EXISTENCE AND STABILITY OF PERIODIC SOLUTIONS FOR A CLASS OF GENERALIZED NONAUTONOMOUS NEURAL NETWORKS WITH DISTRIBUTED DELAYS

YONGKUN LI, LIFEI ZHU, AND WENXIANG LIU

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By using the continuation theorem of coincidence degree theory and Lyapunov functions, we study the existence and global stability of periodic solutions for a class of generalized nonautonomous neural networks with distributed delays.

1. Introduction

The study of the dynamics of neural networks has greatly attracted the attention of the scientific community because of their promising potential for the tasks of classification, associative memory, and parallel computations, and their ability to solve difficult optimization problems. Many papers [1, 4, 5, 8, 9, 10, 12, 13, 14, 15] have been devoted to discussing the stability of neural networks with delays. Recently, the authors of [7] have studied the globally exponential stability of the trivial solution for the following generalized neural networks with distributed delays

$$\begin{aligned}\dot{x}_i(t) = &- d_i(x_i(t)) + \sum_{j=1}^{n} \omega_{ij}(x_1(t),\dots,x_n(t)) f_j(x_j(t)) + \sum_{j=1}^{n} \omega_{ij}^{\tau}(x_1(t),\dots,x_n(t)) \\ &\times \int_{-\infty}^{t} k_{ij}(t-s) f_j(x_j(s))\,ds, \quad i = 1,2,\dots,n,\end{aligned} \tag{1.1}$$

where x_i is the state of the i-neuron at time t, $A = (\omega_{ij})$ and $B = (\omega_{ij}^{\tau})$ are $n \times n$ interconnection matrices, respectively, f_j is an activation function. However, under some practical circumstances, the connection weights, the activation functions, and the rate functions of most neural network models (i.e., ω_{ij}, ω_{ij}^{τ}, f_j, and d_i in system (1.1)) depend not only on the state $x_i(t)$ but also on the time t, so the nonautonomous system can be applied in wider fields. In this paper, we are concerned with the following nonautonomous neural network system

$$\begin{aligned}\dot{x}_i(t) = &- d_i(t,x_i(t)) + \sum_{j=1}^{n} \omega_{ij}(t,x_1(t),\dots,x_n(t)) f_j(t,x_j(t)) + \sum_{j=1}^{n} \omega_{ij}^{\tau}(t,x_1(t),\dots,x_n(t)) \\ &\times \int_{-\infty}^{t} k_{ij}(t-s) f_j(t,x_j(s))\,ds, \quad i = 1,2,\dots,n.\end{aligned} \tag{1.2}$$

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It is well known that studies on neural network dynamical systems not only involve a discussion of stability properties, but also involve many dynamic behaviors such as periodic oscillatory behavior, almost-periodic oscillatory properties, chaos, and bifurcation [11], and to the best of our knowledge, few authors considered the existence of periodic solutions for the model (1.2). Our purpose of this paper is to prove the existence and stability of periodic solutions of (1.2).

Throughout this paper, we assume that

(H1) for each $i = 1,2,\dots,n$, $d_i \in C(R^2,R)$ is T-periodic with respect to its first argument, and $\lim_{u\to+\infty} d_i(t,u) = +\infty$ and $\lim_{u\to-\infty} d_i(t,u) = -\infty$ are uniformly in t, respectively;

(H2) for each $i,j = 1,2,\dots,n$, k_{ij} is real-valued nonnegative continuous function defined on $[0,\infty)$ and $\int_0^\infty k_{ij}(s)ds = 1$;

(H3) for each $i,j = 1,2,\dots,n$, $f_i \in C(R^2,R)$, $\omega_{ij}^\tau, \omega_{ij} \in C(R^{n+1},R)$ are bounded and f_i, ω_{ij}^τ, and ω_{ij} are T-periodic with respect to their first arguments, respectively.

The organization of this paper is as follows. In the second section, we prove the existence of periodic solutions of system (1.2) by applying the continuation theorem of coincidence degree theory. In the third section, some sufficient conditions are obtained to show the global asymptotic stability of periodic solutions of system (1.2).

2. Existence of positive periodic solutions

In this section, based on the Mawhin's continuation theorem, we will study the existence of at least one positive periodic solution of (1.2). First, we will make some preparations.

