\rho = -w(D\rho)/\varepsilon,$$ (2.10)

$$n\vec{h} = (ik_x H_x + ik_y H_y) \vec{q}/\varepsilon,$$ (2.11)

where $D = d/dz$.

Multiplying (2.6) and (2.7) by $-ik_x$ and $-ik_y$ respectively and adding, using (2.9)-

(2.11) and finally, eliminating δp between the resulting equation and (2.8), we

obtain the equation in w:

$$[k^2\rho \ (\frac{n}{\varepsilon} + \frac{\nu}{k_1})w - D \ \{\rho \ (\frac{n}{\varepsilon} + \frac{\nu}{k_1}) \ Dw\}] - \frac{gk^2}{n\varepsilon} \ (D\rho) \ w - \frac{1}{4\pi n\varepsilon} \ (k_x H_x + k_y H_y)^2 (D^2 - k^2)w$$

$$+ \frac{1}{\varepsilon} \ [\mu(D^2 - k^2)^2 w + 2(D\mu)(D^2 - k^2)Dw + (D^2\mu)(D^2 + k^2)w] = 0. \tag{2.12}$$

3.   TWO SUPERPOSED VISCOUS FLUIDS OF UNIFORM DENSITIES.

Here we consider the case when two superposed fluids of uniform densities ρ_1

and ρ_2 and uniform viscosities μ_1 and μ_2 are separated by a horizontal boundary at

z = 0. The subscripts 1 and 2 distinguish the lower and upper fluids respectively.

Then, in each region of constant ρ_1, μ_1 and constant ρ_2, μ_2, equation (2.12) becomes

$$(D^2 - k^2)(D^2 - K^2)w = 0, \tag{3.1}$$

where

$$K^2 = k^2 + \frac{n}{\nu} \ \{1 + \frac{\nu\varepsilon}{nk_1} + \frac{1}{4\pi n^2\rho} \ (k_x H_x + k_y H_y)^2\}. \tag{3.2}$$

Since w must vanish both when $z \to -\infty$ (in the lower fluid) and $z \to +\infty$ (in the

upper fluid), the solutions appropriate to the two regions can be written as

$$w_1 = A_1 e^{+kz} + B_1 e^{+K_1 z} \quad (z<0), \tag{3.3}$$

$$w_2 = A_2 e^{-kz} + B_2 e^{-K_2 z} \quad (z>0), \tag{3.4}$$

where A_1, B_1, A_2, B_2 are constants,

$$K_1 = \sqrt{k^2 + \frac{n}{\nu_1} \ \{1 + \frac{\nu_1\varepsilon}{nk_1} + \frac{1}{4\pi n^2\rho_1} \ (k_x H_x + k_y H_y)^2\}}, \tag{3.5}$$

and

$$K_2 = \sqrt{k^2 + \frac{n}{\nu_2} \ \{1 + \frac{\nu_2\varepsilon}{nk_1} + \frac{1}{4\pi n^2\rho_2} \ (k_x H_x + k_y H_y)^2\}}. \tag{3.6}$$

In writing the solutions (3.3) and (3.4) it is assumed that K_1 and K_2 are so defined that their real parts are positive. The solutions (3.3) and (3.4) must satisfy certain boundary conditions. The boundary conditions to be satisfied at the interface $z = 0$ are (Chandrasekhar [1], p.432)

$$w, \tag{3.7}$$

$$Dw, \tag{3.8}$$

and

$$\mu(D^2+k^2)w, \tag{3.9}$$

must be continuous.

