

MULTIVALUED p -LIENARD SYSTEMS

MICHAEL E. FILIPPAKIS AND NIKOLAOS S. PAPAGEORGIOU

Received 7 October 2003 and in revised form 9 March 2004

We examine p -Lienard systems driven by the vector p -Laplacian differential operator and having a multivalued nonlinearity. We consider Dirichlet systems. Using a fixed point principle for set-valued maps and a nonuniform nonresonance condition, we establish the existence of solutions.

1. Introduction

In this paper, we use fixed point theory to study the following multivalued p -Lienard system:

$$\begin{aligned} (||x'(t)||^{p-2}x'(t))' + \frac{d}{dt}\nabla G(x(t)) + F(t, x(t), x'(t)) \ni 0 \quad \text{a.e. on } T = [0, b], \\ x(0) = x(b) = 0, \quad 1 < p < \infty. \end{aligned} \quad (1.1)$$

In the last decade, there have been many papers dealing with second-order multivalued boundary value problems. We mention the works of Erbe and Krawcewicz [5, 6], Frigon [7, 8], Halidias and Papageorgiou [9], Kandilakis and Papageorgiou [11], Kyritsi et al. [12], Palmucci and Papalini [17], and Pruszek [19]. In all the above works, with the exception of Kyritsi et al. [12], $p = 2$ (linear differential operator), $G = 0$, and $g = 0$. Moreover, in Frigon [7, 8] and Palmucci and Papalini [17], the inclusions are scalar (i.e., $N = 1$). Finally we should mention that recently single-valued p -Lienard systems were studied by Mawhin [14] and Manásevich and Mawhin [13].

In this work, for problem (1.1), we prove an existence theorem under conditions of nonuniform nonresonance with respect to the first weighted eigenvalue of the negative vector ordinary p -Laplacian with Dirichlet boundary conditions [15, 20]. Our approach is based on the multivalued version of the Leray-Schauder alternative principle due to Bader [1] (see Section 2).

2. Mathematical background

In this section, we recall some basic definitions and facts from multivalued analysis, the spectral properties of the negative vector p -Laplacian, and the multivalued fixed point principles mentioned in the introduction. For details, we refer to Denkowski et al. [3] and Hu and Papageorgiou [10] (for multivalued analysis), to Denkowski et al. [2] and Zhang [20] (for the spectral properties of the p -Laplacian), and to Bader [1] (for the multivalued fixed point principle; similar results can also be found in O'Regan and Precup [16] and Precup [18]).

Let (Ω, Σ) be a measurable space and X a separable Banach space. We introduce the following notations:

$$\begin{aligned} P_{f(c)}(X) &= \{A \subseteq X : \text{nonempty, closed (and convex)}\}, \\ P_{(w)k(c)}(X) &= \{A \subseteq X : \text{nonempty, (weakly) compact (and convex)}\}. \end{aligned} \quad (2.1)$$

A multifunction $F : \Omega \rightarrow P_f(X)$ is said to be measurable if, for all $x \in X$, $\omega \rightarrow d(x, F(\omega)) = \inf [\|x - y\| : y \in F(\omega)]$ is measurable. A multifunction $F : \Omega \rightarrow 2^X \setminus \{\emptyset\}$ is said to be “graph measurable” if $\text{Gr}F = \{(\omega, x) \in \Omega \times X : x \in F(\omega)\} \in \Sigma \times B(X)$, with $B(X)$ being the Borel σ -field of X . For $P_f(X)$ -valued multifunctions, measurability implies graph measurability and the converse is true if Σ is complete (i.e., $\Sigma = \hat{\Sigma}$ = the universal σ -field). Let μ be a finite measure on (Ω, Σ) , $1 \leq p \leq \infty$, and $F : \Omega \rightarrow 2^X \setminus \{\emptyset\}$. We introduce the set $S_F^p = \{f \in L^p(\Omega, X) : f(\omega) \in F(\omega) \text{ } \mu\text{-a.e.}\}$. This set may be empty. For a graph-measurable multifunction, it is nonempty if and only if $\inf [\|y\| : y \in F(\omega)] \leq \varphi(\omega)$ μ -a.e. on Ω , with $\varphi \in L^p(\Omega)_+$.

