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A NOTE ON THE k-DOMINATION NUMBER OF A GRAPH

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ABSTRACT. The k-domination number of a graph $G = G(V,E)$, $\gamma_k(G)$, is the least cardinality of a set $X \subset V$ such that any vertex in V X is adjacent to at least k vertices of X.

Extending a result of Cockayne, Gamble and Shepherd [4], we prove that if $\delta(G) \geq \frac{n+1}{n} k-1$, $n \geq 1, k \geq 1$ then, $\gamma_k(G) \leq \frac{np}{n+1}$, where p is the order of G.

KEY WORDS AND PHRASES. k-dominating set and k-domination Number.
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1. INTRODUCTION.

A set X of vertices of a graph $G = G(V,E)$ is k-dominating if each vertex of $V \setminus X$ is adjacent to at least k vertices of X. The k-domination number of a graph G, $\gamma_k(G)$, is the smallest cardinality of a k-dominating set of G.

We write $\delta = \delta(G)$ for the minimum degree of vertices in G and $|G| = p$ is the number of vertices of G.

Several results concerning $y_k(G)$ have been established by Fink and Jacobson [1], [2] who showed that $\gamma_k \geq \frac{kp}{\Delta+k}$, and recently by Favaron [3].

As for the upper bound, Cockayne, Gamble and Shepherd proved the following:

THEOREM 1.1. If G has p vertices and $\delta \geq k$, then $\gamma_k(G) \leq \frac{kp}{k+1}$.

2. MAIN RESULTS.

Our aim in this note is to extend Theorem 1.1 and give a shorter proof of that given in Cockayne, Gamble, and Shepherd [4]. We prove, THEOREM 2.1. Let n, k be positive integers and G a graph such that

$\delta(G) > \frac{n+1}{n} k-1$. Then, $\gamma_k(G) < \frac{np}{n+1}$.

PROOF. Let $V_1, V_2, \ldots, V_{n+1}$ be a partition of V(G) into n+1 subsets which maximizes the number of edges in E' where $E' = E(G) \setminus \bigcup_{i=1}^{n+1} E(\langle V_i \rangle)$ and $\langle V_i \rangle$ is the subgraph induced on the vertex set V_i .

By a classical argument of Erdös [5] we have that for every $x \in V$, $deg_H(x) > [\frac{n}{n+1} deg_G(x)]$, where H = H(V',E'), V' = V, and E' is as above. Hence we conclude that:

$$deg_H(x) > [\frac{n}{n+1} (\frac{n+1}{n} k-1)] = [k- \frac{n}{n+1}] = k.$$

Assume W.L.O.G. that $|V_1| > |V_2| > \ldots > |V_{n+1}|$. Then the set $A = \bigcup_{i=2}^{n+1} V_i$ is a k-dominating set of G since each vertex $x \in V_1$ is adjacent to at least k vertices of A. Thus it follows that $\gamma_k(G) < p - |V_1| < \frac{np}{n+1}$.

COROLLARY 1. [4] If $\delta(G) > k$ then $\gamma_k(G) < \frac{kp}{k+1}$.

PROOF. Take n = k in Theorem 2.1.

COROLLARY 2: If $\delta(G) > 2k - 1$ then $\gamma_k(G) < \frac{p}{2}$.

REMARK. Using a similar argument we can prove the following:

$$\text{If } \delta(G) > k > 1 \text{ and } \chi(G) = n, \text{ then } \gamma_k(G) < \frac{(n-1)p}{n}.$$

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