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Research Article

Stability of the Cauchy-Jensen Functional Equation in C^*-Algebras: A Fixed Point Approach

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we prove the Hyers-Ulam-Rassias stability of C^*-algebra homomorphisms and of generalized derivations on C^*-algebras for the following Cauchy-Jensen functional equation $2f((x+y)/2+z) = f(x)+f(y)+2f(z)$, which was introduced and investigated by Baak (2006). The concept of Hyers-Ulam-Rassias stability originated from the stability theorem of Th. M. Rassias that appeared in (1978).

1. Introduction and preliminaries

The stability problem of functional equations originated from a question of Ulam [1] concerning the stability of group homomorphisms. Hyers [2] gave a first affirmative partial answer to the question of Ulam for Banach spaces. Hyers' theorem was generalized by Aoki [3] for additive mappings and by Rassias [4] for linear mappings by considering an unbounded Cauchy difference.

Theorem 1.1 (see [4]). *Let $f : E \to E'$ be a mapping from a normed vector space E into a Banach space E' subject to the inequality*

$$\|f(x+y) - f(x) - f(y)\| \le \epsilon\left(\|x\|^p + \|y\|^p\right) \tag{1.1}$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then, the limit

$$L(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n} \tag{1.2}$$

exists for all $x \in E$ and $L : E \to E'$ is the unique additive mapping which satisfies

$$\|f(x) - L(x)\| \le \frac{2\epsilon}{2 - 2^p}\|x\|^p \tag{1.3}$$

for all $x \in E$. Also, if for each $x \in E$ the mapping $f(tx)$ is continuous in $t \in \mathbb{R}$, then L is $\mathbb{R}$-linear.

The above inequality (1.1) has provided a lot of influence in the development of what is now known as a *Hyers-Ulam-Rassias stability* of functional equations. A generalization of Th. M. Rassias' theorem was obtained by Găvruţa [5] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias' approach. The result of Găvruţa [5] is a special case of a more general theorem, which was obtained by Forti [6]. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [7–18]).

J. M. Rassias [19] following the spirit of the innovative approach of Th. M. Rassias [4] for the unbounded Cauchy difference proved a similar stability theorem in which he replaced the factor $\|x\|^p + \|y\|^p$ by $\|x\|^p \cdot \|y\|^q$ for $p, q \in \mathbb{R}$ with $p + q \neq 1$ (see also [20] for a number of other new results).

Theorem 1.2 (see [19–21]). *Let X be a real normed linear space and Y a real complete normed linear space. Assume that $f : X \to Y$ is an approximately additive mapping for which there exist constants $\theta \ge 0$ and $p \in \mathbb{R} - \{1\}$ such that f satisfies inequality*

$$\|f(x+y) - f(x) - f(y)\| \le \theta \cdot \|x\|^{p/2} \cdot \|y\|^{p/2} \tag{1.4}$$

for all $x, y \in X$. Then, there exists a unique additive mapping $L : X \to Y$ satisfying

$$\|f(x) - L(x)\| \le \frac{\theta}{|2^p - 2|}\|x\|^p \tag{1.5}$$

for all $x \in X$. If, in addition, $f : X \to Y$ is a mapping such that the transformation $t \to f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$, then L is an $\mathbb{R}$-linear mapping.

We recall two fundamental results in fixed point theory.

Theorem 1.3 (see [22]). *Let (X, d) be a complete metric space and let $J : X \to X$ be strictly contractive, that is,*

$$d(Jx, Jy) \le Lf(x, y), \quad \forall x, y \in X \tag{1.6}$$

for some Lipschitz constant $L < 1$. Then, the following conditions hold.

(1) *The mapping J has a unique fixed point $x^* = Jx^*$.*

(2) *The fixed point x^* is globally attractive, that is,*

$$\lim_{n\to\infty} J^n x = x^* \tag{1.7}$$

for any starting point $x \in X$.

(3) *One has the following estimation inequalities:*

$$d(J^n x, x^*) \le L^n d(x, x^*),$$
$$d(J^n x, x^*) \le \frac{1}{1-L} d(J^n x, J^{n+1} x), \tag{1.8}$$
$$d(x, x^*) \le \frac{1}{1-L} d(x, Jx)$$

for all nonnegative integers n and all $x \in X$.

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a *generalized metric* on X if d satisfies the following conditions:

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(3) $d(x, z) \le d(x, y) + d(y, z)$, for all $x, y, z \in X$.