Let Y, Z be Hausdorff topological spaces. A multifunction $G : Y \rightarrow 2^Z \setminus \{\emptyset\}$ is said to be “upper semicontinuous” (usc for short) if, for all $C \subseteq Z$ closed, $G^-(C) = \{y \in Y : G(y) \cap C \neq \emptyset\}$ is closed or equivalently for all $U \subseteq Z$ open, $G^+ \{y \in Y : G(y) \subseteq U\}$ is open. If Z is a regular space, then a $P_f(Z)$ -valued multifunction which is usc has a closed graph. The converse is true if the multifunction G is locally compact (i.e., for every $y \in Y$, there exists a neighborhood U of y such that $\overline{G(U)}$ is compact in Z). A $P_k(Z)$ -valued multifunction which is usc maps compact sets to compact sets.

Consider the following weighted nonlinear eigenvalue problem in $\mathbb{R}^N$:

$$\begin{aligned} -(|x'(t)|^{p-2}x'(t))' &= \lambda\theta(t)|x(t)|^{p-2}x(t) \quad \text{a.e. on } T = [0, b], \\ x(0) = x(b) &= 0, \quad 1 < p < \infty, \quad \theta \in L^\infty(T), \quad |\{\theta > 0\}|_1 > 0, \quad \lambda \in \mathbb{R}. \end{aligned} \quad (2.2)$$

Here by $|\cdot|_1$ we denote the 1-dimensional Lebesgue measure. The real parameters λ , for which problem (2.3) has a nontrivial solution, are called eigenvalues of the negative vector p -Laplacian with Dirichlet boundary conditions denoted by $(-\Delta_p, W_0^{1,p}(T, \mathbb{R}^N))$, with weight $\theta \in L^\infty(T)$. The corresponding nontrivial solutions are known as eigenfunctions. We know that the eigenvalues of problem (2.3) are the same as those of the corresponding scalar problem [13]. Then from Denkowski et al. [2] and Zhang [20], we know that there exist two sequences $\{\lambda_n(\theta)\}_{n \geq 1}$ and $\{\lambda_{-n}(\theta)\}_{n \geq 1}$ such that $\lambda_n(\theta) > 0$, $\lambda_n(\theta) \rightarrow +\infty$ and $\lambda_{-n}(\theta) < 0$, $\lambda_{-n}(\theta) \rightarrow -\infty$ as $n \rightarrow \infty$. Moreover, if $\theta(t) \geq 0$ a.e. on T with strict inequality on a set of positive Lebesgue measure, then we have only the positive

sequence $\{\lambda_n(\theta)\}_{n \geq 1}$. Also, for $\lambda_1(\theta) > 0$, we have the following variational characterization:

$$\lambda_1(\theta) = \inf \left[\frac{\|x'\|_p^p}{\int_0^b \theta(t) \|x(t)\|^p dt} : x \in W_0^{1,p}(T, \mathbb{R}^N), x \neq 0 \right]. \quad (2.3)$$

The infimum is attained at the normalized principal eigenfunction u_1 ($\lambda_1(\theta) > 0$ is simple) and $u_1(t) \neq 0$ a.e. on T . Also, $\lambda_1(\theta)$ is strictly monotone with respect to θ , namely, if $\theta_1(t) \leq \theta_2(t)$ a.e. on T with strict inequality on a set of positive measure, then $\lambda_1(\theta_2) < \lambda_1(\theta_1)$ (see (3.2)).

Finally we state the multivalued fixed point principle that we will use in the study of problem (1.1). So let Y, Z be two Banach spaces and $C \subseteq Y, D \subseteq Z$ two nonempty closed and convex sets. We consider multifunctions $G : C \rightarrow 2^C \setminus \{\emptyset\}$ which have a decomposition $G = K \circ N$, satisfying the following: $K : D \rightarrow C$ is completely continuous, namely, if $z_n \xrightarrow{w} z$ in D , then $K(z_n) \rightarrow K(z)$ in C and $N : C \rightarrow P_{wkc}(D)$ is usc from C , furnished with the strong topology into D , furnished with the weak topology.

THEOREM 2.1. *If C, D , and $G = K \circ N$ are as above, $0 \in C$, and G is compact (namely, G maps bounded subsets of C into relatively compact subsets of D), then one of the following alternatives holds:*

- (a) $S = \{y \in C : y \in \mu G(y) \text{ for some } \mu \in (0, 1)\}$ is unbounded or
- (b) G has a fixed point, that is, there exists $y \in C$ such that $y \in G(y)$.

Remark 2.2. Evidently this is a multivalued version of the classical Leray-Schauder alternative principle [2, page 206]. In contrast to previous multivalued extensions of the Leray-Schauder alternative principle [4, page 61], **Theorem 2.1** does not require G to have convex values, which is important when dealing with nonlinear problems such as (1.1).