Integrating (2.12) across the interface $z = 0$, we obtain another condition

$$\left\{\rho_2 \left(\frac{n}{\epsilon} + \frac{\nu_2}{k_1}\right) Dw_2 - \frac{\mu_2}{\epsilon} (D^2-k^2)Dw_2 + \frac{1}{4\pi n\epsilon} (k_x H_x + k_y H_y)^2 Dw_2\right\}_{z=0}$$

$$- \left\{\rho_1 \left(\frac{n}{\epsilon} + \frac{\nu_1}{k_1}\right) Dw_1 - \frac{\mu_1}{\epsilon} (D^2-k^2)Dw_1 + \frac{1}{4\pi n\epsilon} (k_x H_x + k_y H_y)^2 Dw_1\right\}_{z=0}$$

$$= -\frac{gk^2}{n\epsilon} (\rho_2-\rho_1)w_0 - \frac{2k^2}{\epsilon} (\mu_2-\mu_1) (Dw)_0, \tag{3.10}$$

where w_0 and $(Dw)_0$ are the unique values of these quantities at $z = 0$.

Applying the conditions (3.7)-(3.10) to the solutions (3.3) and (3.4), we obtain

$$A_1 + B_1 = A_2 + B_2, \tag{3.11}$$

$$kA_1 + K_1 B_1 = -kA_2 - K_2 B_2, \tag{3.12}$$

$$\mu_1 \{2k^2 A_1 + (K_1^2+k^2)B_1\} = \mu_2 \{2k^2 A_2 + (K_2^2+k^2) B_2\}, \tag{3.13}$$

$$-k\rho_2 \left(\frac{1}{\epsilon} + \frac{\nu_2}{nk_1}\right)A_2 - k\rho_1 \left(\frac{1}{\epsilon} + \frac{\nu_1}{nk_1}\right)A_1 - (kA_1+kA_2) \frac{1}{4\pi n^2 \epsilon} (k_x H_x + k_y H_y)^2$$

$$= -\frac{gk^2}{2n^2\epsilon} (\rho_2-\rho_1) (A_1+B_1+A_2+B_2) + \frac{k^2}{n\epsilon} (\mu_1-\mu_2) (kA_1+K_1 B_1 - kA_2 - K_2 B_2). \tag{3.14}$$

Eliminating the constants A_1, A_2, B_1, B_2 from (3.11)-(3.14), we obtain

$$
\begin{vmatrix}
1 & 1 & -1 & -1 \\
k & K_1 & k & K_2 \\
2k^2\mu_1 & \mu_1(K_1^2+k^2) & -2k^2\mu_2 & -\mu_2(K_2^2+k^2) \\
[\tfrac{1}{2}R-C-\dfrac{\alpha_1}{\varepsilon} & [\tfrac{1}{2}R-C\dfrac{K_1}{k} & [\tfrac{1}{2}R+C-\dfrac{\alpha_2}{\varepsilon}-\dfrac{\alpha_2\nu_2}{nk_1} & [\tfrac{1}{2}R+C\dfrac{K_2}{k} \\
-\dfrac{\alpha_1\nu_1}{nk_1}-\dfrac{1}{n^2}(\vec{k}\cdot\vec{V}_A)^2] & +\dfrac{\alpha_1\nu_1}{n}\dfrac{K_1}{k_1}\dfrac{K_1}{k}] & -\dfrac{1}{n^2}(\vec{k}\cdot\vec{V}_A)^2] & +\dfrac{\alpha_2\nu_2}{n}\dfrac{K_2}{k_1}\dfrac{K_2}{k}
\end{vmatrix}
= 0, \quad (3.15)
$$

where

$$
\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1+\rho_2}, \qquad (\vec{k}\cdot\vec{V}_A)^2 = \frac{(k_x H_x+k_y H_y)^2}{4\pi(\rho_1+\rho_2)\varepsilon}.
$$

$$
R = \frac{gk}{\varepsilon n^2}(\alpha_2-\alpha_1), \qquad C = \frac{k^2}{n\varepsilon}\frac{\mu_1-\mu_2}{\rho_1+\rho_2} = \frac{k^2}{n\varepsilon}(\alpha_1\nu_1-\alpha_2\nu_2),
$$

and $\vec{V}_A$ is the Alfvén velocity vector.

