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AXISYMMETRIC LAMB'S PROBLEM IN A SEMI-INFINITE MICROPOLAR VISCOELASTIC MEDIUM

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ABSTRACT. A study is made of the axisymmetric problem of wave propagation under the influence of gravity in a micropolar viscoelastic semi-infinite medium when a time varying axisymmetric loading is applied on the surface of the medium Special attention is given to the effects of gravity which induces a kind of initial stress of a hydrostatic nature on the wave propagation

KEY WORDS AND PHRASES: Axisymmetric Lamb's Problem, micropolar viscoelastic medium, and wave propagation
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1. INTRODUCTION

In classical problems of wave propagation in an elastic medium studied by several authors including Love [1] and De and Sengupta [2], it has been shown that the velocity of Rayleigh waves increases by a significant amount when the wave-length is large due to the influence of gravity Biot [3] investigated the influence of gravity on Rayleigh waves under the assumption that the force of gravity generates an initial stress of a hydrostatic nature so that the medium remains incompressible Nowacki and Nowacki [4] discussed the axisymmetric Lamb's problem in a semi-infinite micropolar elastic solid However, they did not include the effects of gravity in a micropolar viscoelastic solid medium The main purpose of this paper is to consider the axisymmetric Lamb's problem in a semi-infinite micropolar viscoelastic medium under the influence of gravity due to a harmonically oscillating loading acting on the surface of the medium Special attention is given to the effects of gravity which generates an initial stress hydrostatic in nature, on the wave propagation

2. FORMULATION OF THE PROBLEM

We consider a viscoelastic homogeneous isotropic centrosymmetric body and assume that the initial stress due to gravity is hydrostatic in nature Since the initial stress is hydrostatic, stress strain relations in this case will remain the same as in a medium initially stress free The stress and strain relations in the micropolar viscoelastic medium are

$$\sigma_{ji} = \left\{(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\right\}\nu_{ji} + \left\{(\mu_0 - \alpha_0) + (\mu_1 - \alpha_1)\frac{\partial}{\partial t}\right\}\nu_{ij} + \left(\lambda_0 + \lambda_1 \frac{\partial}{\partial t}\right)\nu_{kk}\delta_{ij} \quad (2\ 1)$$

$$\mu_{ji} = \left\{(\nu_0 + \epsilon_0) + (\nu_1 + \epsilon_1)\frac{\partial}{\partial t}\right\}\chi_{ji} + \left\{(\nu_0 - \epsilon_0) + (\nu_1 - \epsilon_1)\frac{\partial}{\partial t}\right\}\chi_{ij} + \left(\beta_0 + \beta_1 \frac{\partial}{\partial t}\right)\chi_{kk}\delta_{ij} \quad (2\ 2)$$

$$\nu_{ji} = u_{i,j} - e_{kji}\omega_k, \qquad \chi_{ji} = \omega_{i,j} \quad (2\ 3ab)$$

where λ_0, μ_0, α_0, β_0, ν_0, ϵ_0 are elastic parameters, λ_1, μ_1, α_1, β_1, ν_1, ϵ_1 are the parameters associated with viscosity

We use the cylindrical polar coordinates (r, θ, z) Without body couples, external loading distributions, body forces, the displacement vector **u**, rotation vector ω depend only on r, z and t because of the axisymmetric configuration The equations of motion in a micropolar viscoelastic solid medium under the influence of gravity are given by

$$\left[(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\right]\left(\nabla^2 u_r - \frac{u_r}{r^2}\right) + \left[(\lambda_0 + \mu_0 - \alpha_0) + (\lambda_1 + \mu_1 - \alpha_1)\frac{\partial}{\partial t}\right]\frac{\partial e}{\partial r}$$
$$- 2\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)\frac{\partial \omega_\theta}{\partial z} + \rho g \frac{\partial u_z}{\partial r} = \rho \ddot{u}_r \quad (2\ 4)$$

$$\left[(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\right]\nabla^2 u_z + \left[(\lambda_0 + \mu_0 - \alpha_0) + (\lambda_1 + \mu_1 - \alpha_1)\frac{\partial}{\partial t}\right]\frac{\partial e}{\partial z}$$
$$+ \frac{2\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)}{r}\frac{\partial}{\partial r}(r\omega_\theta) - \rho g \frac{1}{r}\frac{\partial}{\partial r}(r u_r) = \rho \ddot{u}_z \quad (2\ 5)$$

$$\left[(\nu_0 + \epsilon_0) + (\nu_1 + \epsilon_1)\frac{\partial}{\partial t}\right]\left(\nabla^2 \omega_\theta - \frac{\omega_\theta}{r^2}\right) - 4\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)\omega_\theta$$
$$+ 2\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right) = J\ddot{\omega}_\theta \quad (2\ 6)$$

where

$$e = \frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{\partial u_z}{\partial z}, \qquad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}.$$