3. Nonuniform nonresonance

In this section, we deal with problem (1.1) using a condition of nonuniform nonresonance with respect to the first eigenvalue $\lambda_1(\theta) > 0$. Our hypotheses on the multivalued nonlinearity $F(t, x, y)$ are as follows.

$(H(F)_1)$ $F : T \times \mathbb{R}^N \times \mathbb{R}^N \rightarrow P_{kc}(\mathbb{R}^N)$ is a multifunction such that

- (i) for all $x, y \in \mathbb{R}^N$, $t \rightarrow F(t, x, y)$ is graph measurable;
- (ii) for almost all $t \in T$, $(x, y) \rightarrow F(t, x, y)$ is usc;
- (iii) for every $M > 0$, there exists $\gamma_M \in L^1(T)_+$ such that, for almost all $t \in T$, all $\|x\|, \|y\| \leq M$, and all $u \in F(t, x, y)$, we have $\|u\| \leq \gamma_M(t)$;
- (iv) there exists $\theta \in L^\infty(T)$, $\theta(t) \geq 0$ a.e. on T , with strict inequality on a set of positive measure and

$$\limsup_{\|x\| \rightarrow +\infty} \frac{\sup \{ \langle u, x \rangle_{\mathbb{R}^N} : u \in F(t, x, y), y \in \mathbb{R}^N \}}{\|x\|^p} \leq \theta(t) \quad (3.1)$$

uniformly for almost all $t \in T$ and $\lambda_1(\theta) > 1$.

Remark 3.1. Hypothesis $(H(F)_1)(iv)$ is the nonuniform nonresonance condition. In the literature [15, 20], we encounter the condition $\theta(t) \leq \lambda_1$ a.e. on T with strict inequality on a set of positive measure. Here $\lambda_1 > 0$ is the principal eigenvalue corresponding to the unit weight $\theta = 1$ (i.e., $\lambda_1 = \lambda_1(1)$). Then by virtue of the strict monotonicity property, we have $\lambda_1(\lambda_1) = 1 < \lambda_1(\theta)$, which is the condition assumed in hypothesis $(H(F)_1)(iv)$.

$(H(G)_1)$ $G \in C^2(\mathbb{R}^N, \mathbb{R})$.

Given $h \in L^1(T, \mathbb{R}^N)$, we consider the following Dirichlet problem:

$$\begin{aligned} -(|x'(t)|^{p-2}x'(t))' &= h(t) \quad \text{a.e. on } T = [0, b], \\ x(0) &= x(b) = 0. \end{aligned} \quad (3.2)$$

From Manásevich and Mawhin [13, Lemma 4.1], we know that problem (3.3) has a unique solution $K(h) \in C_0^1(T, \mathbb{R}^N) = \{x \in C^1(T, \mathbb{R}^N) : x(0) = x(b) = 0\}$. So we can define the solution map $K : L^1(T, \mathbb{R}^N) \rightarrow C_0^1(T, \mathbb{R}^N)$.

PROPOSITION 3.2. $K : L^1(T, \mathbb{R}^N) \rightarrow C_0^1(T, \mathbb{R}^N)$ is completely continuous, that is, if $h_n \xrightarrow{w} h$ in $L^1(T, \mathbb{R}^N)$, then $K(h_n) \rightarrow K(h)$ in $C_0^1(T, \mathbb{R}^N)$.

Proof. Let $h_n \xrightarrow{w} h$ in $L^1(T, \mathbb{R}^N)$ and set $x_n = K(h_n)$, $n \geq 1$. We have

$$-(|x'_n(t)|^{p-2}x'_n(t))' = h_n(t) \quad \text{a.e. on } T, \quad x_n(0) = x_n(b) = 0, \quad n \geq 1. \quad (3.3)$$

Taking the inner product with $x_n(t)$, integrating over T , and performing integration by parts, we obtain

$$\|x'_n\|_p^p \leq \|h_n\|_1 \|x_n\|_\infty \leq c_1 \|x'_n\|_p \quad \text{for some } c_1 > 0 \text{ and all } n \geq 1. \quad (3.4)$$

