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ON COINCIDENCE THEOREMS FOR A FAMILY OF MAPPINGS IN CONVEX METRIC SPACES

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ABSTRACT. In this paper, a theorem on common fixed points for a family of mappings defined on convex metric spaces is presented. This theorem is a generalization of the well known fixed point theorem proved by Assad and Kirk. As an application a common fixed point theorem in metric spaces with a convex structure is obtained.

KEY WORDS AND PHRASES. Common fixed points, convex metric spaces, metric spaces with a convex structure.
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1. INTRODUCTION.

Some fixed point theorems and theorems on coincidence points in convex metric spaces or spaces with a convex structure in the sense of Takahashi [1] are obtained by many authors [1-8].

In this paper we shall give a generalization of Theorem 1 from [9] in the case of a convex metric space and as an application we shall obtain a theorem on coincidence points in metric spaces with a convex structure.

First, we shall give two definitions and a proposition which we shall use in the sequel [2].

DEFINITION 1. *A metric space* (M,d) *is convex if for each* $x,y \in M$ *with* $x \neq y$ *there exists* $z \in M$, $x \neq z \neq y$, *such that*

$$d(x,z) + d(z,y) = d(x,y).$$

DEFINITION 2. *Let* (M,d) *be a metric space. The mapping* W *which maps* $M \times M \times [0,1]$ *into* M *is called a convex structure if for all* $x,y,u \in M$ *and* $t \in [0,1]$:

$$d(u,W(x,y,t)) \leq td(u,x) + (1-t)d(u,y) .$$

This definition is similar to the definition of metric spaces of hyperbolic type. The class of metric spaces of hyperbolic type includes all normed linear spaces, as well as all spaces with hyperbolic metric. Some further results on the fixed point theory in such spaces are obtained by W.A. Kirk [5] and K. Goebel and W.A. Kirk [10]. It is known that every metric space with a convex structure belongs to the class of convex metric spaces. The following result is well known [2].

PROPOSITION. *Let* K *be a closed subset of a complete and convex metric space* M. *If* $x \in K$ *and* $y \notin K$, *then there exists a point* $z \in \partial K$ *such that:*

$$d(x,z) + d(z,y) = d(x,y).$$

Let us recall that a pair of mappings (A,S) is *weakly commutative*, where (M,d) is a metric space, $K \subseteq M$ and $A,S : K \to M$, if:

$$Ax, Sx \in K \Rightarrow d(ASx,SAx) \leq d(Sx,Ax) \qquad [11].$$

2. TWO COINCIDENCE THEOREMS.

First, we shall prove a generalization of Theorem 1 from [9] in the case of convex metric spaces. This theorem is also a generalization of a fixed point theorem proved by Assad and Kirk in [2], if the mapping ϕ is single valued.

THEOREM 1. *Let* (M,d) *be a complete, convex metric space,* K *a nonempty closed subset of* M, S *and* T *continuous mappings from* M *into* M *so that* $\partial K \subseteq SK \cap TK$, *for every* $i \in \mathbb{N}$ $A_i : K \to M$ *continuous mapping such that* $A_iK \cap K \subseteq SK \cap TK$, (A_i,S) *and* (A_i,T) *weakly commutative pairs and there exists* $q \in [0,1)$ *so that for every* $x,y \in K$ *and every* $i,j \in \mathbb{N}$ $(i \neq j)$:

$$d(A_ix,A_jy) \leq q\, d(Sx,Ty) .$$

If for every $i \in \mathbb{N}$ *and* $x \in K$:

$$Tx \in \partial K \Rightarrow A_ix \in K \text{ and } Sx \in \partial K \Rightarrow A_ix \in K$$

then there exists $z \in K$ *so that:*

$$z = Tz = Sz = A_iz, \text{ for every } i \in \mathbb{N}$$

and if $Tv = Sv = A_iv$, *for every* $i \in \mathbb{N}$ *then* $Tz = Tv$.