On the free surface $z = 0$, the axially symmetrical and time varying loadings normal and tangential to the boundary surface and moment with a vector tangent to a circle of radius r are applied The displacement components u_r, u_z and rotation component ω_θ are independent of θ

We introduce a scalar potential ϕ and a vector potential ψ and express the displacement components u_r, u_z in terms of these potentials

$$u_r = \frac{\partial \phi}{\partial r} + \frac{\partial^2 \psi}{\partial z \partial r}, \quad u_z = \frac{\partial \phi}{\partial z} - \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\psi. \quad (2\ 7ab)$$

Introducing $\omega_\theta = -\frac{\partial \chi}{\partial r}$ and putting (2 7ab) into (2.4)-(2 6), we obtain the following set of wave equations

$$\left[\nabla^2 - \frac{1}{\left(c_1^2 + c_1'^2 \frac{\partial}{\partial t}\right)} \cdot \frac{\partial^2}{\partial t^2}\right]\phi - \frac{g}{c_1^2 + c_1'^2 \frac{\partial}{\partial t}}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right]\psi = 0, \quad (2\ 8)$$

$$\left(\nabla^2 - \frac{1}{c_2^2 + c_2'^2 \frac{\partial}{\partial t}} \cdot \frac{\partial^2}{\partial t^2}\right)\psi + \frac{2\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)}{(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}}\chi + \frac{g}{\left(c_2^2 + c_2'^2 \frac{\partial}{\partial t}\right)}\phi = 0, \quad (2\ 9)$$

and

$$\left(\nabla^2 - \frac{4\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)}{(\nu_0 + \epsilon_0) + (\nu_1 + \epsilon_1)\frac{\partial}{\partial t}} - \frac{1}{c_4^2 + c_4'^2 \frac{\partial}{\partial t}} \cdot \frac{\partial^2}{\partial t^2}\right)\chi - \frac{2\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)}{(\nu_0 + \epsilon_0) + (\nu_1 + \epsilon_1)\frac{\partial}{\partial t}} \cdot \nabla^2\psi = 0, \quad (2\ 10)$$

where

$$c_1^2 = \frac{2\mu_0 + \lambda_0}{\rho}, \qquad c_1'^2 = \frac{2\mu_1 + \lambda_1}{\rho}, \qquad c_2^2 = \frac{\mu_0 + \lambda_0}{\rho}$$

$$c_2'^2 = \frac{\mu_1 + \lambda_1}{\rho}, \qquad c_4^2 = \frac{\nu_0 + \epsilon_0}{\rho}, \qquad c_4'^2 = \frac{\nu_1 + \epsilon_1}{\rho}.$$

From equations (2 8)-(2 10) we obtain

$$\Bigg[\left(\nabla^2 - \frac{4\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)}{(\nu_0 + \epsilon_0) + (\nu_1 + \epsilon_1)\frac{\partial}{\partial t}} - \frac{1}{c_4^2 + c_4'^2 \frac{\partial}{\partial t}} \cdot \frac{\partial^2}{\partial t^2}\right)$$
$$\left\{\left(\nabla^2 - \frac{1}{c_1^2 + c_1'^2 \frac{\partial}{\partial t}} \cdot \frac{\partial^2}{\partial t^2}\right) \cdot \left(\nabla^2 - \frac{1}{c_2^2 + c_2'^2 \frac{\partial}{\partial t}} \cdot \frac{\partial^2}{\partial t^2}\right) + \frac{g^2}{\left(c_1^2 + c_1'^2 \frac{\partial}{\partial t}\right)\left(c_2^2 + c_2'^2 \frac{\partial}{\partial t}\right)} \cdot \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\right\}$$
$$+ \frac{2\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)}{(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}} \cdot \frac{2\left(\alpha_0 + \alpha_1 \frac{\partial}{\partial t}\right)}{(\nu_0 + \epsilon_0) + (\nu_1 + \epsilon_1)\frac{\partial}{\partial t}}$$
$$\cdot \left(\nabla^2 - \frac{1}{c_1^2 + c_1'^2 \frac{\partial}{\partial t}} \cdot \frac{\partial^2}{\partial t^2}\right)\nabla^2\Bigg](\phi, \psi, \chi) = 0. \quad (2\ 11)$$