Here we have used Hölder and Poincaré inequalities. It follows that

$$\begin{aligned} \{x'_n\}_{n \geq 1} &\subseteq L^p(T, \mathbb{R}^N) \text{ is bounded (since } p > 1) \\ \implies \{x_n\}_{n \geq 1} &\subseteq W_0^{1,p}(T, \mathbb{R}^N) \text{ is bounded (by the Poincaré inequality).} \end{aligned} \quad (3.5)$$

So from (3.22) we infer that

$$\begin{aligned} \{\|x'_n\|^{p-2}x'_n\}_{n \geq 1} &\subseteq W^{1,q}(T, \mathbb{R}^N) \left(\frac{1}{p} + \frac{1}{q} = 1 \right) \text{ is bounded} \\ \implies \{\|x'_n\|^{p-2}x'_n\}_{n \geq 1} &\subseteq C(T, \mathbb{R}^N) \text{ is relatively compact} \end{aligned} \quad (3.6)$$

(recall that $W^{1,q}(T, \mathbb{R}^N)$ is embedded compactly in $C(T, \mathbb{R}^N)$). The map $\varphi_p : \mathbb{R}^N \rightarrow \mathbb{R}^N$, defined by $\varphi_p(y) = \|y\|^{p-2}y$, $y \in \mathbb{R}^N \setminus \{\emptyset\}$, and $\varphi_p(0) = 0$, is a homeomorphism and so $\varphi_p^{-1} : C(T, \mathbb{R}^N) \rightarrow C(T, \mathbb{R}^N)$, defined by $\varphi_p^{-1}(y)(\cdot) = \varphi_p^{-1}(y(\cdot))$, is continuous and bounded. Thus it follows that

$$\begin{aligned} \{x'_n\}_{n \geq 1} &\subseteq C(T, \mathbb{R}^N) \text{ is relatively compact} \\ \implies \{x_n\}_{n \geq 1} &\subseteq C_0^1(T, \mathbb{R}^N) \text{ is relatively compact.} \end{aligned} \quad (3.7)$$

Therefore we may assume that $x_n \rightarrow x$ in $C_0^1(T, \mathbb{R}^N)$. Also $\{\|x'_n\|^{p-2}x'_n\}_{n \geq 1} \subseteq W^{1,q}(T, \mathbb{R}^N)$ is bounded and so we may assume that $\|x'_n\|^{p-2}x'_n \xrightarrow{w} u$ in $W^{1,q}(T, \mathbb{R}^N)$ and $\|x'_n\|^{p-2}x'_n \rightarrow u$ in $C(T, \mathbb{R}^N)$ (because $W^{1,q}(T, \mathbb{R}^N)$ is embedded compactly in $C(T, \mathbb{R}^N)$). It follows that $u = \|x'\|^{p-2}x'$. Hence if in (3.22) we pass to the limit as $n \rightarrow \infty$, we obtain

$$\begin{aligned} -(\|x'(t)\|^{p-2}x'(t))' &= h(t) \quad \text{a.e. on } T = [0, b], \quad x(0) = x(b) = 0 \\ \implies K(h) &= x. \end{aligned} \quad (3.8)$$

Since every subsequence of $\{x_n\}_{n \geq 1}$ has a further subsequence which converges to x in $C_0^1(T, \mathbb{R}^N)$, we conclude that the original sequence converges too. This proves the complete continuity of K . $\square$

Let $N_F : C_0^1(T, \mathbb{R}^N) \rightarrow 2^{L^1(T, \mathbb{R}^N)}$ be the multivalued Nemitsky operator corresponding to F , that is,

$$N_F(x) = \{u \in L^1(T, \mathbb{R}^N) : u(t) \in F(t, x(t), x'(t)) \text{ a.e. on } T\}. \quad (3.9)$$

Also let $N : C_0^1(T, \mathbb{R}^N) \rightarrow 2^{L^1(T, \mathbb{R}^N)}$ be defined by

$$N(x) = \frac{d}{dx} \nabla G(x(\cdot)) + N_F(x). \quad (3.10)$$

This multifunction has the following structure.