Let X,Y be normed vector spaces, $L : \operatorname{Dom} L \subset X \to Y$ a linear mapping, and $N : X \to Y$ a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\operatorname{Ker} L = \operatorname{codim}\operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$, it follows that mapping $L|_{\operatorname{Dom} L\cap\operatorname{Ker} P} : (I - P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that mapping by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Now, we introduce Mawhin's continuation theorem [2, page 40] as follows.

Lemma 2.1. *Let $\Omega \subset X$ be an open bounded set and let $N : X \to Y$ be a continuous operator which is L-compact on $\overline{\Omega}$. Assume*

(a) *for each $\lambda \in (0,1)$, $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda Nx$;*

(b) *for each $x \in \partial\Omega \cap \operatorname{Ker} L$, $QNx \neq 0$;*

(c) $\deg(JNQ,\Omega \cap \operatorname{Ker} L,0) \neq 0$.

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

Theorem 2.2. *Assume that (H1)–(H3) hold. Then the system (1.2) has at least one T-periodic solution.*

Proof. In order to apply the continuation theorem of coincidence degree theory to establish the existence of a T-periodic solution of (1.2), we take

$$X = Y = \{x \in C(R,R^n) : x(t+T) = x(t),\ t \in R\} \tag{2.1}$$

and denote

$$\|x\| = \sup_{t\in[0,T]} \sum_{i=1}^{n} |x_i(t)|, \tag{2.2}$$

then X is a Banach space. Set

$$L : \operatorname{Dom} L \cap X, \quad Lx = \dot{x}(t), \quad x \in X, \tag{2.3}$$

where $\operatorname{Dom} L = \{x \in C^1(R,R^n)\}$ and $N : X \to X$,

$$\begin{aligned}(Nx_i)(t) = &-d_i(t,x_i(t)) + \sum_{j=1}^{n} \omega_{ij}(t,x_1(t),\dots,x_n(t)) f_j(t,x_j(t)) + \sum_{j=1}^{n} \omega_{ij}^{\tau}(t,x_1(t),\dots,x_n(t)) \\ &\times \int_{-\infty}^{t} k_{ij}(t-s) f_j(t,x_j(s))\,ds, \quad i = 1,2,\dots,n.\end{aligned} \tag{2.4}$$

Define two projectors P and Q as

$$Qx = Px = \frac{1}{T}\int_0^T x(s)\,ds, \quad x \in X. \tag{2.5}$$

Clearly, $\operatorname{Ker} L = R^n$, $\operatorname{Im} L = \{(x_1,\dots,x_n)^T \in X : \int_0^T x_i(t)dt = 0,\ i = 1,2,\dots,n\}$ is closed in X and $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L = n$. Hence, L is a Fredholm mapping of index 0. Furthermore, similar to the proof of [6, Theorem 1], one can easily show that N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$.

Corresponding to operator equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, we have

$$\begin{aligned}\frac{dx_i}{dt} = &-\lambda d_i(t,x_i(t)) + \lambda \sum_{j=1}^{n} \omega_{ij}(t,x_1(t),\dots,x_n(t)) f_j(t,x_j(t)) + \lambda \sum_{j=1}^{n} \omega_{ij}^{\tau}(t,x_1(t),\dots,x_n(t)) \\ &\times \int_{-\infty}^{t} k_{ij}(t-s) f_j(t,x_j(s))\,ds, \quad i = 1,2,\dots,n.\end{aligned} \tag{2.6}$$

Suppose that $x = (x_1,\dots,x_n) \in X$ is a solution of (2.6) for some $\lambda \in (0,1)$. Let $\xi_i \in [0,T]$ such that $x_i(\xi_i) = \max_{t\in[0,\omega]} x_i(t)$, $i = 1,2,\dots,n$, then

$$\begin{aligned}&-\lambda d_i(\xi_i,x_i(\xi_i)) + \lambda \sum_{j=1}^{n} \omega_{ij}(\xi_i,x_1(\xi_i),\dots,x_n(\xi_i)) f_j(\xi_i,x_j(\xi_i)) \\ &\quad + \lambda \sum_{j=1}^{n} \omega_{ij}^{\tau}(\xi_i,x_1(\xi_i),\dots,x_n(\xi_i)) \int_{-\infty}^{\xi_i} k_{ij}(\xi_i-s) f_j(\xi_i,x_j(s))\,ds = 0, \quad i = 1,2,\dots,n.\end{aligned} \tag{2.7}$$