3. METHOD OF SOLUTION AND BOUNDARY CONDITIONS

We apply the joint Fourier and Hankel transform (Debnath [5]) of zero order

$$\left(\tilde{\bar{\phi}}, \tilde{\bar{\psi}}, \tilde{\bar{\chi}}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ist} dt \int_0^{\infty} (\phi, \psi, \chi) r J_0(kr) dr, \quad (3\ 1)$$

to (2 11) and solve the transformed system subject to boundedness condition at infinity Thus it turns out that

$$\tilde{\bar{\phi}} = \sum_{j=1}^{3} A_j \exp(-\lambda_j z), \quad \tilde{\bar{\psi}} = \sum_{j=1}^{3} Bj \exp(-\chi_j z) \quad (3\ 2ab)$$

and

$$\tilde{\bar{\chi}} = \sum_{j=1}^{3} C_j \exp(-\lambda_j z), \quad (3\ 2c)$$

where $\sum_{j=1}^{3} \lambda_j^2 = \sum_{j=1}^{3} k_j^2 - pq$,

$$(\lambda_1\lambda_2)^2 + (\lambda_2\lambda_3)^2 + (\lambda_3\lambda_1)^2 = (k_1k_2)^2 + (k_2k_3)^2 + (k_3k_1)^2 - kg^3k_3^2\left(c_1^2 - isc_1'^2\right)^{-1} - pq\left(k_1^2 + k^2\right)$$

$$(\lambda_1\lambda_2\lambda_3)^2 = (k_1k_2k_3)^2 - k^2g^2k_3^2\left(c_1^2 - isc_1'^2\right)^{-1}\left(c_2^2 - isc_2'^2\right)^{-1} - pqkk_1^2$$

$$k_1^2 = k^2 - s^2\left(c_1^2 - isc_1'^2\right)^{-1}, \quad k_2^2 = k^2 - s^2\left(c_2^2 - isc_2'^2\right)^{-1},$$

$$k_3^2 = k^2 + \nu_0^2 - s^2\left(c_3^2 - isc_3'^2\right)^{-1},$$

$$p = \frac{2(\alpha_0 - is\alpha_1)}{(\mu_0 + \alpha_0) - is(\mu_1 + \alpha_1)}, \quad q = \frac{2(\alpha_0 - is\alpha_1)}{(\nu_0 + \epsilon_0) - is(\nu_1 + \epsilon_1)}, \quad \nu_0^2 = 2q.$$

The arbitrary constants A_j, B_j, and C_j are connected by the relations

$$B_j = p_j A_j, \quad C_j = q_j A_j$$

where

$$p_j = -\left(c_1^2 - isc_1'^2\right)\left(\lambda_j^2 - k_1^2\right)\left(gk^2\right)^{-1}$$

$$q_j = -\frac{1}{p}\left[(\lambda_j^2 - k_2^2)p_j + g\left(c_2^2 - isc_2'^2\right)^{-1}\right].$$

The quantities A_j involved in the solutions are determined from the boundary conditions

$$\sigma_{zz} = -f_1(r,t), \quad \sigma_{zr} = -f_2(r,t), \quad \mu_{z\theta} = -f_3(r,t) \quad \text{on} \quad z = 0 \tag{3 3}$$

where $f_i(r,t) > 0$ for $i = 1, 2, 3$, and

$$\sigma_{zz} = 2\left(\mu_0 + \mu_1\frac{\partial}{\partial t}\right)\frac{\partial^2\phi}{\partial z^2} + \left(\lambda_0 + \lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\phi - 2\left(\mu_0 + \mu_1\frac{\partial}{\partial t}\right)\frac{\partial}{\partial z}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\psi$$