PROPOSITION 3.3. *If hypotheses $(H(F)_1)$ and $(H(G)_1)$ hold, then N has values in $P_{wkc}(L^1(T, \mathbb{R}^N))$ and it is usc from $C_0^1(T, \mathbb{R}^N)$ with the norm topology into $L^1(T, \mathbb{R}^N)$ with the weak topology.*

Proof. Clearly N has closed, convex values which are uniformly integrable (see hypothesis $(H(F)_1)(iii)$). Therefore for every $x \in C_0^1(T, \mathbb{R}^N)$, $N(x)$ is convex and w -compact in $L^1(T, \mathbb{R}^N)$. What is not immediately clear is that $N(x) \neq \emptyset$, since hypotheses $(H(F)_1)(i)$ and (ii) in general do not imply the graph measurability of $(t, x, y) \rightarrow F(t, x, y)$ [10, page 227]. To see that $N(x) \neq \emptyset$, we proceed as follows. Let $\{s_n\}_{n \geq 1}, \{r_n\}_{n \geq 1}$ be step functions such that $s_n \rightarrow x$ and $r_n \rightarrow x'$ a.e. on T and $\|s_n(t)\| \leq \|x(t)\|$, $\|r_n(t)\| \leq \|x'(t)\|$ a.e. on T , $n \geq 1$. Then by virtue of hypothesis $(H(F)_1)(i)$, for every $n \geq 1$, the multifunction $t \rightarrow F(t, s_n(t), r_n(t))$ is measurable and so by the Yankon-von Neumann-Aumann selection theorem [10, page 158], we can find $u_n : T \rightarrow \mathbb{R}^N$ a measurable map such that $u_n(t) \in F(t, s_n(t), r_n(t))$ for all $t \in T$. Note that $\|s_n\|_\infty, \|r_n\|_\infty \leq M_1$ for some $M_1 > 0$ and all $n \geq 1$. So $\|u_n(t)\| \leq \gamma_{M_1}(t)$ a.e. on T , with $\gamma_{M_1} \in L^1(T)_+$ (see hypothesis $(H(F)_1)(iii)$). Thus by virtue of the Dunford-Pettis theorem, we may assume that $u_n \xrightarrow{w} u$ in $L^1(T, \mathbb{R}^N)$ as $n \rightarrow \infty$. From Hu and Papageorgiou [10, page 694], we have

$$u(t) \in \overline{\text{conv}} \limsup_{n \rightarrow \infty} F(t, s_n(t), r_n(t)) \subseteq F(t, x(t), x'(t)) \quad \text{a.e. on } T, \quad (3.11)$$

with the last inclusion being a consequence of hypothesis $(H(F)_1)(ii)$. So we have $u \in S_{F(\cdot, x(\cdot), x'(\cdot))}^q$, hence $N(x) \neq \emptyset$.

Next we check the upper semicontinuity of N into $L^1(T, \mathbb{R}^N)_w$ ($L^1(T, \mathbb{R}^N)_w$ equals the Banach space $L^1(T, \mathbb{R}^N)$ furnished with the weak topology). Because of hypothesis $(H(F)_1)(iii)$, N is locally compact into $L^1(T, \mathbb{R}^N)_w$ (recall that uniformly integrable sets are relatively compact in $L^1(T, \mathbb{R}^N)_w$). Also on weakly compact subsets of $L^1(T, \mathbb{R}^N)$, the relative weak topology is metrizable. Therefore to check the upper semicontinuity of N , it suffices to show that $\text{Gr}N$ is sequentially closed in $C_0^1(T, \mathbb{R}^N) \times L^1(T, \mathbb{R}^N)_w$ (see [Section 2](#)). To this end, let $(x_n, f_n) \in \text{Gr}N$, $n \geq 1$, and suppose that $x_n \rightarrow x$ in $C_0^1(T, \mathbb{R}^N)$ and $f_n \xrightarrow{w} f$ in $L^1(T, \mathbb{R}^N)$. For every $n \geq 1$, we have

$$f_n(t) = \frac{d}{dt} \nabla G(x_n(t)) + u_n(t) \quad \text{a.e. on } T, \text{ with } u_n \in S_{F(\cdot, x_n(\cdot), x'_n(\cdot))}^1. \quad (3.12)$$

Because of hypothesis $(H(F)_1)(iii)$, we may assume (at least for a subsequence) that $u_n \xrightarrow{w} u$ in $L^1(T, \mathbb{R}^N)$. As before, from Hu and Papageorgiou [10, page 694], we have

$$u(t) \in \overline{\text{conv}} \limsup_{n \rightarrow \infty} F(t, x_n(t), x'_n(t)) \subseteq F(t, x(t), x'(t)) \quad \text{a.e. on } T \quad (3.13)$$

