

EXISTENCE OF ZEROS FOR OPERATORS TAKING THEIR VALUES IN THE DUAL OF A BANACH SPACE

BIAGIO RICCERI

Received 14 October 2003 and in revised form 21 April 2004

To Professor Giuseppe Pulvirenti, with affection, on his seventieth birthday

Using continuous selections, we establish some existence results about the zeros of weakly continuous operators from a paracompact topological space into the dual of a reflexive real Banach space.

Throughout the sequel, E denotes a reflexive real Banach space and E^* its topological dual. We also assume that E is locally uniformly convex. This means that for each $x \in E$, with $\|x\| = 1$, and each $\epsilon > 0$, there exists $\delta > 0$ such that, for every $y \in E$ satisfying $\|y\| = 1$ and $\|x - y\| \geq \epsilon$, one has $\|x + y\| \leq 2(1 - \delta)$. Recall that any reflexive Banach space admits an equivalent norm with which it is locally uniformly convex [1, page 289]. For $r > 0$, we set $B_r = \{x \in E : \|x\| \leq r\}$.

Moreover, we fix a topology τ on E , weaker than the strong topology and stronger than the weak topology, such that (E, τ) is a Hausdorff locally convex topological vector space with the property that the τ -closed convex hull of any τ -compact subset of E is still τ -compact and the relativization of τ to B_1 is metrizable by a complete metric. In practice, the most usual choice of τ is either the strong topology or the weak topology provided E is also separable.

The aim of this short paper is to establish the following result and present some of its consequences.

THEOREM 1. *Let X be a paracompact topological space and $A : X \rightarrow E^*$ a weakly continuous operator. Assume that there exist a number $r > 0$, a continuous function $\alpha : X \rightarrow \mathbb{R}$ satisfying*

$$|\alpha(x)| \leq r \|A(x)\|_{E^*} \quad (1)$$

for all $x \in X$, a (possibly empty) closed set $C \subset X$, and a τ -continuous function $g : C \rightarrow B_r$ satisfying

$$A(x)(g(x)) = \alpha(x) \quad (2)$$

for all $x \in C$, in such a way that, for every τ -continuous function $\psi : X \rightarrow B_r$ satisfying $\psi|_C = g$, there exists $x_0 \in X$ such that

$$A(x_0)(\psi(x_0)) \neq \alpha(x_0). \quad (3)$$

Then, there exists $x^* \in X$ such that $A(x^*) = 0$.

For the reader's convenience, we recall that a multifunction $F : S \rightarrow 2^V$, between topological spaces, is said to be lower semicontinuous at $s_0 \in S$ if, for every open set $\Omega \subseteq V$ meeting $F(s_0)$, there is a neighborhood U of s_0 such that $F(s) \cap \Omega \neq \emptyset$ for all $s \in U$. F is said to be lower semicontinuous if it is so at each point of S .

The following well-known results will be our main tools.

THEOREM 2 [3]. *Let X be a paracompact topological space and $F : X \rightarrow 2^{B_1}$ a τ -lower semicontinuous multifunction with nonempty τ -closed convex values.*

Then, for each closed set $C \subset X$ and each τ -continuous function $g : C \rightarrow B_1$ satisfying $g(x) \in F(x)$ for all $x \in C$, there exists a τ -continuous function $\psi : X \rightarrow B_1$ such that $\psi|_C = g$ and $\psi(x) \in F(x)$ for all $x \in X$.

THEOREM 3 [4]. *Let X, Y be two topological spaces, with Y connected and locally connected, and let $f : X \times Y \rightarrow \mathbb{R}$ be a function satisfying the following conditions:*

- (a) *for each $x \in X$, the function $f(x, \cdot)$ is continuous, changes sign in Y , and is identically zero in no nonempty open subset of Y ;*
- (b) *the set $\{(y, z) \in Y \times Y : \{x \in X : f(x, y) < 0 < f(x, z)\} \text{ is open in } X\}$ is dense in $Y \times Y$.*

Then, the multifunction $x \rightarrow \{y \in Y : f(x, y) = 0 \text{ and } y \text{ is not a local extremum for } f(x, \cdot)\}$ is lower semicontinuous and its values are nonempty and closed.

