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REAL HYPERSURFACES OF TYPE A IN QUARTERNIONIC PROJECTIVE SPACE

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ABSTRACT. In this paper, under certain conditions on the orthogonal distribution $\mathcal{D}$, we give a characterization of real hypersurfaces of type A in quaternionic projective space QP^m.

KEY WORDS AND PHRASES. Quaternionic projective space, Real hypersurfaces of type A, Orthogonal distribution
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1. Introduction.

Throughout this paper M will denote a connected real hypersurface of the quaternionic projective space QP^m,$m\geq 3$, endowed with the metric g of constant quaternionic sectional curvature 4. Let N be a unit local normal vector field on M and $U_i = -J_iN$, $i = 1,2,3$, where $\{J_i\}_{i=1,2,3}$ is a local basis of the quaternionic structure of QP^m, [2].

Now let us define a distribution $\mathcal{D}$ by $\mathcal{D}(x) = \{X\in T_xM : X\perp U_i(x), i = 1,2,3\}$, $x\in M$, of a real hypersurface M in QP^m, which is orthogonal to the structure vector fields $\{U_1, U_2, U_3\}$ and invariant with respect to the structure tensors $\{\phi_1, \phi_2, \phi_3\}$,and by $\mathcal{D}^\perp = Span\{U_1, U_2, U_3\}$ its orthogonal complement in TM.

There exist many studies about real hypersurfaces of quaternionic projective space QP^m (See [1],[3],[4],[5],[6]). Among them Martinez and the third author [4] have classified real hypersurfaces of QP^m with constant principal curvatures and the distribution $\mathcal{D}$ is invariant by the shape operator A. It was shown that these real hypersurfaces of QP^m could be divided into three types which are said to be of type A_1,A_2, and B.

Without the additional assumption of constant principal curvatures, as a further improvement of this result Berndt [1] showed recently that all real hypersurfaces of QP^m also could be divided into the above three types when two distributions $\mathcal{D}$ and $\mathcal{D}^\perp$ satisfy $g(A\mathcal{D}, \mathcal{D}^\perp) = 0$. Moreover, it is known that the formula $g(A\mathcal{D}, \mathcal{D}^\perp) = 0$ is equivalent to the fact that the distribution $\mathcal{D}$ is invariant by the shape operator A of M.

In a similar notation of Takagi [7] a real hypersurface of type A_1 denotes a geodesic hypersphere or a tube over a totally geodesic hyperplane QP^{m-1} and of type A_2 denotes a tube over a totally geodesic quaternionic projective space QP^k ($1 \leq k \leq m-2$) respectively. Moreover, real hypersurface of type B denotes a tube over a complex projective space CP^m.

Now, let us consider the following conditions that the shape operator A of M in QP^m may satisfy

$$(\nabla_X A)Y = -\Sigma_{i=1}^3 \{f_i(Y)\phi_i X + g(\phi_i X, Y)U_i\}, \tag{1.1}$$

$$g((A\phi_i - \phi_i A)X, Y) = 0, \tag{1.2}$$

for any $i = 1, 2, 3$, and any tangent vector fields X and Y of M.

Pak [5] investigated the above conditions and showed that they are equivalent to each other. Moreover he used the condition (1.1) to find a lower bound of $\|\nabla A\|$ for real hypersurfaces in QP^m. In fact, it was shown that $\|\nabla A\|^2 \geq 24(m-1)$ for such hypersurfaces and the equality holds if and only if the condition (1.1) holds. In this case it was also known that M is locally congruent to a real hypersurface of type A_1 or A_2, which is said to be of type A.