PROOF. Let $p \in \partial K$ and $p_o \in K$ so that $p = Tp_o$. Such p_o exists since $\partial K \subseteq TK$. Further, $Tp_o \in \partial K$ implies that for every $i \in \mathbb{N}$, $A_ip_o \in K$ and so we have that $A_1p_o \in A_1K \cap K \subseteq SK$. Let $p_1 \in K$ be such that $Sp_1 = A_1p_o \in K$ and $p_1' = A_1p_o$, $p_2' = A_2p_1$. If $p_2' \in K$ then from $A_2p_1 \in A_2K \cap K \subseteq TK$ it follows that there exists $p_2 \in K$ so that $Tp_2 = A_2p_1$. Suppose now that $p_2' \notin K$. Then from the Proposition it follows that there exists $q \in \partial K$ so that:

$$d(Sp_1,Tp_2) + d(Tp_2,A_2p_1) = d(Sp_1,A_2p_1) \quad \text{where } q = Tp_2.$$

Such element $p_2 \in K$ exists since $\partial K \subseteq TK$. In this way we obtain two sequences $\{p_i\}_{i \in \mathbb{N}}$ and $\{p_i'\}_{i \in \mathbb{N}}$ so that for every $n \in \mathbb{N}$ $p_n \in K$, $p_{n+1}' = A_{n+1}p_n$ and the following implications hold:

(i) $p_{2n}' \in K \Rightarrow p_{2n}' = Tp_{2n}$.

$p_{2n}' \notin K \Rightarrow Tp_{2n} \in \partial K$ and

$$d(Sp_{2n-1},Tp_{2n}) + d(Tp_{2n},A_{2n}p_{2n-1}) = d(Sp_{2n-1},A_{2n}p_{2n-1}).$$

(ii) $p'_{2n+1} \in K \Rightarrow p'_{2n+1} = Sp_{2n+1}$.

$p'_{2n+1} \notin K \Rightarrow Sp_{2n+1} \in \partial K$ and

$d(Tp_{2n},Sp_{2n+1}) + d(Sp_{2n+1},A_{2n+1}p_{2n}) = d(Tp_{2n},A_{2n+1}p_{2n})$.

Let:

$$P_o = \{p_{2n} \in \{p_n | n \in \mathbb{N}\},\ p'_{2n} = Tp_{2n},\ n \in \mathbb{N}\} ,$$

$$P_1 = \{p_{2n} \in \{p_n | n \in \mathbb{N}\},\ p'_{2n} \neq Tp_{2n},\ n \in \mathbb{N}\} ,$$

$$Q_o = \{p_{2n+1} \in \{p_n | n \in \mathbb{N}\},\ p'_{2n+1} = Sp_{2n+1},\ n \in \mathbb{N}\} ,$$

$$Q_1 = \{p_{2n+1} \in \{p_n | n \in \mathbb{N}\},\ p'_{2n+1} \neq Sp_{2n+1},\ n \in \mathbb{N}\} .$$

Let us prove that there exists $z \in K$ such that:

$$z = \lim_{n\to\infty} Tp_{2n} = \lim_{n\to\infty} Sp_{2n+1} .$$

Suppose that $p_{2n} \in P_1$. Then $Tp_{2n} \in \partial K$ and so $A_{2n+1}p_{2n} = p'_{2n+1} \in K$ which implies that $p'_{2n+1} = Sp_{2n+1}$ and so $p_{2n+1} \in Q_o$. So we have the following possibilities:

$$(p_{2n},p_{2n+1}) \in P_o x Q_o;\ (p_{2n},p_{2n+1}) \in P_o x Q_1;\ (p_{2n},p_{2n+1}) \in P_1 x Q_o$$

a) $(p_{2n},p_{2n+1}) \in P_o x Q_o$.

Then

$$d(Tp_{2n},Sp_{2n+1}) = d(A_{2n}p_{2n-1},A_{2n+1}p_{2n}) \leq q\, d(Sp_{2n-1},Tp_{2n}).$$

b) $(p_{2n},p_{2n+1}) \in P_o x Q_1$.