(again the last inclusion follows from hypothesis $(H(F)_1)(ii)$). So $u \in S_{F(\cdot, x(\cdot), x'(\cdot))}^1$. Also by virtue of hypothesis $(H(G)_1)$, we have

$$\begin{aligned} \frac{d}{dt} \nabla G(x_n(t)) &= G''(x_n(t))x'_n(t) \rightarrow G''(x(t))x'(t) = \frac{d}{dt} \nabla G(x(t)), \quad \forall t \in T \\ \Rightarrow \frac{d}{dt} \nabla G(x_n(\cdot)) &\rightarrow \frac{d}{dt} \nabla G(x(\cdot)) \quad \text{in } L^1(T, \mathbb{R}^N) \\ &\quad (\text{by the dominated convergence theorem}). \end{aligned} \quad (3.14)$$

So in the limit as $n \rightarrow \infty$, we have

$$\begin{aligned} f &= \frac{d}{dt} \nabla G(x(\cdot)) + u \quad \text{with } u \in N_F(x) \\ &\Rightarrow (x, f) \in \text{Gr}N. \end{aligned} \quad (3.15)$$

This proves the desired upper semicontinuity of N . □

PROPOSITION 3.4. *There exists $\xi > 0$ such that, for all $x \in W_0^{1,p}(T, \mathbb{R}^N)$,*

$$\|x'\|_p^p - \int_0^b \theta(t) \|x(t)\|^p dt \geq \xi \|x'\|_p^p. \quad (3.16)$$

Proof. Let $\eta : W_0^{1,p}(T, \mathbb{R}^N) \rightarrow \mathbb{R}$ be the functional defined by

$$\eta(x) = \|x'\|_p^p - \int_0^b \theta(t) \|x(t)\|^p dt. \quad (3.17)$$

From the variational characterization of $\lambda_1(\theta) > 1$, we see that $\eta(x) > 0$ for all $x \in W_0^{1,p}(T, \mathbb{R}^N)$, $x \neq 0$. Suppose that the proposition was not true. Then by virtue of the p -homogeneity of η , we can find $\{x_n\}_{n \geq 1} \subseteq W_0^{1,p}(T, \mathbb{R}^N)$ such that $\|x'_n\|_p = 1$ and $\eta(x_n) \downarrow 0$.

By the Poincaré inequality, the sequence $\{x_n\}_{n \geq 1} \subseteq W_0^{1,p}(T, \mathbb{R}^N)$ is bounded and so we may assume that

$$x_n \xrightarrow{w} x \quad \text{in } W_0^{1,p}(T, \mathbb{R}^N), \quad x_n \rightarrow x \quad \text{in } C_0(T, \mathbb{R}^N). \quad (3.18)$$

Also exploiting the weak lower semicontinuity of the norm functional in a Banach space, we obtain

$$\|x'\|_p^p \leq \int_0^b \theta(t) \|x(t)\|^p dt \Rightarrow \lambda_1(\theta) \leq 1, \quad (3.19)$$

a contradiction to our hypothesis that $\lambda_1(\theta) > 1$. $\square$

We introduce the set

$$S = \{x \in C_0^1(T, \mathbb{R}^N) : x \in \lambda KN(x), 0 < \lambda < 1\}. \quad (3.20)$$

PROPOSITION 3.5. *If hypotheses $(H(F)_1)$ and $(H(G)_1)$ hold, then $S \subseteq C_0^1(T, \mathbb{R}^N)$ is bounded.*

Proof. Let $x \in S$. We have

$$\begin{aligned} \frac{1}{\lambda} x &\in KN(x) \quad \text{with } 0 < \lambda < 1 \\ \Rightarrow \frac{1}{\lambda^{p-1}} (\|x'(t)\|^{p-2} x'(t))' + \frac{d}{dt} \nabla G(x(t)) + u(t) &= 0 \quad \text{a.e. on } T, \text{ with } u \in S_{F(\cdot, x(\cdot), x'(\cdot))}^1 \\ \Rightarrow (\|x'(t)\|^{p-2} x'(t))' + \lambda^{p-1} \frac{d}{dt} \nabla G(x(t)) + \lambda^{p-1} u(t) &= 0 \quad \text{a.e. on } T. \end{aligned} \quad (3.21)$$

Taking the inner product with $x(t)$, integrate over T , and perform integration by parts, we obtain

$$-\|x'\|_p^p - \lambda^{p-1} \int_0^b (\nabla G(x(t)), x'(t))_{\mathbb{R}^N} dt + \lambda^{p-1} \int_0^b (u(t), x(t))_{\mathbb{R}^N} dt = 0. \quad (3.22)$$