Evaluating the determinant (3.15), we obtain the following characteristic equation

$$
(K_1-k)\ [2k^2(\alpha_1\nu_1-\alpha_2\nu_2)\ \{\frac{C}{k}(K_2-k)+\alpha_2\ (\frac{1}{\varepsilon}+\frac{\nu_2}{nk_1})+\frac{1}{n^2}(\vec{k}\cdot\vec{V}_A)^2\}
$$

$$
+\{\alpha_2 n+\frac{\alpha_2\nu_2\varepsilon}{k_1}+\frac{\varepsilon}{n}(\vec{k}\cdot\vec{V}_A)^2\}\ \{R-\frac{1}{\varepsilon}-\frac{(\alpha_1\nu_1+\alpha_2\nu_2)}{nk_1}-\frac{2}{n^2}(\vec{k}\cdot\vec{V}_A)^2\}]
$$

$$
-2k[\{\alpha_1 n+\frac{\alpha_1\nu_1\varepsilon}{k_1}+\frac{\varepsilon}{n}(\vec{k}\cdot\vec{V}_A)^2\}\{\alpha_2(\frac{1}{\varepsilon}+\frac{\nu_2}{nk_1})+\frac{C}{k}(K_2-k)+\frac{1}{n^2}(\vec{k}\cdot\vec{V}_A)^2\}
$$

$$
+\{\alpha_2 n+\frac{\alpha_2\nu_2\varepsilon}{k_1}+\frac{\varepsilon}{n}(\vec{k}\cdot\vec{V}_A)^2\}\ \{\alpha_1(\frac{1}{\varepsilon}+\frac{\nu_1}{nk_1})-\frac{C}{k}(K_1-k)+\frac{1}{n^2}(\vec{k}\cdot\vec{V}_A)^2\}\ ]
$$

$$
+(K_2-k)\ [\{\alpha_1 n+\frac{\alpha_1\nu_1\varepsilon}{k_1}+\frac{\varepsilon}{n}(\vec{k}\cdot\vec{V}_A)^2\}\ \{R-\frac{1}{\varepsilon}-\frac{\alpha_1\nu_1+\alpha_2\nu_2}{nk_1}-\frac{2}{n^2}(\vec{k}\cdot\vec{V}_A)^2\}
$$

$$
-2k^2(\alpha_1\nu_1-\alpha_2\nu_2)\{\alpha_1(\frac{1}{\varepsilon}+\frac{\nu_1}{nk_1})-\frac{C}{k}(K_1-k)+\frac{1}{n^2}(\vec{k}\cdot\vec{V}_A)^2\}] = 0. \qquad (3.16)
$$

Since K_1 and K_2 involve square roots, the dispersion relation (3.16) is quite

complex. We therefore carry out the stability analysis for highly viscous fluids.

Under this assumption of highly viscous fluids, we have

$$K = k[1 + \frac{n}{\nu k^2} + \frac{\varepsilon}{k^2 k_1} + \frac{\varepsilon(k_x H_x + k_y H_y)^2}{4\pi\rho\nu nk^2}]^{\frac{1}{2}}$$

$$= k [1 + \frac{n}{2\nu k^2} + \frac{\varepsilon}{2k^2 k_1} + \frac{\varepsilon(\rho_1 + \rho_2)}{2nk^2\rho\nu} (\vec{k} \cdot \vec{v}_A)^2], \qquad (3.17)$$

so that

$$K_1 - k = \frac{n}{2\nu_1 k} + \frac{\varepsilon}{2kk_1} + \frac{\varepsilon}{2nk\alpha_1\nu_1} (\vec{k} \cdot \vec{v}_A)^2, \qquad (3.18)$$

and

$$K_2 - k = \frac{n}{2\nu_2 k} + \frac{\varepsilon}{2kk_1} + \frac{\varepsilon}{2nk\alpha_2\nu_2} (\vec{k} \cdot \vec{v}_A)^2. \qquad (3.19)$$