If we restrict the properties (1.1) and (1.2) to the orthogonal distribution $\mathcal{D}$, then for any vector fields X and Y in $\mathcal{D}$ the shape operator A of M satisfies the following conditions

$$(\nabla_X A)Y = -\Sigma_{i=1}^3 g(\phi_i X, Y)U_i \tag{1.3}$$

and

$$g((A\phi_i - \phi_i A)X, Y) = 0 \tag{1.4}$$

for any $i = 1, 2, 3$. Thus the above conditions (1.3) and (1.4) are weaker than the conditions (1.1) and (1.2) respectively. Thus it is natural that real hypersurfaces of type A should satisfy (1.3) and (1.4). From this point of view we give a characterization of real hypersurfaces of type A in QP^m as the following

THEOREM. Let M be a real hypersurface in QP^m, $m \geq 3$, satisfying (1.3) and (1.4) for all X, Y in $\mathcal{D}$ and any $i = 1, 2, 3$. Then M is congruent to an open subset of a tube of radius r over the canonically (totally geodesic) embedded quaternionic projective space QP^k, for some $k \in \{0, 1, ..., m-1\}$, where $0 < r < \frac{\pi}{2}$.

2. Preliminaries.

Let X be a tangent field to M. We write $J_i X = \phi_i X + f_i(X)N, i = 1, 2, 3$, where $\phi_i X$ is the tangent component of $J_i X$ and $f_i(X) = g(X, U_i), i = 1, 2, 3$. As $J_i^2 = -id$, $i = 1, 2, 3$, where id denotes the identity endomorphism on TQP^m, we get

$$\phi_i^2 X = -X + f_i(X)U_i, \quad f_i(\phi_i X) = 0, \quad \phi_i U_i = 0, \quad i = 1, 2, 3 \tag{2.1}$$

for any X tangent to M. As $J_i J_j = -J_j J_i = J_k$, where (i, j, k) is a cyclic permutation of $(1, 2, 3)$ we obtain

$$\phi_i X = \phi_j \phi_k X - f_k(X)U_j = -\phi_k \phi_j X + f_j(X)U_k \tag{2.2}$$

and

$$f_i(X) = f_j(\phi_k X) = -f_k(\phi_j X) \tag{2.3}$$

for any vector field X tangent to M, where (i, j, k) is a cyclic permutation of $(1, 2, 3)$. It is also easy to see that for any X, Y tangent to M and $i = 1, 2, 3$

$$g(\phi_i X, Y) + g(X, \phi_i Y) = 0, \quad g(\phi_i X, \phi_i Y) = g(X, Y) - f_i(X) f_i(Y) \tag{2.4}$$

and

$$\phi_i U_j = -\phi_j U_i = U_k \tag{2.5}$$

(i, j, k) being a cyclic permutation of $(1,2,3)$. From the expression of the curvature tensor of QP^m, $m \geq 2$, we have that the equations of Gauss and Codazzi are respectively given by

$$\begin{aligned} R(X,Y)Z =& g(Y,Z)X - g(X,Z)Y + \Sigma_{i=1}^3 \{g(\phi_i Y, Z)\phi_i X - g(\phi_i X, Z)\phi_i Y \\ &+ 2g(X, \phi_i Y)\phi_i Z\} + g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \tag{2.6}$$

and

$$(\nabla_X A)Y - (\nabla_Y A)X = \Sigma_{i=1}^3 \{f_i(X)\phi_i Y - f_i(Y)\phi_i X + 2g(X, \phi_i Y)U_i\} \tag{2.7}$$

for any X, Y, Z tangent to M, where R denotes the curvature tensor of M, See [4].

From the expressions of the covariant derivatives of J_i, $\quad i = 1,2,3$, it is easy to see that

$$\nabla_X U_i = -p_j(X)U_k + p_k(X)U_j + \phi_i AX \tag{2.8}$$

and

$$(\nabla_X \phi_i)Y = -p_j(X)\phi_k Y + p_k(X)\phi_j Y + f_i(Y)AX - g(AX, Y)U_i \tag{2.9}$$

for any X, Y tangent to M, $\quad (i, j, k)$ being a cyclic permutation of $(1,2,3)$ and p_i, $\quad i = 1,2,3$, local 1-forms defined on M.