Theorem 1.4 (see [23]). *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d(J^n x, J^{n+1} x) = \infty \tag{1.9}$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) $d(J^n x, J^{n+1} x) < \infty$, *for all $n \ge n_0$;*

(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*

(3) *y^* is the unique fixed point of J in the set $Y = \{y \in X \mid d(J^{n_0} x, y) < \infty\}$;*

(4) $d(y, y^*) \le (1/(1-L)) d(y, Jy)$, *for all $y \in Y$.*

This paper is organized as follows. In Section 2, using the fixed point method, we prove the Hyers-Ulam-Rassias stability of C^*-algebra homomorphisms for the Cauchy-Jensen functional equation.

In Section 3, using the fixed point method, we prove the Hyers-Ulam-Rassias stability of generalized derivations on C^*-algebras for the Cauchy-Jensen functional equation.

Throughout this paper, assume that A is a C^*-algebra with norm $\|\cdot\|_A$ and that B is a C^*-algebra with norm $\|\cdot\|_B$.

2. Stability of C^*-algebra homomorphisms

For a given mapping $f : A \to B$, we define

$$C_\mu f(x, y, z) := 2\mu f\left(\frac{x+y}{2} + z\right) - f(\mu x) - f(\mu y) - 2f(\mu z), \tag{2.1}$$

for all $\mu \in \mathbb{T}^1 := \{\nu \in \mathbb{C} : |\nu| = 1\}$ and all $x, y, z \in A$.

We prove the Hyers-Ulam-Rassias stability of C^*-algebra homomorphisms for the functional equation $C_\mu f(x, y, z) = 0$.

Theorem 2.1. *Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A^3 \to [0, \infty)$ such that*

$$\lim_{j\to\infty} \frac{1}{2^j}\varphi(2^j x, 2^j y, 2^j z) = 0, \tag{2.2}$$

$$\|C_\mu f(x, y, z)\|_B \le \varphi(x, y, z), \tag{2.3}$$

$$\|f(xy) - f(x)f(y)\|_B \le \varphi(x, y, 0), \tag{2.4}$$

$$\|f(x^*) - f(x)^*\|_B \le \varphi(x, x, x) \tag{2.5}$$

for all $\mu \in \mathbb{T}^1$ and all $x, y, z \in A$. If there exists an $L < 1$ such that $\varphi(x, x, x) \le 2L\varphi(x/2, x/2, x/2)$ for all $x \in A$, then there exists a unique C^*-algebra homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\|_B \le \frac{1}{4 - 4L}\varphi(x, x, x) \tag{2.6}$$

for all $x \in A$.

Proof. Consider the set

$$X := \{g : A \to B\} \tag{2.7}$$

and introduce the *generalized metric* on X as follows:

$$d(g, h) = \inf\{C \in \mathbb{R}_+ : \|g(x) - h(x)\|_B \le C\varphi(x, x, x),\ \forall x \in A\}. \tag{2.8}$$

It is easy to show that (X, d) is complete.

Now, we consider the linear mapping $J : X \to X$ such that

$$Jg(x) := \frac{1}{2}g(2x) \tag{2.9}$$

for all $x \in A$.

By [22, Theorem 3.1],

$$d(Jg, Jh) \le Ld(g, h) \tag{2.10}$$

for all $g, h \in X$.

Letting $\mu = 1$ and $y = z = x$ in (2.3), we get

$$\|2f(2x) - 4f(x)\|_B \le \varphi(x, x, x) \tag{2.11}$$

for all $x \in A$. So

$$\left\|f(x) - \frac{1}{2}f(2x)\right\|_B \le \frac{1}{4}\varphi(x, x, x) \tag{2.12}$$

for all $x \in A$. Hence, $d(f, Jf) \le 1/4$.

By Theorem 1.4, there exists a mapping $H : A \to B$ such that the following conditions hold.

(1) H is a fixed point of J, that is,

$$H(2x) = 2H(x) \tag{2.13}$$

for all $x \in A$. The mapping H is a unique fixed point of J in the set

$$Y = \{g \in X : d(f,g) < \infty\}. \tag{2.14}$$

This implies that H is a unique mapping satisfying (2.13) such that there exists $C \in (0,\infty)$ satisfying

$$\|H(x) - f(x)\|_B \leq C\varphi(x,x,x) \tag{2.15}$$

for all $x \in A$.

(2) $d(J^n f, H) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} \frac{f(2^n x)}{2^n} = H(x) \tag{2.16}$$

for all $x \in A$.