Substituting the values of K_1 and K_2 from (3.18), (3.19) in (3.16) and putting

$\nu_1 = \nu_2 = \nu$ (the case of equal kinematic viscosities, for mathematical simplicity,

as in Chandrasekhar [1]), we obtain the following dispersion relation

$$\frac{1}{\varepsilon} n^6 + [\frac{3\nu}{k_1} + \frac{2\alpha_1\alpha_2}{\varepsilon} + (\alpha_1 - \alpha_2)^2 (\frac{1}{2} + \frac{k^2\nu}{\varepsilon} \overline{2-\varepsilon})]n^5 + [\frac{(\vec{k} \cdot \vec{v}_A)^2}{\alpha_1\alpha_2} + A + \frac{3\nu^2\varepsilon}{k_1^2} + \frac{4k^2\nu^2}{k_1} (\alpha_1 - \alpha_2)^2 \times$$

$$\times(1-\varepsilon) + \frac{\nu}{k_1} \{4\alpha_1\alpha_2 + \varepsilon(\alpha_1 - \alpha_2)^2\}]n^4 + [\frac{\nu\varepsilon}{k_1} \{2A + \frac{\nu^2\varepsilon}{k_1^2} + \frac{2k^2\nu^2}{k_1} (\alpha_1 - \alpha_2)^2 (1-\varepsilon) + 2\alpha_1\alpha_2\nu + \frac{\nu\varepsilon}{2k_1}$$

$$+ (\vec{k} \cdot \vec{v}_A)^2 \{2 + \frac{\varepsilon}{2\alpha_1\alpha_2} (\alpha_1 - \alpha_2)^2 + \frac{2\nu\varepsilon}{k_1\alpha_1\alpha_2} + \frac{2k^2\nu}{\alpha_1\alpha_2} (\alpha_1 - \alpha_2)^2 (1-\varepsilon)\}]n^3$$

$$+ [\frac{\varepsilon(\vec{k} \cdot \vec{v}_A)^2}{\alpha_1\alpha_2} \{A + (\vec{k} \cdot \vec{v}_A)^2 + \frac{\nu^2\varepsilon}{k_1^2} (\alpha_1^2 + \alpha_2^2) + \frac{2k^2\nu^2}{k_1} (\alpha_1 - \alpha_2)^2 (1-\varepsilon)\}$$

$$+ \frac{\nu\varepsilon}{k_1} \{2(\vec{k} \cdot \vec{v}_A)^2 (1 + \frac{\nu\varepsilon}{k_1}) + \frac{\nu\varepsilon A}{k_1} + \frac{\varepsilon}{2\alpha_1\alpha_2} (\vec{k} \cdot \vec{v}_A)^2 (\alpha_1 - \alpha_2)^2\}]n^2$$

$$+ [\varepsilon(\vec{k} \cdot \vec{v}_A)^4 \{2 + \frac{\varepsilon}{2\alpha_1\alpha_2} (\alpha_1 - \alpha_2)^2\} + \frac{\varepsilon}{\alpha_1\alpha_2} (\vec{k} \cdot \vec{v}_A)^2 \{\frac{\nu\varepsilon}{k_1} (\vec{k} \cdot \vec{v}_A)^2 + \frac{\nu\varepsilon A}{k_1}$$

$$+ \frac{2k^2 \nu (\alpha_1 - \alpha_2)^2 (\vec{k} \cdot \vec{V}_A)^2}{\alpha_1 \alpha_2} \quad (1-\varepsilon) \}] n + \frac{\varepsilon}{\alpha_1 \alpha_2} A \quad (\vec{k} \cdot \vec{V}_A)^4 = 0, \tag{3.20}$$

where we have written $2(\vec{k} \cdot \vec{V}_A)^2 + \frac{gk}{\varepsilon} (\alpha_1 - \alpha_2) = A$.

(i) Stable case

For the potentially stable arrangement $\alpha_1 > \alpha_2$, we find by applying Hurwitz' criterion to (3.20), that (as all the coefficients in (3.20) are then positive) all the roots of n are either real and negative or there are complex roots with negative real parts. The system is therefore stable in each case. The potentially stable configuration, therefore, remains stable whether the effects of viscosity and medium porosity are included or not.