3. Proof of the Theorem.

Let M be a real hypersurface in a quaternionic projective space QP^m, and let $\mathcal{D}$ be a distribution defined by $\mathcal{D}(x) = \{X \in T_x M : X \perp U_i(x), i = 1,2,3\}$. Now we prove the theorem in the introduction. In order to prove this Theorem we should verify that $g(A\mathcal{D}, \mathcal{D}^{\perp}) = 0$ from the conditions (1.3) and (1.4). Then by using a theorem of Berndt [1] we can prove that a real hypersurface M satisfying (1.3) and (1.4) is locally congruent to one of type A_1, or A_2 in the Theorem.

Namely we can obtain another new characterization of real hypersurfaces of type A in a quaternionic projective space QP^m. For this purpose we need a lemma obtained from the restricted condition (1.4) as the following

LEMMA 3.1. Let M be a real hypersurface of QP^m. If it satisfies the condition (1.4) for all X, Y in $\mathcal{D}$ and any $i = 1,2,3$, then we have

$$g((\nabla_X A)Y, Z) = \mathfrak{S} g(AX, Y) g(Z, V_i), \quad i = 1,2,3, \tag{3.1}$$

where $\mathfrak{S}$ denotes the cyclic sum with respect to X, Y and Z in $\mathcal{D}$ and V_i stands for the vector field defined by $\phi_i AU_i$.

PROOF. Differentiating the condition (1.4) covariantly, for any vector fields X, Y and Z in $\mathcal{D}$ we get

$$\begin{aligned} g((\nabla_X A)\phi_i Y + A(\nabla_X \phi_i)Y + A\phi_i \nabla_X Y - (\nabla_X \phi_i)AY - \phi_i(\nabla_X A)Y, Z) \\ - g(\phi_i A \nabla_X Y, Z) + g((A\phi_i - \phi_i A)Y, \nabla_X Z) = 0. \end{aligned}$$

Now let us consider the following for a case where $i = 1$

$$g((\nabla_X A)Y, \phi_1 Z) + g((\nabla_X A)Z, \phi_1 Y) = -g((\nabla_X \phi_1)Y, AZ) - g(\phi_1 \nabla_X Y, AZ) + g((\nabla_X \phi_1)AY, Z) - g(A\nabla_X Y, \phi_1 Z) + \Sigma_i \theta_i(Y) g(\phi_i AX, Z),$$

where $g((A\phi_1 - \phi_1 A)Y, U_i)$ is denoted by $\theta_i(Y)$ and we have used the fact that

$$\begin{aligned} g((A\phi_1 - \phi_1 A)Y, \nabla_X Z) &= \Sigma_i \theta_i(Y) g(U_i, \nabla_X Z) \\ &= -\Sigma_i \theta_i(Y) g(\nabla_X U_i, Z) \\ &= -\Sigma_i \theta_i(Y) g(\phi_i AX, Z). \end{aligned}$$

Then by taking account of (2.8) and (2.9) and using the condition (1.4) again, we have

$$\begin{aligned} g((\nabla_X A)Y, \phi_1 Z) + g((\nabla_X A)Z, \phi_1 Y) = f_1(AZ) g(AX, Y) + f_1(AY) g(AX, Z) \\ + \Sigma_i \theta_i(Z) g(\phi_i AX, Y) + \Sigma_i \theta_i(Y) g(\phi_i AX, Z). \end{aligned} \tag{3.2}$$

In this equation we shall replace X, Y and Z in $\mathcal{D}$ cyclically and we shall then add the second equation to (3.2), from which we subtract the third one. Consequently, by means of Codazzi equation (2.7) we get

$$\begin{aligned} g((\nabla_X A)Y, \phi_1 Z) = & f_1(AZ) g(AX, Y) + \Sigma_i \theta_i(X) g(A\phi_i Y, Z) \\ & + \Sigma_i \theta_i(Y) g(A\phi_i X, Z). \end{aligned}$$