$$\sigma_{zr} = 2\left(\mu_0 + \mu_1\frac{\partial}{\partial t}\right)\frac{\partial^2\phi}{\partial z\partial r} + \left\{(\mu_0 + \alpha_0) + (\mu_1 + \alpha_1)\frac{\partial}{\partial t}\right\}\frac{\partial^3\psi}{\partial r\partial z^2} - 2\left\{(\mu_0 - \alpha) + (\mu_1 - \alpha_1)\frac{\partial}{\partial t}\right\}\frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\psi + 2\left(\alpha_0 + \alpha_1\frac{\partial}{\partial t}\right)\frac{\partial\chi}{\partial r}$$

and

$$\mu_{z\phi} = -\left\{(\nu_0 + \epsilon_0) + (\nu_1 + \epsilon_1)\frac{\partial}{\partial t}\right\}\frac{\partial^2\chi}{\partial r\partial z}.$$

The quantities A_j found from the boundary conditions (3 3) are as follows

$$A_j = (-1)^j\frac{\triangle_j}{\triangle} \tag{3 4}$$

where

$$\begin{aligned}
\triangle_1 &= f_1(b_2c_3 - b_3c_2) + f_2(c_2a_3 - c_3a_2) + f_3(a_2b_3 - a_3b_2)\\
\triangle_2 &= f_1(b_1c_3 - b_3c_1) + f_2(c_1a_3 - c_3a_1) + f_3(a_1b_3 - a_3b_1)\\
\triangle_3 &= f_1(b_1c_2 - b_2c_1) + f_2(c_1a_2 - c_2a_1) + f_3(a_1b_2 - a_2b_1)\\
\triangle &= a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1)\\
a_j &= 2(\mu_0 - is\mu_1)\lambda_j^2 + (\lambda_0 - is\mu_1)(\lambda_j^2 - k^2) - 2(\mu_0 - is\mu_1)p_j\lambda_jk^2\\
b_j &= 2(\mu_0 - is\mu_1)\lambda_jk - k\left[\{(\mu_0 + \alpha_0) - is(\mu_1 + \alpha_1)\}\lambda_j^2 + \{(\mu_0 - \alpha_0)\right.\\
&\qquad \left. - is(\mu_1 - \alpha_1)k^2\}\right]p_j + 2(\alpha_0 - is\alpha_1)kq_j
\end{aligned}$$

and

$$c_j = [(\nu_0 + \epsilon_0) - is(\nu_1 + \epsilon_1)]\lambda_j q_j k.$$

In view of the inverse Fourier and Hankel transformations combined with relations (2 4)-(2 6) and (2 7ab) we get

$$u_r = -\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ist}ds\int_0^{\infty}\sum_{j=1}^{3}(1-\lambda_j p_j)a_j\exp(-\lambda_j z)k^2 J_1(kr)dk \quad (3\ 5)$$

$$\omega_\theta = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ist}ds\int_0^{\infty}\sum_{j=1}^{3}\{q_j A_j\exp(-\lambda_j z)\}k^2 J_1(kr)dk \quad (3\ 6)$$

and

$$u_z = -\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ist}ds\int_0^{\infty}\sum_{j=1}^{3}(\lambda_j - k^2 p_j)A_j\exp(-\lambda_j z)kJ_0(kr)dk, \quad (3\ 7)$$

where A_j are given by (3 4) Hence, utilizing results (2 1)-(2 2) we can find the state of strain and the state of stress in the semi infinite space

When the viscosity and gravity are not taken into account, that is, when $\lambda_1, \mu_1, \alpha_1, \beta_1, \gamma_1, \epsilon_1$ are equal to zero and $g = 0$, relations (3 5)-(3 7) for displacement components and rotation component reduce to

$$u_r = -\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ist}ds\int_0^{\infty} k^2\left\{A_1\exp(-\lambda_1 z) - \sum_{j=2}^{3}A_j\exp(-\lambda_j z)\right\}J_1(kr)dk \quad (3\ 8)$$

$$\omega_\theta = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ist}ds\int_0^{\infty}\sum_{j=2}^{3}x\lambda_j A_j\exp(-\lambda_j z)k^2 J_1(kr)dk \quad (3\ 9)$$

and

$$u_z = -\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ist}ds\int_0^{\infty} k\left\{\lambda_1 A_1\exp(-\lambda_1 z) - k^2\sum_{j=2}^{3}A_j\exp(-\lambda_j z)\right\}J_0(kr)dk \quad (3\ 10)$$