Proof of Theorem 1. Arguing by contradiction, assume that $A(x) \neq 0$ for all $x \in X$. For each $x \in X, y \in B_1$, put

$$\begin{aligned} f(x, y) &= A(x)(y) - \frac{\alpha(x)}{r}, \\ F(x) &= \{z \in B_1 : f(x, z) = 0\}. \end{aligned} \quad (4)$$

Also, set

$$X_0 = \{x \in X : |\alpha(x)| < r\|A(x)\|_{E^*}\}. \quad (5)$$

Since A is weakly continuous, the function $x \rightarrow \|A(x)\|_{E^*}$, as a supremum of a family of continuous functions, is lower semicontinuous. From this, it follows that the set X_0 is open. For each $x \in X_0$, the function $f(x, \cdot)$ is continuous and has no local, nonabsolute extrema, being affine. Moreover, it changes sign in B_1 since $A(x)(B_1) = [-\|A(x)\|_{E^*}, \|A(x)\|_{E^*}]$ (recall that E is reflexive). Since $f(\cdot, y)$ is continuous for all $y \in B_1$, we then realize that the restriction of f to $X_0 \times B_1$ satisfies the hypotheses of Theorem 3, B_1 being considered with the relativization of the strong topology. Hence, the multifunction $F|_{X_0}$ is lower semicontinuous. Consequently, since X_0 is open, the multifunction F is lower

semicontinuous at each point of X_0 . Now, fix $x_0 \in X \setminus X_0$. So, $|\alpha(x_0)| = r\|A(x_0)\|_{E^*}$. Let $y_0 \in F(x_0)$ and $\epsilon > 0$. Clearly, since y_0 is an absolute extremum of $A(x_0)$ in B_1 , one has $\|y_0\| = 1$. Choose $\delta > 0$ so that, for each $y \in E$ satisfying $\|y\| = 1$ and $\|y - y_0\| \geq \epsilon$, one has $\|y + y_0\| \leq 2(1 - \delta)$. By semicontinuity, the function $x \rightarrow (\|A(x)\|_{E^*})^{-1}$ is bounded in some neighborhood of x_0 , and so, since the functions α and $A(\cdot)(y_0)$ are continuous, it follows that

$$\lim_{x \rightarrow x_0} \frac{|A(x)(y_0) - \alpha(x)/r|}{\|A(x)\|_{E^*}} = 0. \quad (6)$$

So, there is a neighborhood U of x_0 such that

$$\frac{|A(x)(y_0) - \alpha(x)/r|}{\|A(x)\|_{E^*}} < \frac{\epsilon\delta}{2} \quad (7)$$

for all $x \in U$. Fix $x \in U$. Pick $z \in E$, with $\|z\| = 1$, in such a way that $|A(x)(z)| = \|A(x)\|_{E^*}$ and

$$\left(A(x)(z) - \frac{\alpha(x)}{r}\right) \left(A(x)(y_0) - \frac{\alpha(x)}{r}\right) \leq 0. \quad (8)$$

From this choice, it follows, of course, that the segment joining y_0 and z meets the hyperplane $(A(x))^{-1}(\alpha(x)/r)$. In other words, there is $\lambda \in [0, 1]$ such that

$$A(x)(\lambda z + (1 - \lambda)y_0) = \frac{\alpha(x)}{r}. \quad (9)$$

So, if we put $y = \lambda z + (1 - \lambda)y_0$, we have $y \in F(x)$ and

$$\|y - y_0\| = \lambda\|z - y_0\|. \quad (10)$$

We claim that $\|y - y_0\| < \epsilon$. This follows at once from (10) if $\lambda < \epsilon/2$. Thus, assume $\lambda \geq \epsilon/2$. In this case, to prove our claim, it is enough to show that

$$2(1 - \delta) < \|z + y_0\| \quad (11)$$

since (11) implies $\|z - y_0\| < \epsilon$. To this end, note that by (9), one has

$$\frac{|A(x)(y_0) - \alpha(x)/r|}{\|A(x)\|_{E^*}} = \frac{\lambda|A(x)(z - y_0)|}{\|A(x)\|_{E^*}}, \quad (12)$$

and so, from (7), it follows that

$$\frac{|A(x)(z - y_0)|}{\|A(x)\|_{E^*}} < \delta. \quad (13)$$

Suppose $A(x)(z) = \|A(x)\|_{E^*}$. Then, from (13), we get

$$1 - \delta < \frac{A(x)(y_0)}{\|A(x)\|_{E^*}}. \quad (14)$$

On the other hand, we also have

$$1 + \frac{A(x)(y_0)}{\|A(x)\|_{E^*}} = \frac{A(x)(z + y_0)}{\|A(x)\|_{E^*}} \leq \|z + y_0\|. \quad (15)$$