From this, replacing Z by $\phi_1 Z$, we have

$$\begin{aligned} g((\nabla_X A)Y, Z) = & g(V_1, Z) g(AX, Y) - \Sigma_i \theta_i(X) g(A\phi_i Y, \phi_1 Z) \\ & - \Sigma_i \theta_i(Y) g(A\phi_i X, \phi_1 Z). \end{aligned} \tag{3.3}$$

where V_1 denotes $\phi_1 AU_1$ and the second term of the right side are given by the following

$$\begin{aligned} \Sigma_i \theta_i(X) g(A\phi_i Y, \phi_1 Z) = & - g(X, \phi_1 AU_1) g(AY, Z) + \{g(A\phi_1 X, U_2) \\ & + g(AX, U_3)\} g(AY, \phi_3 Z) - \{g(A\phi_1 X, U_3) \\ & - g(AX, U_2)\} g(AY, \phi_2 Z), \end{aligned}$$

from this, the third term can be given by exchanging X and Y. Thus substituting this into (3.3), we have

$$g((\nabla_X A)Y, Z) = \mathfrak{S} g(V_1, Z) g(AX, Y) + \alpha(X, Y, Z) + \alpha(Y, X, Z), \tag{3.4}$$

where $\mathfrak{S}$ denotes the cyclic sum with respect to X, Y and Z in $\mathcal{D}$ and $\alpha(X, Y, Z)$ denotes

$$-\{g(A\phi_1 X, U_2) + g(AX, U_3)\} g(AY, \phi_3 Z) + \{g(A\phi_1 X, U_3) - g(AX, U_2)\} g(AY, \phi_2 Z).$$

Then by virtue of the assumption $\alpha(X, Y, Z)$ is skew-symmetric with respect to Y and Z in $\mathcal{D}$.

Now firstly let us take cyclic sum of the both sides of (3.4) one more time. Next using the skew-symmetry of $\alpha(X, Y, Z)$ to the right and the equation of Codazzi (2.7) to the left of the obtained equation respectively, we have the above result for $i = 1$. For a case where $i = 2$ or 3 by using the same method we can also prove the above result. □

PROOF OF THE THEOREM. From the assumption (1.3) we know that the shape operator A is η-parallel, that is, $g((\nabla_X A)Y, Z) = 0$ for any X, Y and Z in $\mathcal{D}$. From this, by Lemma 3.1 we have for a case where $i = 1$

$$g(V_1, Z)g(AX, Y) + g(V_1, Y)g(AZ, X) + g(V_1, X)g(AZ, Y) = 0. \tag{3.5}$$

Thus in order to prove $g(A\mathcal{D}, \mathcal{D}^\perp) = 0$, we suppose that there is a point p at which $g(A\mathcal{D}, \mathcal{D}^\perp)_p \neq 0$. Then there exists a neighborhood $\mathcal{U} = \{p \in M : g(A\mathcal{D}, \mathcal{D}^\perp)_p \neq 0\}$ on which there exist such a distribution $\mathcal{D}$. Now let us denote AU_i by

$$AU_i = W_i + \Sigma_j \alpha_{ij} U_j, \tag{3.6}$$

where $W_i, i = 1,2,3$ denote certain vectors in $\mathcal{D}$. Since on this neighborhood $\mathcal{U}$ we have $g(A\mathcal{D}, \mathcal{D}^\perp) \neq 0$, at least one of the vectors $W_i, i = 1,2,3$ should not be vanishing. Thus for a convenience sake let us assume that W_1 is a non zero vector on this neighborhood $\mathcal{U}$. Then it follows that

$$V_1 = \phi_1 AU_1 = \phi_1 W_1 + \Sigma_j \alpha_{1j} \phi_1 U_j, \quad W_1 \in \mathcal{D},$$

so that, (3.5) gives the following for any X, Y and Z in $\mathcal{D}$

$$g(\phi_1 W_1, Z)g(AX, Y) + g(\phi_1 W_1, Y)g(AZ, X) + g(\phi_1 W_1, X)g(AZ, Y) = 0.$$

From this, putting $Z = \phi_1 W_1$, then for any X, Y in $\mathcal{D}$

$$\|W_1\|^2 g(AX, Y) + g(\phi_1 W_1, Y)g(A\phi_1 W_1, X) + g(\phi_1 W_1, X)g(A\phi_1 W_1, Y) = 0, \tag{3.7}$$

so that, putting $Y = \phi_1 W_1$ gives

$$2\|W_1\|^2 g(AX, \phi_1 W_1) + g(\phi_1 W_1, X)g(A\phi_1 W_1, \phi_1 W_1) = 0. \tag{3.8}$$