Then:

$$d(Tp_{2n},Sp_{2n+1}) = d(Tp_{2n},A_{2n+1}p_{2n}) - d(Sp_{2n+1},A_{2n+1}p_{2n}) \leq d(Tp_{2n},A_{2n+1}p_{2n})$$

$$= d(A_{2n}p_{2n-1},A_{2n+1}p_{2n}) \leq q\, d(Sp_{2n-1},Tp_{2n}) .$$

c) $(p_{2n},p_{2n+1}) \in P_1 x Q_o \Rightarrow d(Tp_{2n},Sp_{2n+1}) \leq q\, d(Tp_{2n-2},Sp_{2n-1})$.

In this case we have:

$$d(Tp_{2n},Sp_{2n+1}) \leq d(Tp_{2n},A_{2n}p_{2n-1}) + d(A_{2n}p_{2n-1},Sp_{2n+1}) \leq$$

$$\leq d(Tp_{2n},A_{2n}p_{2n-1}) + d(A_{2n}p_{2n-1},A_{2n+1}p_{2n}) \leq$$

$$\leq d(Tp_{2n},A_{2n}p_{2n-1}) + q\, d(Sp_{2n-1},Tp_{2n}) \leq$$

$$\leq d(Sp_{2n-1},Tp_{2n}) + d(Tp_{2n},A_{2n}p_{2n-1}) = d(Sp_{2n-1},A_{2n}p_{2n-1}) .$$

Since $p_{2n} \in P_1$ we have that $p_{2n-1} \in Q_o$ and so $Sp_{2n-1} = A_{2n-1}p_{2n-2}$. Further

$$(p_{2n-1},p_{2n}) \in Q_o x P_o \Rightarrow d(Sp_{2n-1},Tp_{2n}) \leq q\, d(Tp_{2n-2},Sp_{2n-1}),$$

$$(p_{2n-1},p_{2n}) \in Q_1 x P_o \Rightarrow d(Sp_{2n-1},Tp_{2n}) \leq q\, d(Tp_{2n-2},Sp_{2n-3}),$$

$$(p_{2n-1},p_{2n}) \in Q_o x P_1 \Rightarrow d(Sp_{2n-1},Tp_{2n}) \leq q\, d(Tp_{2n-2},Sp_{2n-1})$$

If $r = \max\{d(Tp_2,Sp_3),d(Tp_2Sp_1)\}$ then we can easily prove that:

$$d(Tp_{2n},Sp_{2n+1}) \leqq q^{n-1}\cdot r \quad \text{and} \quad d(Sp_{2n+1},Tp_{2n+2}) \leqq q^{n}\cdot r$$

for every $n \in \mathbb{N}$.
This implies that for every $n \in \mathbb{N}$:

$$d(Tp_{2n},Tp_{2n+2}) \leqq r(q^{n-1} + q^{n}) \ .$$

Hence, the sequence $\{Tp_{2n}\}_{n\in\mathbb{N}}$ is a Cauchy sequence and since M is complete and K is closed it follows that there exists $z \in K$ so that $z = \lim_{n\to\infty} Tp_{2n} = \lim_{n\to\infty} Sp_{2n+1}$.

We shall prove that $Tz = Sz = A_m z$, for every $m \in \mathbb{N}$. It is obvious that there exists a sequence $\{n_k\}_{k\in\mathbb{N}}$ in $\mathbb{N}$ such that $Tp_{2n_k} = A_{2n_k}p_{2n_k-1}$, for every $k \in \mathbb{N}$ or $Sp_{2n_k-1} = A_{2n_k-1}p_{2n_k-2}$, for every $k \in \mathbb{N}$. Suppose that $Tp_{2n_k} = A_{2n_k}p_{2n_k-1}$, for every $k \in \mathbb{N}$. Then for every $k \in \mathbb{N}$ we have:

$$d(ST_{p_{2n_k}},A_m z) = d(SA_{2n_k}p_{2n_k-1},A_m z) \quad \text{for every } m \in \mathbb{N}. \text{ Hence:}$$

$$d(STp_{2n_k},A_m z) \leqq d(SA_{2n_k}p_{2n_k-1},A_{2n_k}Sp_{2n_k-1}) + d(A_{2n_k}Sp_{2n_k-1},A_m z) \leqq$$

$$\leqq d(A_{2n_k}p_{2n_k-1},Sp_{2n_k-1}) + q\, d(TSp_{2n_k-1},Sz) \quad (m \neq 2n_k)$$

and when $k \to \infty$ we obtain that:

$$d(Sz,A_m z) \leqq q\, d(Tz,Sz), \text{ for every } m \in \mathbb{N}. \tag{2.1}$$

If $T = S$ the proof of the relation $Sz = Tz = A_m z$ is complete.
Let us remark that (2.1) holds also in the case when $S,T : K \to M$. Further, we have:

$$d(A_m p_{2n_k},Tp_{2n_k}) = d(A_m p_{2n_k},A_{2n_k}p_{2n_k-1}) \leqq q\, d(Tp_{2n_k},Sp_{2n_k-1})$$

$(m \neq 2n_k)$ and if $k \to \infty$ we obtain that $\lim_{k\to\infty} A_m p_{2n_k} = z$. Further, $Sz = z$ since $d(A_m p_{2n_k},A_{2n_k}Sp_{2n_k-1}) \leqq q\, d(Tp_{2n_k},SSp_{2n_k-1})$ $(m \neq 2n_k)$ implies that $d(z,Sz) \leqq q\, d(z,Sz)$ where we use that (A_{2n_k},S) is weakly commutative.

Thus we obtain:

$$Tz = T(\lim_{k\to\infty} A_m p_{2n_k}) = \lim_{k\to\infty} T(A_m p_{2n_k}) \ . \tag{2.2}$$

Since (A_m,T) is a weakly commutative pair of mappings we have that $d(T(A_m p_{2n_k}), A_m(Tp_{2n_k})) \leqq d(A_m p_{2n_k},Tp_{2n_k})$ which implies that $\lim_{k\to\infty} A_m(Tp_{2n_k}) = \lim_{k\to\infty} T(A_m p_{2n_k})$ and so from (2.2) we obtain that:

$$Tz = \lim_{k\to\infty} A_m(Tp_{2n_k}) = A_m(\lim_{k\to\infty} Tp_{2n_k}) = A_m z \ .$$

Using (2.1) we conclude that:

$$d(Sz,A_m z) = d(Sz,Tz) \leq q\ d(Tz,Sz)$$

and so $Sz = A_m z = Tz$, for every $m \in \mathbb{N}$.
Let $u \in K$ be such that $Tu = Su = A_m u$, for every $m \in \mathbb{N}$. Then $d(Tu,Tz) = d(A_m u,A_{m+1}z) \leq q\ d(Tu,Tz)$ which implies that $Tu = Tz$.

REMARK 1. If z is an interior point in K it is enough to suppose that $S,T : K \to M$ since from $\lim_{k\to\infty} A_m p_{2n_k} = z$ it follows that there exists $k_o \in \mathbb{N}$ such that for all $k \geq k_o$, $A_m p_{2n_k} \in K$. In this case $T(A_m p_{2n_k})$ is defined for every $k \geq k_o$.