So, (11) follows from (14) and (15). Now, suppose $A(x)(z) = -\|A(x)\|_{E^*}$. Then, from (13), we get

$$1 - \delta < -\frac{A(x)(y_0)}{\|A(x)\|_{E^*}}. \quad (16)$$

On the other hand, we have

$$1 - \frac{A(x)(y_0)}{\|A(x)\|_{E^*}} = -\frac{A(x)(z + y_0)}{\|A(x)\|_{E^*}} \leq \|z + y_0\|. \quad (17)$$

So, in the present case, (11) is a consequence of (16) and (17). In such a manner, we have proved that F is lower semicontinuous at x_0 . Hence, it remains proved that F is lower semicontinuous in X with respect to the strong topology and so, a fortiori, with respect to τ . Since F is also with nonempty τ -closed convex values and g/r is a τ -continuous selection of it over the closed set C , by Theorem 2, F admits a τ -continuous selection ω in X such that $\omega|_C = g/r$. At this point, if we put $\psi = r\omega$, it follows that ψ is a τ -continuous function, from X into B_r , such that $\psi|_C = g$ and $A(x)(\psi(x)) = \alpha(x)$ for all $x \in X$, against the hypotheses. This concludes the proof. $\square$

Remark 4. From the proof, it clearly follows that if the assumption $|\alpha(x)| \leq r\|A(x)\|_{E^*}$ for all $x \in X$ is replaced by the more restrictive $|\alpha(x)| < r\|A(x)\|_{E^*}$ for all $x \in X \setminus A^{-1}(0)$, then the restrictions made on E and its norm become superfluous and, furthermore, the continuity assumption on A can be weakened to supposing that the function $x \rightarrow A(x)(y)$ is continuous for each y in a dense subset of E . Likewise, essentially the same proof gives the following version of Theorem 1, for $r = \infty$.

THEOREM 5. *Let X be a paracompact topological space, Y a real Banach space, and $A : X \rightarrow Y^*$ an operator such that the set*

$$\{y \in Y : x \rightarrow A(x)(y) \text{ is continuous}\} \quad (18)$$

is dense in Y . Assume that there exist a continuous function $\alpha : X \rightarrow \mathbb{R}$, a (possibly empty) closed set $C \subset X$, and a continuous function $g : C \rightarrow Y$ satisfying $A(x)(g(x)) = \alpha(x)$ for all $x \in C$, in such a way that, for every continuous function $\psi : X \rightarrow Y$ satisfying $\psi|_C = g$, there exists $x_0 \in X$ such that $A(x_0)(\psi(x_0)) \neq \alpha(x_0)$. Then, there exists $x^ \in X$ such that $A(x^*) = 0$.*

Sketch of proof. Arguing by contradiction, assume that $A^{-1}(0) = \emptyset$. For each $x \in X$, put

$$F(x) = \{y \in Y : A(x)(y) = \alpha(x)\}. \quad (19)$$

Thanks to Theorem 3, the multifunction F is lower semicontinuous. Since F is also with nonempty closed convex values and g is a continuous selection of it over the closed set C ,

by Michael's theorem, F admits a continuous selection ψ in X such that $\psi|_C = g$, against the hypotheses. $\square$

We now point out an interesting alternative coming from [Theorem 5](#). The spaces $C^0(X, Y)$ and $C^0(X)$ that will appear are considered with the sup-norm. We recall that a subset D of a topological space S is a retract of S if there exists a continuous function $h : S \rightarrow D$ such that $h(s) = s$ for all $s \in D$.

THEOREM 6. *Let X be a compact Hausdorff topological space, Y a real Banach space, with $\dim(Y) \geq 2$, and $A : X \rightarrow Y^*$ a continuous operator.*

Then, at least one of the following assertions holds:

- (a) *there exists $x^* \in X$ such that $A(x^*) = 0$;*
- (b) *there exists $\epsilon > 0$ such that, for every Lipschitzian operator $J : C^0(X, Y) \rightarrow C^0(X)$, with Lipschitz constant less than ϵ , the set*

$$\{\psi \in C^0(X, Y) : A(x)(\psi(x)) = J(\psi)(x) \quad \forall x \in X\} \quad (20)$$

is an unbounded retract of $C^0(X, Y)$.