(ii) Unstable case

For the potentially unstable arrangement $\alpha_2 > \alpha_1$, the system is unstable in the hydrodynamic case for all wave numbers k in the presence of viscosity effects and in the absence of porosity effects (Chandrasekhar [1]). Also the system, in the present case, is unstable if

$$2(\vec{k} \cdot \vec{V}_A)^2 < \frac{gk}{\varepsilon} (\alpha_2 - \alpha_1).$$

In the present hydromagnetic case we find, by applying Hurwitz' criterion to (3.20) when $\alpha_2 > \alpha_1$, that the system is stable for all wave numbers which satisfy the inequality

$$2(\vec{k} \cdot \vec{V}_A)^2 > \frac{gk}{\varepsilon} (\alpha_2 - \alpha_1), \tag{3.21}$$

i.e. $2k(V_1 \cos\theta + V_2 \sin\theta)^2 > \frac{g}{\varepsilon} (\alpha_2 - \alpha_1),$ \hfill (3.22)

where V_1 and V_2 are the Alfvén velocities in the x and y directions and θ is the angle between $\vec{k}$ and H_x.

The stability criterion (3.22) is independent of the effects of viscosity and medium porosity. The magnetic field stabilizes a certain wave number range k > k* where

$$k^* = g(\alpha_2 - \alpha_1)/2\varepsilon(V_1\cos\theta + V_2\sin\theta)^2,\tag{3.23}$$

of the unstable configuration even in the presence of the effects of viscosity and medium porosity. The critical wave number k^*, above which the system is stabilized, is dependent on the magnitudes V_1 and V_2 of the magnetic field as well as the orientation of the magnetic field θ.

We now examine the behaviour of growth rates with respect to viscosity, porosity and medium permeability analytically. Since for $\alpha_2 > \alpha_1$ and $2(\vec{k}\cdot\vec{V}_A)^2 <$ $\frac{gk}{\varepsilon}(\alpha_2 - \alpha_1)$, (3.20) has one positive root, let n_o denote the positive root. Then (3.20) is satisfied if n_o is substituted in place of n. To study the behaviour of growth rates with respect to viscosity and medium permeability, we examine the natures of $dn_o/d\nu$ and dn_o/dk_1. It follows that

$$(6A_6\,n_o^5 + 5A_5\,n_o^4 + 4A_4\,n_o^3 + 3A_3\,n_o^2 + 2A_2\,n_o + A_1)\,\frac{dn_o}{dk_1}$$

$$= \frac{3\nu}{k_1^2}n_o^5 + \frac{1}{k_1^2}\left\{\frac{6\nu^2\varepsilon}{k_1} + 4k^2\nu^2\,(\alpha_1-\alpha_2)^2\,(1-\varepsilon) + \nu\,\{4\alpha_1\alpha_2 + \varepsilon(\alpha_1-\alpha_2)^2\}\right\}n_o^4$$

$$+ \frac{1}{k_1^2}\left\{\frac{2\nu\varepsilon}{\alpha_1\alpha_2}(\vec{k}\cdot\vec{V}_A)^2 - 2A\nu\varepsilon + \frac{3\nu^3\varepsilon^2}{k_1^2} + \frac{4k^2\nu^3\varepsilon(\alpha_1-\alpha_2)^2(1-\varepsilon)}{k_1} + 2\alpha_1\alpha_2\nu^2\varepsilon + \frac{\nu^2\varepsilon^2}{k_1}\right\}n_o^3$$

$$+ \left[\frac{2\nu^2\varepsilon(\vec{k}\cdot\vec{V}_A)^2}{\alpha_1\alpha_2\,k_1^2}\left\{\frac{\varepsilon(\alpha_1^2+\alpha_2^2)}{k_1} + k^2(\alpha_1-\alpha_2)^2\,(1-\varepsilon)\right\} + 2(\vec{k}\cdot\vec{V}_A)^2\,\frac{\nu\varepsilon}{k_1^2}\,\left(1 + \frac{2\nu\varepsilon}{k_1}\right)\right.$$

$$\left. - \frac{2\nu^2\varepsilon^2 A}{k_1^3} + \frac{\nu\varepsilon^2}{2\alpha_1\alpha_2 k_1^2}\,(\vec{k}\cdot\vec{V}_A)^2\,(\alpha_1-\alpha_2)^2\right]n_o^2$$