From this, putting $X = \phi_1 W_1$, by virtue of $\|W_1\| \neq 0$ we have

$$g(A\phi_1 W_1, \phi_1 W_1) = 0.$$

From this together with (3.8) we have

$$g(AX, \phi_1 W_1) = 0.$$

for any X in $\mathcal{D}$. Thus it can be written

$$A\phi_1 W_1 \in \mathcal{D}^\perp.$$

From this together with (3.7) it follows that for any X, Y in $\mathcal{D}$

$$g(AX, Y) = 0,$$

where we also have used the fact $\|W_1\| \neq 0$ on a neighborhood $\mathcal{U}$. Unless otherwise stated let us continue our discussion on this open set $\mathcal{U}$. Accordingly, by (3.6) we know for any $X \in \mathcal{D}$

$$\begin{aligned} AX &= \Sigma_i g(AX, U_i)U_i \\ &= \Sigma_i g(X, AU_i)U_i \\ &= \Sigma_i g(W_i, X)U_i. \end{aligned} \tag{3.9}$$

On the other hand, from the condition (1.3) let us put

$$\begin{aligned}(\nabla_X A)Y &= -\Sigma_{i=1}^3 g(\phi_i X, Y)U_i \\ &= \lambda_1(X,Y)U_1 + \lambda_2(X,Y)U_2 + \lambda_3(X,Y)U_3.\end{aligned} \tag{3.10}$$

for any X, Y in $\mathcal{D}$. Since we have put $AU_1 = W_1 + \Sigma_j \alpha_{1j} U_j$, from which it follows

$$\begin{aligned}(\nabla_X A)U_1 =& \nabla_X W_1 + \Sigma_j X(\alpha_{1j})U_j \\ &+ \Sigma_j \alpha_{1j}\{-p_k(X)U_i + p_i(X)U_k + \phi_j AX\} \\ &- A\{-p_2(X)U_3 + p_3(X)U_2 + \phi_1 AX\}.\end{aligned}$$

Then for any X, Y in $\mathcal{D}$ the function $\lambda_1(X,Y)$ is given by

$$\begin{aligned}\lambda_1(X,Y) =& g((\nabla_X A)U_1, Y) \\ =& g(\nabla_X W_1, Y) + \Sigma_j \alpha_{1j} g(\phi_j AX, Y) + p_2(X) g(AU_3, Y) \\ &- p_3(X) g(AU_2, Y) - g(A\phi_1 AX, Y).\end{aligned} \tag{3.11}$$

When we put $X = W_1$ and $Y = \phi_1 W_1$ in (3.10), we get

$$\lambda_1(W_1, \phi_1 W_1) = -\|W_1\|^2. \tag{3.12}$$

On the other hand, by the equation of Codazzi (2.7) and using (3.6) and (3.9) we have

$$\begin{aligned}(\nabla_{U_1} A)W_1 - (\nabla_{W_1} A)U_1 = \phi_1 W_1 & \\ =& \nabla_{U_1}(AW_1) - A\nabla_{U_1} W_1 - \nabla_{W_1}(AU_1) + A\nabla_{W_1} U_1 \\ =& \Sigma_i U_1(g(W_i, W_1))U_i + \Sigma_i g(W_i, W_1)\nabla_{U_1} U_i \\ &- A\nabla_{U_1} W_1 - \nabla_{W_1} W_1 - \Sigma_j W_1(\alpha_{1j})U_j \\ &- \Sigma_j \alpha_{1j}\{-p_k(W_1)U_i + p_i(W_1)U_k + \phi_j AW_1\} \\ &+ A\{-p_2(W_1)U_3 + p_3(W_1)U_2 + \phi_1 AW_1\}.\end{aligned}$$