Remark that

$$\int_0^b (\nabla G(x(t)), x'(t))_{\mathbb{R}^N} dt = \int_0^b \frac{d}{dt} G(x(t)) dt = G(x(b)) - G(x(0)) = 0. \quad (3.23)$$

By virtue of hypotheses $(H(F)_1)$ (iii) and (iv), given $\varepsilon > 0$, we can find $\gamma_\varepsilon \in L^1(T)_+$ such that for almost all $t \in T$, all $x, y \in \mathbb{R}^N$, and all $u \in F(t, x, y)$, we have

$$(u, x)_{\mathbb{R}^N} \leq (\theta(t) + \varepsilon) \|x\|^p + \gamma_\varepsilon(t). \quad (3.24)$$

So we have

$$\int_0^b (u(t), x(t))_{\mathbb{R}^N} dt \leq \int_0^b \theta(t) \|x(t)\|^p dt + \varepsilon \|x\|_p^p + \|\gamma_\varepsilon\|_1. \quad (3.25)$$

Using (3.24) and (3.27) in (3.23), we obtain

$$\begin{aligned} \|x'\|_p^p &\leq \int_0^b \theta(t) \|x(t)\|^p dt + \varepsilon \|x\|_p^p + \|\gamma_\varepsilon\|_1 \\ &\Rightarrow \xi \|x'\|_p^p - \frac{\varepsilon}{\lambda_1} \|x'\|_p^p \leq \|\gamma_\varepsilon\|_1 \end{aligned} \quad (3.26)$$

(see Proposition 3.5 and recall that $\lambda_1 \|x\|_p^p \leq \|x'\|_p^p$, $\lambda_1 = \lambda_1(1)$).

Choose $\varepsilon > 0$ so that $\varepsilon < \lambda_1 \xi$. Then from the last inequality, we infer that

$$\begin{aligned} \{x'\}_{x \in S} &\subseteq L^p(T, \mathbb{R}^N) \text{ is bounded} \\ &\Rightarrow S \subseteq W_0^{1,p}(T, \mathbb{R}^N) \text{ is bounded (by Poincaré's inequality)} \\ &\Rightarrow S \subseteq C_0(T, \mathbb{R}^N) \text{ is relatively compact.} \end{aligned} \quad (3.27)$$

Also we have

$$\begin{aligned} &\|(|x'(t)|^{p-2} x'(t))'\| \\ &\leq \|G''(x(t))\|_{\mathcal{L}} \|x'(t)\| + \|u(t)\| \text{ a.e. on } T \\ &\leq M_2 (\|x'(t)\| + \theta(t) + \varepsilon + \gamma_\varepsilon(t)) \text{ a.e. on } T \text{ for some } M_2 > 0 \text{ (see (3.25))} \\ &\Rightarrow \{\|x'\|^{p-2} x'\}_{x \in S} \subseteq W^{1,1}(T, \mathbb{R}^N) \text{ is bounded} \\ &\Rightarrow \{\|x'\|^{p-2} x'\}_{x \in S} \subseteq C(T, \mathbb{R}^N) \text{ is bounded} \\ &\quad (\text{since } W^{1,1}(T, \mathbb{R}^N) \text{ is embedded continuously but not compactly in } C(T, \mathbb{R}^N)) \\ &\Rightarrow \{x'\}_{x \in S} \subseteq C(T, \mathbb{R}^N) \text{ is bounded.} \end{aligned} \quad (3.28)$$

From (3.28) and (3.29), we conclude that $S \subseteq C_0^1(T, \mathbb{R}^N)$ is bounded. $\square$

Propositions 3.2, 3.3, and 3.5 permit the use of Theorem 2.1. So we obtain the following existence result for problem (1.1).

THEOREM 3.6. *If hypotheses $(H(F)_1)$ and $(H(G)_1)$ hold, then problem (1.1) has a solution $x \in C_0^1(T, \mathbb{R}^N)$ with $\|x'\|^{p-2} x' \in W^{1,1}(T, \mathbb{R}^N)$.*

As an application of this theorem, we consider the following system:

$$\begin{aligned} &(|x'(t)|^{p-2} x'(t))' + \|x(t)\|^{p-2} Ax(t) + F(t, x(t)) \ni e(t) \quad \text{a.e. on } T = [0, b], \\ &x(0) = x(b) = 0, \quad e \in L^1(T, \mathbb{R}^N). \end{aligned} \quad (3.29)$$

Our hypotheses on the data of problem (3.29) are the following.