In view of (H2) and (H3), we have

$$\begin{aligned}
d_i(\xi_i,x_i(\xi_i)) \le& \sum_{j=1}^{n} |\omega_{ij}(\xi_i,x_1(\xi_i),\dots,x_n(\xi_i))|\,|f_j(\xi_i,x_j(\xi_i))| \\
&+\sum_{j=1}^{n} |\omega_{ij}^{\tau}(\xi_i,x_1(\xi_i),\dots,x_n(\xi_i))| \\
&\times\left|\int_{-\infty}^{\xi_i} k_{ij}(\xi_i-s)f_j(\xi_i,x_j(s))\,ds\right| \\
\le& n\overline{\omega}\overline{f}+n\overline{\omega^{\tau}}\overline{f},\quad i=1,2,\dots,n,
\end{aligned}\tag{2.8}$$

where $\overline{\omega}=\max\{|\omega_{ij}(t,v_1,\dots,v_n)|,(t,v_1,\dots,v_n)^T\in R^{n+1}, i,j=1,2,\dots,n\}$, $\overline{\omega^{\tau}}=\max\{|\omega_{ij}^{\tau}(t,v_1,\dots,v_n)|,(t,v_1,\dots,v_n)^T\in R^{n+1}, i,j=1,2,\dots,n\}$, $\overline{f}=\max\{|f_j(t,\mu_j)|, (t,\mu_j)^T\in R^2, j=1,2,\dots,n\}$. According to (H1) and (2.8), we know that there exists a constant $A_1>0$ such that

$$x_i(\xi_i)\le A_1,\quad i=1,2,\dots,n.\tag{2.9}$$

Similarly, let $\eta_i\in[0,\omega]$ such that $x_i(\eta_i)=\min_{t\in[0,\omega]}x_i(t)$, $i=1,2,\dots,n$, then

$$\begin{aligned}
&-\lambda d_i(\eta_i,x_i(\eta_i))+\lambda\sum_{j=1}^{n}\omega_{ij}(\eta_i,x_1(\eta_i),\dots,x_n(\eta_i))f_j(\eta_i,x_j(\eta_i)) \\
&\quad+\lambda\sum_{j=1}^{n}\omega_{ij}^{\tau}(\eta_i,x_1(\eta_i),\dots,x_n(\eta_i))\int_{-\infty}^{\eta_i}k_{ij}(\eta_i-s)f_j(\eta_i,x_j(s))\,ds=0,\quad i=1,2,\dots,n.
\end{aligned}\tag{2.10}$$

Then,

$$\begin{aligned}
d_i(\eta_i,x_i(\eta_i)) \ge& -\sum_{j=1}^{n} |\omega_{ij}(\eta_i,x_1(\eta_i),\dots,x_n(\eta_i))|\,|f_j(\eta_i,x_j(\eta_i))| \\
&-\sum_{j=1}^{n} |\omega_{ij}^{\tau}(\eta_i,x_1(\eta_i),\dots,x_n(\eta_i))| \\
&\times\left|\int_{-\infty}^{\eta_i} k_{ij}(\eta_i-s)f_j(\eta_i,x_j(s))\,ds\right| \\
\ge& -n\overline{\omega}\overline{f}-n\overline{\omega^{\tau}}\overline{f},\quad i=1,2,\dots,n,
\end{aligned}\tag{2.11}$$

where $\overline{\omega},\overline{f},\overline{\omega^{\tau}}$ is the same as those in (2.8). Therefore, there exists a constant $A_2>0$ such that

$$x_i(\eta_i)\ge -A_2,\quad i=1,2,\dots,n.\tag{2.12}$$

Denote $D = \max\{nA_1, nA_2\} + E$, where E is a positive constant, clearly, D is independent of λ. Now, we take $\Omega = \{x \in X, \|x\| < D\}$. This Ω satisfies condition (a) in Lemma 2.1.