We shall give some conditions when we can also suppose that S and T are defined only on K.

a) $d(Tu,Su) \leq t_m d(Su,A_m u)$, for some $m \in \mathbb{N}$, where $qt_m \in [0,1)$ and u belongs to the boundary of K. Then $d(Sz,A_m z) \leq q\ d(Tz,Sz) \leq qt_m\ d(Sz,A_m z)$ and so $Sz = A_m z = Tz$.

b) $d(Tu,A_m u) \leq r_m\ d(Tu,Su)$, for some $m \in \mathbb{N}$, where $(r_m + q) < 1$ and u belongs to the boundary of K. Then $d(Tz,Sz) \leq d(Tz,A_m z) + d(A_m z,A_{2n_k} Sp_{2n_k-1}) + $
$+ d(A_{2n_k} Sp_{2n_k-1},Sz) \leq d(Tz,A_m z) + q\ d(Sz,TSp_{2n_k-1}) + d(A_{2n_k} Sp_{2n_k-1},Sz)$ $(m \neq 2n_k)$ and if $k \to \infty$ we obtain that $d(Tz,Sz) \leq d(Tz,A_m z) + q\ d(Sz,Tz) \leq$ $(r_m + q)d(Sz,Tz)$ which implies that $Tz = Sz = A_m z$, for every $m \in \mathbb{N}$.

c) $d(Tu,Su) \leq s_m\ d(Tu,A_m u)$, for some $m \in \mathbb{N}$, where $s_m(1+q) < 1$ and u belongs to the boundary of K. We have that $d(Tz,A_m z) \leq d(Tz,Sz) + d(Sz,A_{2n_k} Sp_{2n_k-1}) +$
$+ d(A_{2n_k} Sp_{2n_k-1},A_m z)$ $(m \neq 2n_k)$ and if $k \to \infty$ we obtain that:

$$d(Tz,A_m z) \leq (1+q)d(Tz,Sz) \leq s_m(1+q)d(Tz,A_m z)\ .$$

From this we conclude that $Tz = A_m z = Sz$, for every $m \in \mathbb{N}$.

REMARK 2. Suppose that $z \in K$ is such that $Tz = Sz = A_m z$, for every $m \in \mathbb{N}$ and that $Tz \in K$. In this case, we can prove that Tz is a common fixed point for S, T and A_m $(m \in \mathbb{N})$. Let $m \neq n$. Then $d(A_m A_m z,A_n z) \leq q\ d(TA_m z,Sz) \leq$ $\leq q[d(TA_m z,A_m Tz) + d(A_m Tz,A_n z)] \leq q\ d(A_m z,Tz) + q\ d(A_m Tz,A_n z) = q\ d(A_m Tz,A_n z) =$ $= q\ d(A_m A_m z,A_n z)$ and so $A_m Tz = Tz$, for every $m \in \mathbb{N}$. From $TA_m z = A_m Tz$ and $SA_m z = A_m Sz$ $(m \in \mathbb{N})$ it follows that Tz is a fixed point for T, S and A_m $(m \in \mathbb{N})$. It is easy to prove the uniqueness of Tz as a coincidence point.

The next Theorem is an existence theorem for a coincidence point in metric spaces with a convex structure.

Let (M,d) be a metric space with convex structure W. If for all $(x,y,z,t) \in M \times M \times M \times [0,1)$:

$$d(W(x,z,t),W(y,z,t)) \leq t\ d(x,y)$$

then W satisifes *condition* II [7]. Let $x_o \in M$ and $S : M \to M$. The mapping S is said to be (W,x_o)-*convex* if for every $z \in X$ and every $t \in (0,1) : W(Sz,x_o,t) =$ $S(W(z,x_o,t))$. If M is a normed space and $W(x,y,t) = tx + (1-t)y(x,y \in M,\ t \in [0,1])$ then every homogeneous mapping $S : M \to M$ is $(W,0)$-convex.

Let α be the Kuratowski measure of noncompactness on M and K a nonempty subset of M. If $A,S : K \to M$ we say that A is (α,S)-*densifying* if for every $B \subseteq K$ such that $S(B)$ and $A(B)$ are bounded the implication:

$$\alpha(S(B)) \leq \alpha(A(B)) \Rightarrow \bar{B} \text{ is compact}$$

holds. In the next theorem we suppose that W satisfies condition II.