(3) $d(f,H) \leq (1/(1-L))d(f,Jf)$, which implies the inequality

$$d(f,H) \leq \frac{1}{4-4L}. \tag{2.17}$$

This implies that inequality (2.6) holds.

It follows from (2.2), (2.3), and (2.16) that

$$\begin{aligned}
&\left\|2H\left(\frac{x+y}{2}+z\right) - H(x) - H(y) - 2H(z)\right\|_B \\
&\quad = \lim_{n\to\infty} \frac{1}{2^n} \left\|2f(2^{n-1}(x+y) + 2^n z) - f(2^n x) - f(2^n y) - 2f(2^n z)\right\|_B \\
&\quad \leq \lim_{n\to\infty} \frac{1}{2^n} \varphi(2^n x, 2^n y, 2^n z) = 0
\end{aligned} \tag{2.18}$$

for all $x, y, z \in A$. So

$$2H\left(\frac{x+y}{2}+z\right) = H(x) + H(y) + 2H(z) \tag{2.19}$$

for all $x, y, z \in A$. By [24, Lemma 2.1], the mapping $H : A \to B$ is Cauchy additive, that is, $H(x+y) = H(x) + H(y)$, for all $x, y \in A$.

By a similar method to the proof of [11], one can show that the mapping $H : A \to B$ is $\mathbb{C}$-linear.

It follows from (2.4) that

$$\begin{aligned}
\|H(xy) - H(x)H(y)\|_B &= \lim_{n\to\infty} \frac{1}{4^n} \left\|f(4^n xy) - f(2^n x)f(2^n y)\right\|_B \\
&\leq \lim_{n\to\infty} \frac{1}{4^n} \varphi(2^n x, 2^n y, 0) \leq \lim_{n\to\infty} \frac{1}{2^n} \varphi(2^n x, 2^n y, 0) = 0
\end{aligned} \tag{2.20}$$

for all $x, y \in A$. So

$$H(xy) = H(x)H(y) \tag{2.21}$$

for all $x, y \in A$.

It follows from (2.5) that

$$\left\|H(x^*) - H(x)^*\right\|_B = \lim_{n\to\infty}\frac{1}{2^n}\left\|f(2^n x^*) - f(2^n x)^*\right\|_B \le \lim_{n\to\infty}\frac{1}{2^n}\varphi(2^n x, 2^n x, 2^n x) = 0 \tag{2.22}$$

for all $x \in A$. So

$$H(x^*) = H(x)^* \tag{2.23}$$

for all $x \in A$.

Thus, $H : A \to B$ is a C^*-algebra homomorphism satisfying (2.6), as desired. □

Corollary 2.2. *Let $r < 1$ and θ be nonnegative real numbers, and let $f : A \to B$ be a mapping such that*

$$\begin{gathered}\left\|C_\mu f(x, y, z)\right\|_B \le \theta\left(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r\right),\\ \left\|f(xy) - f(x)f(y)\right\|_B \le \theta\left(\|x\|_A^r + \|y\|_A^r\right),\\ \left\|f(x^*) - f(x)^*\right\|_B \le 3\theta\|x\|_A^r\end{gathered} \tag{2.24}$$

for all $\mu \in \mathbb{T}^1$ and all $x, y, z \in A$. Then, there exists a unique C^*-algebra homomorphism $H : A \to B$ such that

$$\left\|f(x) - H(x)\right\|_B \le \frac{3\theta}{4 - 2^{r+1}}\|x\|_A^r \tag{2.25}$$

for all $x \in A$.

Proof. The proof follows from Theorem 2.1 by taking

$$\varphi(x, y, z) := \theta\left(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r\right) \tag{2.26}$$

for all $x, y, z \in A$. Then, $L = 2^{r-1}$ and we get the desired result. □

Theorem 2.3. *Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A^3 \to [0, \infty)$ satisfying* (2.3)*,* (2.4)*, and* (2.5) *such that*

$$\lim_{j\to\infty} 4^j \varphi\left(\frac{x}{2^j}, \frac{y}{2^j}, \frac{z}{2^j}\right) = 0 \tag{2.27}$$

for all $x, y, z \in A$. If there exists an $L < 1$ such that $\varphi(x, x, x) \le (1/2)L\varphi(2x, 2x, 2x)$ for all $x \in A$, then there exists a unique C^*-algebra homomorphism $H : A \to B$ such that

$$\left\|f(x) - H(x)\right\|_B \le \frac{L}{4 - 4L}\varphi(x, x, x) \tag{2.28}$$

for all $x \in A$.