Proof. Assume that $A(x) \neq 0$ for all $x \in X$. For each $\psi \in C^0(X, Y)$ and $x \in X$, put

$$T(\psi)(x) = A(x)(\psi(x)). \quad (21)$$

Since A is continuous and bounded (due to the compactness of X), the function $T(\psi)(\cdot)$ is continuous (see the proof of [Theorem 12](#)). So, T turns out to be a continuous linear operator from $C^0(X, Y)$ into $C^0(X)$. Due to [Theorem 5](#) (applied taking $C = \emptyset$), $A^{-1}(0) \neq \emptyset$ if (and only if) the operator T is not surjective. Thus, since we are supposing that $A^{-1}(0) = \emptyset$, the operator T is surjective. Furthermore, note that T is not injective. Indeed, if we fix any $x_0 \in X$ and choose $y_0 \in Y \setminus \{0\}$ so that $A(x_0)(y_0) = 0$ (recall that $\dim(Y) \geq 2$), by [Theorem 5](#) again (applied taking $C = \{x_0\}$), there is $\psi \in C^0(X, Y)$ such that $T(\psi) = 0$ and $\psi(x_0) = y_0$. Finally, set

$$\epsilon = \frac{1}{\sup_{\|\varphi\|_{C^0(X)} \leq 1} \text{dist}(0, T^{-1}(\varphi))}. \quad (22)$$

Due to this choice, by [\[5, Théorème 2\]](#), for every Lipschitzian operator $J : C^0(X, Y) \rightarrow C^0(X)$, with Lipschitz constant less than ϵ , the set

$$\Gamma := \{\psi \in C^0(X, Y) : T(\psi) = J(\psi)\} \quad (23)$$

turns out to be a retract of $C^0(X, Y)$. Moreover, from the proof of [\[5, Théorème 2\]](#), it follows that the multifunction $\psi \rightarrow T^{-1}(J(\psi))$ is a multivalued contraction, and so, since its values are closed and unbounded, the set of its fixed points (which agrees with Γ) is unbounded too by [\[7, Corollary 9\]](#). $\square$

We now indicate two reasonable ways to apply [Theorem 1](#). The first one is based on the Tychonoff fixed point theorem.

THEOREM 7. Assume that E is a separable Hilbert space with inner product $\langle \cdot, \cdot \rangle$. Let $r > 0$ and let $A : B_r \rightarrow E$ be a continuous operator from the weak to the strong topology. Assume that there exist a weakly continuous function $\alpha : B_r \rightarrow \mathbb{R}$ satisfying $|\alpha(x)| \leq r\|A(x)\|$ for all $x \in B_r$, and a weakly continuous function $g : C \rightarrow B_r$ such that

$$\langle A(x), g(x) \rangle = \alpha(x), \quad g(x) \neq x, \quad (24)$$

for all $x \in C$, where

$$C = \{x \in B_r : \langle A(x), x \rangle = \alpha(x)\}. \quad (25)$$

Then, there exists $x^* \in B_r$ such that $A(x^*) = 0$.

Proof. Identifying E with E^* , we apply [Theorem 1](#) taking $X = B_r$, with the relativization of the weak topology of E , and taking τ as the weak topology of E . Due to the kind of continuity we are assuming for A , the function $x \rightarrow \langle A(x), x \rangle$ turns out to be weakly continuous (see the proof of [Theorem 12](#)), and so the set C is weakly closed. Now, let $\psi : B_r \rightarrow B_r$ be any weakly continuous function such that $\psi|_C = g$. By the Tychonoff fixed point theorem, there is $x_0 \in B_r$ such that $\psi(x_0) = x_0$. Since g has no fixed points in C , it follows that $x_0 \notin C$, and so

$$\langle A(x_0), \psi(x_0) \rangle = \langle A(x_0), x_0 \rangle \neq \alpha(x_0). \quad (26)$$

Hence, all the assumptions of [Theorem 1](#) are satisfied and the conclusion follows from it. $\square$

It is worth noticing the following consequences of [Theorem 7](#).