From this, substituting (2.8) and taking the inner product with $\phi_1 W_1$ and using (3.6), we have

$$\begin{aligned}g(\nabla_{W_1} W_1, \phi_1 W_1) =& \|W_1\|^2(\|W_1\|^2 - 1) - g(A\nabla_{U_1} W_1, \phi_1 W_1) - \Sigma_j \alpha_{1j} g(\phi_j AW_1, \phi_1 W_1) \\ &- p_2(W_1) g(AU_3, \phi_1 W_1) + p_3(W_1) g(AU_2, \phi_1 W_1) \\ &+ g(A\phi_1 AW_1, \phi_1 W_1).\end{aligned} \tag{3.13}$$

On the other hand, it can be easily verified that

$$\begin{aligned}g(A\nabla_{U_1} W_1, \phi_1 W_1) =& g(\nabla_{U_1} W_1, A\phi_1 W_1) \\ =& \Sigma_i g(W_i, \phi_1 W_1) g(\nabla_{U_1} W_1, U_i) \\ =& -\Sigma_i g(W_i, \phi_1 W_1) g(W_1, \phi_i AU_1) \\ =& 0,\end{aligned}$$

where we have used (3.9) and (2.8) to the second and the third equality respectively. Moreover, the facts that $AW_1 = \Sigma_i g(W_i, W_1)U_i \in \mathcal{D}^\perp$ and $\phi_1 W_1 \in \mathcal{D}$ imply

$$\Sigma_j \alpha_{1j} g(\phi_j AW_1, \phi_1 W_1) = 0. \tag{3.14}$$

By virtue of these formulae (3.13) can be rewritten as

$$\begin{aligned} g(\nabla_{W_1} W_1, \phi_1 W_1) =& \|W_1\|^2(\|W_1\|^2 - 1) - p_2(W_1) g(AU_3, \phi_1 W_1) \\ &+ p_3(W_1) g(AU_2, \phi_1 W_1) + g(A\phi_1 A W_1, \phi_1 W_1). \end{aligned} \tag{3.15}$$

Now putting $X = W_1$ and $Y = \phi_1 W_1$ in (3.11), from which substituting (3.15) and using (3.14), we have

$$\lambda_1(W_1, \phi_1 W_1) = \|W_1\|^2(\|W_1\|^2 - 1).$$

From this and (3.12) we know $\|W_1\| = 0$, which makes a contradiction on $\mathcal{U}$. Using the same method for the cases where W_2 or W_3 are non vanishing, we can also prove $W_2 = 0$ or $W_3 = 0$ respectively. This makes a contradiction. From this we know that there does not exist such a neighborhood $\mathcal{U}$ on M. Thus we can conclude $g(A\mathcal{D}, \mathcal{D}^\perp) = 0$. Then from [1] M is congruent to an open part of either a tube of radius r, $0 < r < \frac{\pi}{2}$ over the canonically (totally geodesic) embedded quaternionic projective space QP^k, $k \in \{0, 1, ..., m-1\}$ or a tube of radius r, $0 < r < \frac{\pi}{4}$, over the canonically (totally geodesic) embedded complex projective space CP^m.

Let us consider the second kind of tubes. The principal curvatures on $\mathcal{D}^\perp$ and $\mathcal{D}$ of such a tube are given by $\alpha_1 = 2cot2r$, $\alpha_2 = \alpha_3 = -2tan2r$, $\lambda = cotr$ and $\mu = -tanr$, with multiplicities 1, 2, 2$(m-1)$ and 2$(m-1)$ respectively ([1],[4]). Moreover, it is also known that

$$A\phi_i X = \frac{\lambda\alpha_i + 2}{2\lambda - \alpha_i}\phi_i X, \quad i = 1, 2, 3$$

for a principal vector X in $\mathcal{D}$ with principal curvature λ. When we consider for the cases where $\alpha_2 = \alpha_3 = -2tan2r$, we have

$$(A\phi_i - \phi_i A)X = -(cotr + tanr)\phi_i X, \quad i = 2, 3$$

for any X in $\mathcal{D}$ with principal curvature $cotr$. Then from (1.4) we have $-tanr - cotr = 0$. This implies that $cot^2 r = -1$, which is impossible. Thus the second kind of tubes can not satisfy (1.4). This completes the proof of the Theorem. □

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