Proof. We consider the linear mapping $J : X \to X$ such that

$$Jg(x) := 2g\left(\frac{x}{2}\right) \tag{2.29}$$

for all $x \in A$.

It follows from (2.11) that

$$\left\| f(x) - 2f\left(\frac{x}{2}\right) \right\|_B \le \frac{1}{2}\varphi\left(\frac{x}{2}, \frac{x}{2}, \frac{x}{2}\right) \le \frac{L}{4}\varphi(x, x, x) \tag{2.30}$$

for all $x \in A$. Hence $d(f, Jf) \le L/4$.

By Theorem 1.4, there exists a mapping $H : A \to B$ such that the following conditions hold.

(1) H is a fixed point of J, that is,

$$H(2x) = 2H(x) \tag{2.31}$$

for all $x \in A$. The mapping H is a unique fixed point of J in the set

$$Y = \{g \in X : d(f, g) < \infty\}. \tag{2.32}$$

This implies that H is a unique mapping satisfying (2.31) such that there exists $C \in (0, \infty)$ satisfying

$$\|H(x) - f(x)\|_B \le C\varphi(x, x, x) \tag{2.33}$$

for all $x \in A$.

(2) $d(J^n f, H) \to 0$ as $n \to \infty$. This implies the equality

$$\lim_{n\to\infty} 2^n f\left(\frac{x}{2^n}\right) = H(x) \tag{2.34}$$

for all $x \in A$.

(3) $d(f, H) \le (1/(1-L))d(f, Jf)$, which implies the inequality

$$d(f, H) \le \frac{L}{4 - 4L}, \tag{2.35}$$

which implies that inequality (2.28) holds.

The rest of the proof is similar to the proof of Theorem 2.1. □

Corollary 2.4. *Let $r > 2$, let θ be nonnegative real numbers, and let $f : A \to B$ be a mapping satisfying* (2.24). *Then, there exists a unique C^*-algebra homomorphism $H : A \to B$ such that*

$$\|f(x) - H(x)\|_B \le \frac{3\theta}{2^{r+1} - 4}\|x\|_A^r \tag{2.36}$$

for all $x \in A$.

Proof. The proof follows from Theorem 2.3 by taking

$$\varphi(x, y, z) := \theta\left(\|x\|_A^r + \|y\|_A^r + \|z\|_A^r\right) \tag{2.37}$$

for all $x, y, z \in A$. Then, $L = 2^{1-r}$ and we get the desired result. □

3. Stability of generalized derivations on C^*-algebras

For a given mapping $f : A \to A$, we define

$$C_\mu f(x,y,z) := 2\mu f\left(\frac{x+y}{2}+z\right) - f(\mu x) - f(\mu y) - 2f(\mu z) \tag{3.1}$$

for all $\mu \in \mathbb{T}^1$ and all $x,y,z \in A$.

Definition 3.1 (see [25]). A generalized derivation $\delta : A \to A$ is involutive $\mathbb{C}$-linear and fulfills

$$\delta(xyz) = \delta(xy)z - x\delta(y)z + x\delta(yz) \tag{3.2}$$

for all $x,y,z \in A$.

We prove the Hyers-Ulam-Rassias stability of derivations on C^*-algebras for the functional equation $C_\mu f(x,y,z) = 0$.

Theorem 3.2. *Let $f : A \to A$ be a mapping for which there exists a function $\varphi : A^3 \to [0,\infty)$ satisfying* (2.2) *such that*

$$\left\|C_\mu f(x,y,z)\right\|_A \le \varphi(x,y,z), \tag{3.3}$$

$$\left\|f(xyz) - f(xy)z + xf(y)z - xf(yz)\right\|_A \le \varphi(x,y,z), \tag{3.4}$$

$$\left\|f(x^*) - f(x)^*\right\|_A \le \varphi(x,x,x) \tag{3.5}$$

for all $\mu \in \mathbb{T}^1$ and all $x,y,z \in A$. If there exists an $L < 1$ such that $\varphi(x,x,x) \le 2L\varphi(x/2,x/2,x/2)$ for all $x \in A$, then there exists a unique generalized derivation $\delta : A \to A$ such that

$$\left\|f(x) - \delta(x)\right\|_A \le \frac{1}{4-4L}\varphi(x,x,x) \tag{3.6}$$

for all $x \in A$.