THEOREM 8. Let E and A be as in [Theorem 7](#). Assume that for each $x \in B_r$, with $\|A(x)\| > r$,

$$\left\| A \left(\frac{rA(x)}{\|A(x)\|} \right) \right\| \leq r. \quad (27)$$

Then, the operator A has either a zero or a fixed point.

Proof. Define the function $\alpha : B_r \rightarrow \mathbb{R}$ by

$$\alpha(x) = \begin{cases} \|A(x)\|^2 & \text{if } \|A(x)\| \leq r, \\ r\|A(x)\| & \text{if } \|A(x)\| > r. \end{cases} \quad (28)$$

Clearly, the function α is weakly continuous and satisfies $|\alpha(x)| \leq r\|A(x)\|$ for all $x \in B_r$. Put $C = \{x \in B_r : \langle A(x), x \rangle = \alpha(x)\}$. Note that if $x \in C$, then $\|A(x)\| \leq r$. Indeed, otherwise, we would have $\langle A(x), x \rangle = r\|A(x)\|$, and so, necessarily, $x = rA(x)/\|A(x)\|$, against [\(27\)](#). Hence, we have $\langle A(x), A(x) \rangle = \alpha(x)$ for all $x \in C$. At this point, the conclusion follows at once from [Theorem 7](#), taking $g = A|_C$. $\square$

Remark 9. It would be interesting to know whether [Theorem 8](#) can be improved assuming that A is a compact operator (i.e., continuous and with relatively compact range).

Remark 10. Note that [Theorem 8](#) can be compared with the classical Rothe's theorem which assures the existence of a fixed point of A provided that it is compact and maps ∂B_r into B_r . [Theorem 8](#) tells us that, under a more severe continuity assumption (see, however, [Remark 9](#)) and the condition $A^{-1}(0) = \emptyset$, the key Rothe's condition can be remarkably weakened to

$$A\left(\bigcup_{\lambda>0} \lambda A(A^{-1}(E \setminus B_r)) \cap \partial B_r\right) \subseteq B_r. \quad (29)$$

THEOREM 11. *Let E and A be as in [Theorem 7](#). Assume that there exists $w \in B_r$, with $\langle A(w), w \rangle \neq 0$, such that $\langle A(x), w \rangle = 0$ for all $x \in B_r$ satisfying $\langle A(x), x \rangle = 0$.*

Then, there exists $x^ \in B_r$ such that $A(x^*) = 0$.*

Proof. Apply [Theorem 7](#) taking $\alpha(x) = 0$ and $g(x) = w$ for all $x \in B_r$. $\square$

The second application of [Theorem 1](#) is based on connectedness arguments. For other results of this type, we refer to [6] (see also [2]).

THEOREM 12. *Let X be a connected paracompact topological space and $A : X \rightarrow E^*$ a weakly continuous and locally bounded operator. Assume that there exist $r > 0$, a closed set $C \subset X$, a continuous function $g : C \rightarrow B_r$, and an upper semicontinuous function $\beta : X \rightarrow \mathbb{R}$, with $|\beta(x)| \leq r\|A(x)\|_{E^*}$ for all $x \in X$, such that $g(C)$ is disconnected,*

$$\beta(x) \leq A(x)(g(x)) \quad (30)$$

for all $x \in C$, and

$$A(x)(y) < \beta(x) \quad (31)$$

for all $x \in X \setminus C$ and $y \in B_r \setminus g(C)$.

Then, there exists $x^ \in C$ such that $A(x^*) = 0$.*

Proof. First, note that the function $x \rightarrow A(x)(g(x))$ is continuous in C . To see this, let $x_1 \in C$ and let $\{x_\gamma\}_{\gamma \in D}$ be any net in C converging to x_1 . By assumption, there are $M > 0$ and a neighborhood U of x_1 such that $\|A(x)\|_{E^*} \leq M$ for all $x \in U$. Let $\gamma_0 \in D$ be such that $x_\gamma \in U$ for all $\gamma \geq \gamma_0$. Thus, for each $\gamma \geq \gamma_0$, one has

$$\begin{aligned} & |A(x_\gamma)(g(x_\gamma)) - A(x_1)(g(x_1))| \\ & \leq M\|g(x_\gamma) - g(x_1)\| + |A(x_\gamma)(g(x_1)) - A(x_1)(g(x_1))| \end{aligned} \quad (32)$$

