A COUNTEREXAMPLE TO THE ARTICLE "ON THE FIXED POINTS OF AFFINE NONEXPANSIVE MAPPINGS"

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We give a counterexample to the article "On the fixed points of affine nonexpansive mappings" (2001).

Let K be a nonempty, closed convex subset of a real Banach space E. A mapping $T : K \to K$ is said to be *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in K$. T is said to be *affine* if for each $x, y \in K$ and $0 < \lambda < 1$, $T(\lambda x + (1-\lambda)y) = \lambda Tx + (1-\lambda)Ty$.

In the main theorem of the above-referenced paper [2, Theorem 2.4], the author proves that when K is a nonempty, closed convex and bounded subset of E and $T : K \to K$ is a nonexpansive and affine mapping, then it has a fixed point in K.

Here, we give an example to show that the mentioned theorem above *is not correct*.

1. Counterexample

We consider c_0, the real Banach space of all sequences $(x_1, x_2, \ldots, x_n, \ldots)$ such that $\lim_{n\to\infty} x_n = 0$, equipped by the maximum norm (i.e., $\|(x_1, x_2, \ldots, x_n, \ldots)\| := \max_n |x_n|$). Define $T : B_1 \to B_1$ by $T(x_1, x_2, \ldots) := (1, x_1, x_2, \ldots)$ for each $x = (x_1, x_2, \ldots)$ in B_1, where B_1 is the closed unit ball in c_0. It is easy to show that $\|Tx - Ty\| = \|x - y\|$, for every x, y in B_1 and also that T is affine. Therefore, the conditions of the main theorem of [2] hold. However, T does not have a fixed point.

It is worth mentioning that if we impose weak compactness on K, then the theorem will be true. For details and some other related results, it is convenient to see [1, 3] and most importantly [4].

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