where

$$\begin{aligned}
&a_1 = 2\mu_0\lambda_1^2 + \lambda_0(\lambda_1^2 - k^2), \quad a_j = -2\mu_0 k^2\lambda_j, \quad (j = 2,3)\\
&c_j = (\nu_0 + \epsilon_0)k\lambda_j q_j, \qquad q_j = x\lambda_j = -\frac{1}{p}(\lambda_j^2 - k_2^2)\\
&\lambda_1^2 = k^2 - \frac{s^2}{c_1^2}, \qquad \lambda_2^2 + \lambda_3^2 = k_2^2 + k_3^2 - p_0 s_0\\
&\lambda_2^2\lambda_3^2 = k_2^2 k_3^2 - p_0 s_0 k^2, \qquad p_0 = \frac{2\alpha_0}{\mu_0 + \alpha_0}, \qquad s_0 = \frac{2\alpha_0}{\gamma_0 + \epsilon_0}.
\end{aligned} \quad (3\ 11)$$

Relations (3 8)-(3 10) are in agreement with those obtained by Nowacki and Nowacki [4]

4. PARTICULAR CASE

We now consider a particular case of loading on the semi-infinite space boundary, that is, the loading oscillating harmonically in time, the medium being stationary for $t < 0$

The boundary conditions on the surface $z = 0$ are

$$\sigma_{zz} = Qe^{-i\omega t}f(r), \quad \sigma_{zr} = 0, \quad \mu_{z\theta} = 0. \quad (4\ 1)$$

Now the constants A_j in the equations (3 4) reduce to

$$A_j = (-1)^j \frac{\triangle_j^0}{\triangle}, \tag{4 2}$$

where

$$\triangle_1^0 = \tilde{\bar{f}}\,(b_2c_3 - b_3c_2), \quad \triangle_2^0 = \tilde{\bar{f}}\,(b_1c_3 - b_3c_1), \quad \triangle_3^0 = \tilde{\bar{f}}\,(b_1c_2 - b_2c_1)$$

and

$$\tilde{\bar{f}}\,(s,k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ist}dt \int_0^{\infty} f(r)e^{-i\omega t} r J_0(kr)dr = \sqrt{2\pi}\,\delta(s-\omega)\tilde{\bar{f}}\,(k). \tag{4 3}$$

Putting $f(r) = F_0 \sin \xi r$, we have

$$\bar{f}\,(k) = -\frac{d}{d\xi}\left\{\frac{H(k-\xi)}{\sqrt{k^2-\xi^2}}\right\}. \tag{4 4}$$

Thus it turns out that

$$u_r = e^{-i\omega t}\frac{d}{d\xi}\int_{\xi}^{\infty} k^2 \sum_{j=1}^{3}(1-\lambda_j' p_j')A_j' \cdot (k^2-\xi^2)^{-\frac{1}{2}} \exp(-\lambda_j' z) J_1(kr)dk \tag{4 5}$$

$$u_z = e^{-i\omega t}\frac{d}{d\xi}\int_{a}^{\infty} k \sum_{j=1}^{3}\left(\lambda_j'^2 - k^2 p_j'\right)A_j' \cdot (k^2-\xi^2)^{-\frac{1}{2}} \exp(-\lambda_j' z) J_0(kr)dk \tag{4 6}$$

$$\omega_\theta = -e^{-i\omega t}\int_{a}^{\infty} k^2 \left(\sum_{j=1}^{3} q_j' A_j'\right)(k^2-\xi^2)^{-\frac{1}{2}} \exp(-\lambda_j' z) J_1(kr)dk \tag{4 7}$$

where dashed quantities represent the value of the function at $s = \omega$

Results (3 8)-(3 10) show the striking difference between the displacement and rotation due to the influences of gravity when the effects of viscosity and gravity are neglected In the absence of gravity, λ_1^2, λ_2^2, λ_3^2 are given in (3 11), and these quantities are given in Section 3 where the effects of gravity and viscosity are included

In conclusion, we state that λ_j for the present case also depend on gravity and the corresponding results are changed from those where the effects of gravity and viscosity are neglected Further, the displacement field and rotation are correspondingly modified with increasing depth The modification is due to the pressure of viscosity and gravity Finally, in the absence of gravity with very small viscosity, the results reduce to those of the classical theory of elasticity due to Ghosh [6,7]

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