from which, of course, it follows that $\lim_\gamma A(x_\gamma)(g(x_\gamma)) = A(x_1)(g(x_1))$. Next, observe that the multifunction $x \rightarrow [\beta(x), r\|A(x)\|_{E^*}]$ is lower semicontinuous and that the function $x \rightarrow A(x)(g(x))$ is a continuous selection of it in C . Hence, by Michael's theorem, there is a continuous function $\alpha : X \rightarrow \mathbb{R}$ such that $\alpha(x) = A(x)(g(x))$ for all $x \in C$ and $\beta(x) \leq \alpha(x) \leq r\|A(x)\|_{E^*}$ for all $x \in X$. Now, let $\psi : X \rightarrow B_r$ be any continuous function such that $\psi|_C = g$. Since X is connected, $\psi(X)$ is connected too. But then, since $g(C)$ is disconnected and $g(C) \subset \psi(X)$, there exists $y_0 \in \psi(X) \setminus g(C)$. Let $x_0 \in X \setminus C$ be such that $\psi(x_0) = y_0$.

So, by hypothesis, we have

$$A(x_0)(\psi(x_0)) = A(x_0)(y_0) < \beta(x_0) \leq \alpha(x_0). \quad (33)$$

Hence, taking τ as the strong topology of E , all the assumptions of [Theorem 1](#) are satisfied and the conclusion follows from it. $\square$

Remark 13. Observe that when X is first-countable, the local boundedness of A follows automatically from its weak continuity. This follows from the fact that, in a Banach space, any weakly convergent sequence is bounded.

It is worth noticing the corollary of [Theorem 12](#) which comes out taking $X = B_r$, $\beta = 0$, and $g = \text{identity}$.

THEOREM 14. *Let E be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$. Let $r > 0$ and let $A : B_r \rightarrow E$ be a continuous operator from the strong to the weak topology. Assume that the set $C = \{x \in B_r : \langle A(x), x \rangle \geq 0\}$ is disconnected and that, for each $x, y \in B_r \setminus C$, $\langle A(x), y \rangle < 0$.*

Then, there exists $x^ \in C$ such that $A(x^*) = 0$.*

Acknowledgment

The author wishes to thank Prof. J. Saint Raymond for useful correspondence.

References

- [1] R. Deville, G. Godefroy, and V. Zizler, *Smoothness and Renormings in Banach Spaces*, Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 64, Longman Scientific & Technical, Harlow, 1993.
- [2] A. J. B. Lopes-Pinto, *On a new result on the existence of zeros due to Ricceri*, J. Convex Anal. **5** (1998), no. 1, 57–62.
- [3] E. Michael, *A selection theorem*, Proc. Amer. Math. Soc. **17** (1966), 1404–1406.
- [4] B. Ricceri, *Applications de théorèmes de semi-continuité inférieure [Applications of lower semi-continuity theorems]*, C. R. Acad. Sci. Paris Sér. I Math. **295** (1982), no. 2, 75–78 (French).
- [5] ———, *Structure, approximation et dépendance continue des solutions de certaines équations non linéaires [Structure, approximation and continuous dependence of the solutions of certain nonlinear equations]*, C. R. Acad. Sci. Paris Sér. I Math. **305** (1987), no. 2, 45–47 (French).
- [6] ———, *Existence of zeros via disconnectedness*, J. Convex Anal. **2** (1995), no. 1-2, 287–290.
- [7] J. Saint Raymond, *Multivalued contractions*, Set-Valued Anal. **2** (1994), no. 4, 559–571.

Biagio Ricceri: Department of Mathematics, University of Catania, 95125 Catania, Italy
E-mail address: ricceri@dmf.unict.it

Special Issue on Intelligent Computational Methods for Financial Engineering

Call for Papers

As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods:** artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning

- **Application fields:** asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects:** decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

Authors should follow the Journal of Applied Mathematics and Decision Sciences manuscript format described at the journal site <http://www.hindawi.com/journals/jamds/>. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at <http://mts.hindawi.com/>, according to the following timetable:

Manuscript Due	December 1, 2008
First Round of Reviews	March 1, 2009
Publication Date	June 1, 2009

Guest Editors

Lean Yu, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; yulean@amss.ac.cn

Shouyang Wang, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; sywang@amss.ac.cn

K. K. Lai, Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; mskkklai@cityu.edu.hk