THEOREM 2. *Let* (M,d) *be a complete metric space with a convex structure* W, K *a nonempty, closed subset of* M, $x_o \in K$ *and for every* $x \in K$ *and every* $t \in (0,1)$, $W(x,x_o,t) \in K$. *Let, further,* S *and* T *be continuous,* (W,x_o)*-convex mappings from* M *into* M *such that* $K \subseteq SK \cap TK$, *for every* $i \in N$, $A_i : K \to M$ *continuous mapping,* $A_i(K)$ *a bounded set and the following implications hold for every* $i \in \mathbb{N}$:

$$Sx \in K \Rightarrow SA_ix = A_iSx; \quad Sx \in \partial K \Rightarrow A_ix \in K;$$

$$Tx \in K \Rightarrow TA_ix = A_iTx, \quad Tx \in \partial K \Rightarrow A_ix \in K.$$

If there exists $i_o \in N$ *such that* A_{i_o} *is* (α,I_M) *or* (α,S) *or* (α,T) *densifying and:*

$$d(A_ix,A_jy) \leq d(Sx,Ty), \text{ for every } x,y \in K \text{ and } i,j \in \mathbb{N}(i \neq j)$$

then there exists $z \in K$ *such that* $z = Tz = Sz = A_iz$, *for every* $i \in \mathbb{N}$.

PROOF: Let, for every $n \in \mathbb{N}$, $r_n \in (0,1)$ and $\lim_{n\to\infty} r_n = 1$.
For every $(i,n) \in \mathbb{N} \times \mathbb{N}$ and every $x \in K$ let:

$$A_{i,n}x = W(A_ix,x_o,r_n) .$$

Then for every $n \in \mathbb{N}$, the family $\{A_{i,n}\}_{i \in \mathbb{N}}$, S and T satisfy all the conditions of Theorem 1, which will be proved.

First, we have that for every $i,j \in \mathbb{N}$ $(i \neq j)$ and every $n \in \mathbb{N}$:

$$d(A_{i,n}x,A_{j,n}y) = d(W(A_ix,x_o,r_n),W(A_jy,x_o,r_n)) \leq$$

$$\leq r_nd(A_ix,A_jy) \leq r_nd(Sx,Ty), \text{ for every } x,y \in K.$$

Further, if $Sx \in K$ we have that:

$$SA_{i,n}x = SW(A_ix,x_o,r_n) = W(SA_ix,x_o,r_n) = W(A_iSx,x_o,r_n) = A_{i,n}Sx$$

and similarly $Tx \in K \Rightarrow TA_{i,n}x = A_{i,n}Tx$. Let $Sx \in \partial K$. Then $A_ix \in K$ and this implies that for every $n \in \mathbb{N}$:

$$W(A_ix,x_o,r_n) = A_{i,n}x \in K, \text{ for every } i \in \mathbb{N}$$

Similarly $Tx \in \partial K \Rightarrow A_{i,n}x \in K$, for every $(i,n) \in \mathbb{N} \times \mathbb{N}$.

Thus, for every $n \in \mathbb{N}$ there exists $x_n \in K$ so that:

$$x_n = Sx_n = Tx_n = A_{i,n}x_n, \text{ for every } i \in \mathbb{N}. \tag{2.3}$$

From (2.3) we obtain that:

$$d(x_n,A_ix_n) = d(Sx_n,A_ix_n) = d(Tx_n,A_ix_n) = d(A_{i,n}x_n,A_ix_n) =$$

$$= d(W(A_ix_n,x_o,r_n),A_ix_n) \le r_nd(A_ix_n,A_ix_n) + (1-r_n)d(A_ix_n,x_o)$$

for every $(i,n) \in \mathbb{N}$. Since A_iK is bounded for every $i \in \mathbb{N}$ it follows that:

$$\lim_{n\to\infty} d(x_n,A_ix_n) = \lim_{n\to\infty} d(Sx_n,A_ix_n) = \lim_{n\to\infty} d(Tx_n,A_ix_n) = 0.$$

Suppose that there exists i_o such that A_{i_o} is (α,S)-densifying. The proof is similar if A_{i_o} is (α,I_M) or (α,T)-densifying.