When $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap R^n$, x is a constant vector in R^n with $\|x\| = D$. Then

$$\begin{aligned} x^T QNx = &-\frac{1}{T}\sum_{i=1}^{n} x_i \int_0^T \Big[d_i(t,x_i) - \sum_{j=1}^{n}\omega_{ij}(t,x_1,\dots,x_n)f_j(t,x_j) \\ &\qquad - \sum_{j=1}^{n}\omega_{ij}^{\tau}(t,x_1,\dots,x_n)\int_{-\infty}^{t}k_{ij}(t-s)f_j(t,x_j)ds\Big]dt \\ \le &-\frac{1}{T}\sum_{i=1}^{n} x_i \int_0^T [d_i(t,x_i) - n\overline{\omega}\overline{f} - n\overline{\omega^{\tau}}\overline{f}]dt \\ < &\ 0, \quad i = 1,2,\dots,n. \end{aligned} \tag{2.13}$$

If necessary, we can let E be greater such that $-(1/T)\sum_{i=1}^{n} x_i \int_0^T [d_i(t,x_i) - n\overline{\omega}\overline{f} - n\overline{\omega^{\tau}}\overline{f}]dt < 0$. This prove that condition (b) in Lemma 2.1 is satisfied.

Finally, we will prove that condition (c) in Lemma 2.1 is also satisfied. Let $\psi(\nu;x) = -\nu x + (1-\nu)QNx$, then for any $x \in \partial\Omega \cap \operatorname{Ker} L$, $x^T\psi(\nu,x) < 0$, we get

$$\deg(JQM, \Omega \cap \operatorname{Ker} L, 0) \ne 0. \tag{2.14}$$

Thus, by Lemma 2.1, we conclude that $Lx = Nx$ has at least one solution in X, that is, (1.2) has at least one positive T-periodic solution. The proof is complete. □

3. Global asymptotic stability of periodic solutions

Let $x(t) = (x_1(t),\dots,x_n(t))$ be any solution of (1.2) and $x^*(t) = (x_1^*(t),\dots,x_n^*(t))$ a T-periodic solution of (1.2). Set $u(t) = x(t) - x^*(t)$, then

$$\frac{du_i(t)}{dt} = -\alpha_i(u_i(t)) + \beta_i(u_i(t)) + \gamma_i(u_i(t)), \quad i = 1,2,\dots,n, \tag{3.1}$$

where

$$\begin{aligned} \alpha_i(u_i(t)) =& d_i(t,x_i(t)) - d_i(t,x_i^*(t)), \\ \beta_i(u_i(t)) =& \sum_{j=1}^{n}\omega_{ij}(t,x_1(t),\dots,x_n(t))f_j(t,x_j(t)) - \sum_{j=1}^{n}\omega_{ij}(t,x_1^*(t),\dots,x_n^*(t))f_j(t,x_j^*(t)), \\ \gamma_i(u_i(t)) =& \sum_{j=1}^{n}\omega_{ij}^{\tau}(t,x_1(t),\dots,x_n(t))\int_{-\infty}^{t}k_{ij}(t-s)f_j(t,x_j(s))ds \\ &- \sum_{j=1}^{n}\omega_{ij}^{\tau}(t,x_1^*(t),\dots,x_n^*(t))\int_{-\infty}^{t}k_{ij}(t-s)f_j(t,x_j^*(s))ds. \end{aligned} \tag{3.2}$$

In the sequel, we will use the following notations:

$$\overline{\omega}_{ij} = \max\{\omega_{ij}(t,\nu_1,\dots,\nu_n),\ (t,\nu_1,\dots,\nu_n)^T \in R^{n+1}\}, \qquad \overline{f}_j = \max\{f_j(t,\mu),\ (t,\mu)^T \in R^2\}. \tag{3.3}$$

Theorem 3.1. *Assume that (H1)–(H3) hold. Furthermore, assume that*

(H4) *for each $i = 1,2,\dots,n$, $f_i : R^2 \to R$ is globally Lipschitz continuous with a Lipschitz constant F_i with respect to its second argument,*