Proof. By the same reasoning as the proof of Theorem 2.1, there exists a unique involutive $\mathbb{C}$-linear mapping $\delta : A \to A$ satisfying (3.6). The mapping $\delta : A \to A$ is given by

$$\delta(x) = \lim_{n\to\infty}\frac{f(2^n x)}{2^n} \tag{3.7}$$

for all $x \in A$.

It follows from (3.4) that

$$\begin{aligned} &\left\|\delta(xyz) - \delta(xy)z + x\delta(y)z - x\delta(yz)\right\|_A \\ &\quad = \lim_{n\to\infty}\frac{1}{8^n}\left\|f(8^n xyz) - f(4^n xy)\cdot 2^n z + 2^n x f(2^n y)\cdot 2^n z - 2^n x f(4^n yz)\right\|_A \\ &\quad \le \lim_{n\to\infty}\frac{1}{8^n}\varphi(2^n x, 2^n y, 2^n z) \le \lim_{n\to\infty}\frac{1}{2^n}\varphi(2^n x, 2^n y, 2^n z) = 0 \end{aligned} \tag{3.8}$$

for all $x,y,z \in A$. So

$$\delta(xyz) = \delta(xy)z - x\delta(y)z + x\delta(yz) \tag{3.9}$$

for all $x,y,z \in A$. Thus, $\delta : A \to A$ is a generalized derivation satisfying (3.6). □

Corollary 3.3. *Let r <1, Let θ be nonnegative real numbers, and let $f : A \to A$ be a mapping such that*

$$\begin{gathered}\|C_\mu f(x,y,z)\|_A \le \theta\cdot\|x\|_A^{r/3}\cdot\|y\|_A^{r/3}\cdot\|z\|_A^{r/3},\\ \|f(xyz)-f(xy)z+xf(y)z-xf(yz)\|_A \le \theta\cdot\|x\|_A^{r/3}\cdot\|y\|_A^{r/3}\cdot\|z\|_A^{r/3},\\ \|f(x^*)-f(x)^*\|_A \le \theta\cdot\|x\|_A^r\end{gathered} \tag{3.10}$$

for all $\mu \in \mathbb{T}^1$ and all $x, y, z \in A$. Then, there exists a unique generalized derivation $\delta : A \to A$ such that

$$\|f(x)-\delta(x)\|_A \le \frac{\theta}{4-2^{r+1}}\|x\|_A^r \tag{3.11}$$

for all $x \in A$.

Proof. The proof follows from Theorem 3.2 by taking

$$\varphi(x,y,z) := \theta\cdot\|x\|_A^{r/3}\cdot\|y\|_A^{r/3}\cdot\|z\|_A^{r/3} \tag{3.12}$$

for all $x, y, z \in A$. Then, $L = 2^{r-1}$ and we get the desired result. □

Theorem 3.4. *Let $f : A \to A$ be a mapping for which there exists a function $\varphi : A^3 \to [0,\infty)$ satisfying* (3.3), (3.4), *and* (3.5) *such that*

$$\lim_{j\to\infty} 8^j \varphi\left(\frac{x}{2^j},\frac{y}{2^j},\frac{z}{2^j}\right) = 0 \tag{3.13}$$

for all $x, y, z \in A$. If there exists an $L < 1$ such that $\varphi(x,x,x) \le (1/2)L\varphi(2x,2x,2x)$ for all $x \in A$, then there exists a unique generalized derivation $\delta : A \to A$ such that

$$\|f(x)-\delta(x)\|_A \le \frac{L}{4-4L}\varphi(x,x,x) \tag{3.14}$$

for all $x \in A$.

Proof. The proof is similar to the proofs of Theorems 2.3 and 3.2. □

Corollary 3.5. *Let $r > 3$, let θ be nonnegative real numbers, and let $f : A \to A$ be a mapping satisfying* (3.10). *Then, there exists a unique generalized derivation $\delta : A \to A$ such that*

$$\|f(x)-\delta(x)\|_A \le \frac{\theta}{2^{r+1}-4}\|x\|_A^r \tag{3.15}$$

for all $x \in A$.

Proof. The proof follows from Theorem 3.4 by taking

$$\varphi(x,y,z) := \theta\cdot\|x\|_A^{r/3}\cdot\|y\|_A^{r/3}\cdot\|z\|_A^{r/3} \tag{3.16}$$

for all $x, y, z \in A$. Then, $L = 2^{1-r}$ and we get the desired result. □

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