Since $\lim_{n\to\infty} d(Sx_n,A_{i_o}x_n) = 0$ it follows that for every $\varepsilon > 0$ there exists $n_o(\varepsilon) \in \mathbb{N}$ so that:

$$\{Sx_n|n \ge n_o(\varepsilon)\} \subseteq \bigcup_{y\in A_{i_o}B} L(y,\varepsilon), \ B = \{x_n|n \in \mathbb{N}\}. \tag{2.4}$$

Relation (2.4) implies that:

$$\alpha(\{Sx_n|n \ge n_o(\varepsilon)\}) \ge \alpha(A_{i_o}B) + 2\varepsilon.$$

Since $\alpha(SB) = \alpha(\{Sx_n|n \ge n_o(\varepsilon)\})$ we obtain that:

$$\alpha(SB) \le \alpha(A_{i_o}B) + 2\varepsilon.$$

Because $\varepsilon > 0$ is an arbitrary positive number we obtain that $\alpha(SB) \le \alpha(A_{i_o}B)$ and since A_{i_o} is (α,S)-densifying we obtain that B is relatively compact. Suppose that $\lim_{k\to\infty} x_{n_k} = z$.

Then we obtain that:

$$d(z,A_iz) = \lim_{k\to\infty} d(x_{n_k},A_ix_{n_k}) = d(A_iz,Sz) = \lim_{k\to\infty} d(Sx_{n_k},A_ix_{n_k}) =$$

$$= d(A_iz,Tz) = \lim_{k\to\infty} d(Tx_{n_k},A_ix_{n_k}) = 0$$

and so $z = Sz = Tz = A_iz$, for every $i \in \mathbb{N}$.

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REFERENCES

1. TAKAHASHI, W. A Convexity in Metric Space and Nonexpansive Mappings, I, Kodai Math. Sem. Rep. 22(1970), 142-149.
2. ASSAD, N.A. and KIRK, W.A. Fixed Point Theorems for Set-Valued Mappings of Contractive Type, Pacific J. Math. 43(1972), 553-562.
3. ITOH, S. Multivalued Generalized Contractions and Fixed Point Theorems, Comm. Math. Univ. Carolinae 18(2) (1977)
4. KHAN, M.S. Common Fixed Point Theorems for Multivalued Mappings, Pacific J. Math. 95(2) (1981), 337-347.
5. KIRK, W.A. Krasnoselskii's Iteration Process in Hyperbolic Spaces, Numerical Functional Analysis and Optimization 4(1982), 371-381.
6. NAIMPALLY, S.A., SINGH, K.L. and WHITFIELD, J.H.M. Common Fixed Points for Nonexpansive and Asymptotically Nonexpansive Mappings, Comm. Math. Univ. Carolinae 24,2(1983), 287-300.
7. RHOADES, B.E., SINGH, K.L. and WHITFIELD, J.H.M. FIxed Point for Generalized Nonexpansive Mappings, Comm. Math. Univ. Carolinae 23,3(1982), 443-451.

8. TALMAN, L.A. Fixed Points for Condensing Multifunctions in Metric Spaces with Convex Structure, Kodai Math. Sem. Rep. 29(1977), 62-70.

9. HADZIC, O. Common Fixed Point Theorems for a Family of Mappings in Complete Metric Spaces, Math. Japonica 29(1984), 127-134.

10. GOEBEL, K. and KIRK, W.A. Iteration Processes for Nonexpansive Mappings, Contemporary Mathematics 21(1983), 115-123.

11. SESSA, S. On a Weak Commutativity Condition in Fixed Point Considerations, Publ. Inst. Math. 32(46) (1982), 149-153.

12. FISHER, B. Mapping With a Common Fixed Point, Math. Sem. Notes, Kobe Univ., 8 (1980), 81-84.