$$+ \left[\frac{\varepsilon}{\alpha_1\alpha_2 k_1^2}\,(\vec{k}\cdot\vec{V}_A)^2\,\{\nu\varepsilon(\vec{k}\cdot\vec{V}_A)^2 - \nu\varepsilon A\}\right]n_o,\tag{3.24}$$

and

$$-(6A_6 n_o^5 + 5A_5 n_o^4 + 4A_4 n_o^3 + 3A_3 n_o^2 + 2A_2 n_o + A_1)\,\frac{dn_o}{d\nu} =$$

$$= [\frac{3}{k_1} + \frac{k^2}{\varepsilon}(2-\varepsilon)(\alpha_1-\alpha_2)^2]n_o^5 + [\frac{6\nu\varepsilon}{k_1^2} + \frac{8\nu k^2}{k_1}(\alpha_1-\alpha_2)^2(1-\varepsilon) + \frac{1}{k_1}\{4\alpha_1\alpha_2+\varepsilon(\alpha_1-\alpha_2)^2\}]n_o^4$$

$$+ [\frac{3\varepsilon^2\nu^2}{k_1^2} - \frac{2A\varepsilon}{k_1} + \frac{6k^2\varepsilon\nu^2}{k_1^2}(\alpha_1-\alpha_2)^2(1-\varepsilon) + \frac{4\alpha_1\alpha_2\varepsilon\nu}{k_1} + \frac{\nu\varepsilon^2}{k_1^2} + \frac{2\varepsilon(\vec{k}\cdot\vec{V}_A)^2}{\alpha_1\alpha_2 k_1}$$

$$+ \frac{2k^2(\alpha_1-\alpha_2)^2(1-\varepsilon)}{\alpha_1\alpha_2}(\vec{k}\cdot\vec{V}_A)^2]n_o^3$$

$$+ \frac{2\nu\varepsilon^2(\vec{k}\cdot\vec{V}_A)^2}{\alpha_1\alpha_2 k_1^2}(\alpha_1^2+\alpha_2^2) + \frac{4\nu k^2\varepsilon(\vec{k}\cdot\vec{V}_A)^2}{\alpha_1\alpha_2 k_1}(\alpha_1-\alpha_2)^2(1-\varepsilon) + 2(\vec{k}\cdot\vec{V}_A)^2\frac{\varepsilon}{k_1}$$

$$+ \frac{4\nu\varepsilon^2}{k_1^2}(\vec{k}\cdot\vec{V}_A)^2 - \frac{2\nu\varepsilon^2 A}{k_1^2} + \frac{\varepsilon^2}{2k_1\alpha_1\alpha_2}(\vec{k}\cdot\vec{V}_A)^2(\alpha_1-\alpha_2)^2]n_o^2$$

$$+ [\frac{\varepsilon^2}{\alpha_1\alpha_2 k_1}(\vec{k}\cdot\vec{V}_A)^4 - \frac{\varepsilon^2 A}{\alpha_1\alpha_2 k_1}(\vec{k}\cdot\vec{V}_A)^2 + \frac{2k^2\varepsilon}{\alpha_1^2\alpha_2^2}(\vec{k}\cdot\vec{V}_A)^4(\alpha_1-\alpha_2)^2(1-\varepsilon)]n_o, \qquad (3.25)$$

where A_5, A_4, A_3, A_2 and A_1 are the coefficients of n_o^5, n_o^4, n_o^3, n_o^2 and n_o

respectively in (3.20) where n_o is substituted for n. It is evident from (3.24)

and (3.25) that dn_o/dk_1 and $dn_o/d\nu$ may be both positive or negative. Similarly

it can be shown that $dn_o/d\varepsilon$ may be both positive or negative. Thus the growth

rates both increase or decrease with the increase in viscosity, porosity, and medium

permeability. The viscosity, porosity, and medium permeability therefore have both

stabilizing as well as destabilizing tendencies on the growth rates.

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