(H5) *for each $i = 1,2,\dots,n$, $d_i \in C^1(R^2,R)$ and there exists a constant $D_i \geq 0$ such that*

$$\left[d_i(t,u)\right]_u' \geq D_i, \quad u \in R, \tag{3.4}$$

(H6) *for each $i,j = 1,2,\dots,n$, there exist constants ${}^lB_{ij} \geq 0$ and ${}^lB_{ij}^\tau \geq 0$ such that*

$$\begin{aligned} \left|\omega_{ij}(t,x_1(t),\dots,x_n(t)) - \omega_{ij}(t,y_1(t),\dots,y_n(t))\right| &\leq \sum_{l=1}^n {}^lB_{ij}\left|x_l(t) - y_l(t)\right|, \quad t \in R, \\ \left|\omega_{ij}^\tau(t,x_1(t),\dots,x_n(t)) - \omega_{ij}^\tau(t,y_1(t),\dots,y_n(t))\right| &\leq \sum_{l=1}^n {}^lB_{ij}^\tau\left|x_l(t) - y_l(t)\right|, \quad t \in R, \end{aligned} \tag{3.5}$$

(H7) *for each $i = 1,2,\dots,n$,*

$$M_i := D_i - F_i\sum_{j=1}^n\left(\overline{\omega}_{ji} + \overline{\omega}_{ji}^\tau\right) - \sum_{j=1}^n\sum_{l=1}^n\left({}^iB_{lj} + {}^iB_{lj}^\tau\right)\overline{f}_j > 0, \tag{3.6}$$

then (1.2) has a unique T-periodic solution which is globally asymptotically stable.

Proof. We consider the Lyapunov function

$$V(t) = V_1(t) + V_2(t), \tag{3.7}$$

where

$$\begin{aligned} V_1(t) &= \sum_{i=1}^n\left|u_i(t)\right|, \\ V_2(t) &= \sum_{i=1}^n\sum_{j=1}^n\overline{\omega}_{ij}^\tau F_j\int_0^\infty K_{ij}(s)\int_{t-s}^t\left|u_j(z)\right|dz\,ds. \end{aligned} \tag{3.8}$$

Calculating the derivatives of V_1 and V_2 along the solution of (3.1), respectively,

$$
\begin{aligned}
\frac{du}{dt}\bigg|_{(3.1)} &= \sum_{i=1}^{n} \operatorname{sign}(u_i(t))\dot{u}_i(t) \\
&= \sum_{i=1}^{n} \operatorname{sign}(u_i(t))\{-\alpha_i(u_i(t)) + \beta_i(u_i(t)) + \gamma_i(u_i(t))\} \\
&\le \sum_{i=1}^{n} \{-D_i|u_i(t)| + |\beta_i(u_i(t))| + |\gamma_i(u_i(t))|\} \\
&\le \sum_{i=1}^{n} \bigg\{-D_i|u_i(t)| + \sum_{j=1}^{n} |\omega_{ij}(t,x_1(t),\dots,x_n(t))|\,|f_j(t,x_j(t)) - f_j(t,x_j^*(t))| \\
&\qquad + \sum_{j=1}^{n} |\omega_{ij}(t,x_1(t),\dots,x_n(t)) - \omega_{ij}(t,x_1^*(t),\dots,x_n^*(t))|\,|f_j(t,x_j^*(t))| \\
&\qquad + \sum_{j=1}^{n} |\omega_{ij}^{\tau}(t,x_1(t),\dots,x_n(t))| \int_{-\infty}^{t} k_{ij}(t-s) \\
&\qquad\qquad \times |f_j(t,x_j(s)) - f_j(t,x_j^*(s))|\,ds \\
&\qquad + \sum_{j=1}^{n} |\omega_{ij}^{\tau}(t,x_1(t),\dots,x_n(t)) \\
&\qquad\qquad - \omega_{ij}^{\tau}(t,x_1^*(t),\dots,x_n^*(t))| \left|\int_{-\infty}^{t} k_{ij}(t-s) f_j(t,x_j^*(s))\,ds\right|\bigg\}, \\
&\le \sum_{i=1}^{n} \bigg\{-D_i|u_i(t)| + \sum_{j=1}^{n} \overline{\omega}_{ij}F_j|u_j(t)| + \sum_{j=1}^{n}\sum_{l=1}^{n} {}^{l}B_{ij}|u_l(t)|\overline{f}_j \\
&\qquad + \sum_{j=1}^{n} \overline{\omega}_{ij}^{\tau} \int_{-\infty}^{t} k_{ij}(t-s)F_j|u_j(s)|\,ds + \sum_{j=1}^{n}\sum_{l=1}^{n} {}^{l}B_{ij}^{\tau}|u_l(t)|\overline{f}_j\bigg\} \\
&= \sum_{i=1}^{n} \bigg\{-D_i|u_i(t)| + \sum_{j=1}^{n} \overline{\omega}_{ij}F_j|u_j(t)| + \sum_{j=1}^{n}\sum_{l=1}^{n} {}^{l}B_{ij}|u_l(t)|\overline{f}_j \\
&\qquad + \sum_{j=1}^{n} \overline{\omega}_{ij}^{\tau}F_j \int_{0}^{\infty} k_{ij}(s)|u_j(t-s)|\,ds + \sum_{j=1}^{n}\sum_{l=1}^{n} {}^{l}B_{ij}^{\tau}|u_l(t)|\overline{f}_j\bigg\},
\end{aligned}
\tag{3.9}
$$

$$
\frac{dV_2}{dt}\bigg|_{(3.1)} = \sum_{i=1}^{n}\left(\sum_{j=1}^{n} \overline{\omega}_{ij}^{\tau}F_j\int_{0}^{\infty} k_{ij}(s)|u_j(t)|\,ds - \sum_{j=1}^{n} \overline{\omega}_{ij}^{\tau}F_j\int_{0}^{\infty} k_{ij}(s)|u_j(t-s)|\,ds\right).
\tag{3.10}
$$

So,

$$
\begin{aligned}
\frac{dV(t)}{dt}\bigg|_{(3.1)} \le \sum_{i=1}^{n} \bigg\{&-D_i|u_i(t)| + \sum_{j=1}^{n} \overline{\omega}_{ij}F_j|u_j(t)| + \sum_{j=1}^{n}\sum_{l=1}^{n} ({}^{l}B_{ij} + {}^{l}B_{ij}^{\tau})|u_l(t)|\overline{f}_j \\
&+ \sum_{j=1}^{n} \overline{\omega}_{ij}^{\tau}F_j|u_j(t)|\bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{n}\Bigg\{-D_i\,|\,u_i(t)\,|+\sum_{j=1}^{n}(\overline{\omega}_{ij}+\overline{\omega}_{ij}^{\tau})F_j\,|\,u_j(t)\,| \\
&\qquad + \sum_{j=1}^{n}\sum_{l=1}^{n}({}^{l}B_{ij}+{}^{l}B_{ij}^{\tau})\,|\,u_l(t)\,|\,\overline{f}_j\Bigg\} \\
&\le \sum_{i=1}^{n}\Bigg\{-D_i+F_i\sum_{j=1}^{n}(\overline{\omega}_{ji}+\overline{\omega}_{ji}^{\tau})+\sum_{j=1}^{n}\sum_{l=1}^{n}({}^{i}B_{lj}+{}^{i}B_{lj}^{\tau})\overline{f}_j\Bigg\}\,|\,u_i(t)\,| \\
&\le 0.
\end{aligned}
\tag{3.11}
$$

In view of (3.7) and (3.10), we see that $\sum_{i=1}^{n}|u_i(t)|$ is bounded for all $t \ge 0$. For the proof of Theorem 2.2, it follows that for each $i = 1,2,\dots,n$, $x_i^*(t)$ is bounded. Hence the solutions of (1.2) exist and are bounded for all $t \ge 0$. Integrating both sides of (3.11) from 0 to t, we get

$$
V(t)+\int_0^t\sum_{i=1}^{n}M_i\,|\,u_i(s)\,|\,ds \le V(0),
\tag{3.12}
$$

which implies $u_i(t) \in L^1[0,\infty)$. Therefore, by Barbalatt's lemma [3, Lemma 1.2.2, page 4], we have

$$
\lim_{t\to\infty}u_i(t)=0.
\tag{3.13}
$$

This completes the proof. □

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Yongkun Li: Department of Mathematics, Yunnan University, Kunming, Yunnan 650091, China
E-mail address: yklie@ynu.edu.cn

Lifei Zhu: Department of Mathematics and Statistics, McGill University, Montreal, QC, Canada H3A 2K6
E-mail address: zhu@math.mcgill.ca

Wenxiang Liu: Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, AB, Canada T6G 2G1
E-mail address: wliu@math.ualberta.ca