$(H(A))$ A is an $N \times N$ matrix such that for all $x \in \mathbb{R}^N$ we have $\langle Ax, x \rangle_{\mathbb{R}^N} \leq \theta \|x\|^2$ with $\theta < (\pi_p/b)^p$.

Remark 3.7. The quantity π_p is defined by $\pi_p = 2(p-1)^{1/p} \int_0^1 (1/(1-t)^{1/p}) dt = 2(p-1)^{1/p} ((\pi/p)/\sin(\pi/p))$. If $p = 2$, then $\pi_2 = \pi$. Recall that the eigenvalues of $(-\Delta_p, W_0^{1,p}(T, \mathbb{R}^N))$ are $\lambda_n = (n\pi_p/b)^p$, $n \geq 1$ [13]. So in hypothesis $(H(A))$, we have $\theta < \lambda_1$.

$(H(F)'_1)$ $F : T \times \mathbb{R}^N \rightarrow P_{kc}(\mathbb{R}^N)$ is a multifunction such that

- (i) for all $x \in \mathbb{R}^N$, $t \rightarrow F(t, x)$ is graph measurable;
- (ii) for almost all $t \in T$, $x \rightarrow F(t, x)$ is usc;
- (iii) for every $M > 0$, there exists $\gamma_M \in L^1(T)_+$ such that for almost all $t \in T$, all $\|x\| \leq M$, and all $u \in F(t, x)$, we have $\|u\| \leq \gamma_M(t)$;
- (iv) $\lim_{\|x\| \rightarrow \infty} ((u, x)_{\mathbb{R}^N} / \|x\|^p) = 0$ uniformly for almost all $t \in T$ and all $u \in F(t, x)$.

Invoking Theorem 3.6, we obtain the following existence result for problem (3.29).

THEOREM 3.8. *If hypotheses $(H(A))$ and $(H(F)'_1)$ hold, then for every $e \in L^1(T, \mathbb{R}^N)$, problem (3.29) has a solution $x \in C_0^1(T, \mathbb{R}^N)$ with $\|x'\|^{p-2}x' \in W^{1,1}(T, \mathbb{R}^N)$.*

Remark 3.9. Theorem 3.8 extends Theorem 7.1 of Manásevich and Mawhin [13].

Acknowledgments

The authors wish to thank a very knowledgeable referee for pointing out an error in the first version of the paper and for constructive remarks. Michael E. Filippakis was supported by a grant from the National Scholarship Foundation of Greece (IKY).

References

- [1] R. Bader, *A topological fixed-point index theory for evolution inclusions*, Z. Anal. Anwendungen **20** (2001), no. 1, 3–15.
- [2] Z. Denkowski, S. Migórski, and N. S. Papageorgiou, *An Introduction to Nonlinear Analysis: Applications*, Kluwer Academic Publishers, Massachusetts, 2003.
- [3] ———, *An Introduction to Nonlinear Analysis: Theory*, Kluwer Academic Publishers, Massachusetts, 2003.
- [4] J. Dugundji and A. Granas, *Fixed Point Theory. I*, Monografie Matematyczne, vol. 61, Państwowe Wydawnictwo Naukowe (PWN), Warsaw, 1982.
- [5] L. H. Erbe and W. Krawcewicz, *Boundary value problems for differential inclusions*, Differential Equations (Colorado Springs, Colo, 1989), Lecture Notes in Pure and Appl. Math., vol. 127, Dekker, New York, 1991, pp. 115–135.
- [6] ———, *Nonlinear boundary value problems for differential inclusions $y'' \in F(t, y, y')$* , Ann. Polon. Math. **54** (1991), no. 3, 195–226.
- [7] M. Frigon, *Application de la théorie de la transversalité topologique à des problèmes non linéaires pour des équations différentielles ordinaires* [Applications of the theory of topological transversality to nonlinear problems for ordinary differential equations], Dissertationes Math. (Rozprawy Mat.) **296** (1990), 75 pp. (French).
- [8] ———, *Théorèmes d'existence de solutions d'inclusions différentielles* [Existence theorems for solutions of differential inclusions], Topological Methods in Differential Equations and Inclusions (Montreal, PQ, 1994) (A. Granas, M. Frigon, and G. Sabidussi, eds.), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., vol. 472, Kluwer Academic Publishers, Dordrecht, 1995, pp. 51–87.
- [9] N. Halidias and N. S. Papageorgiou, *Existence and relaxation results for nonlinear second-order multivalued boundary value problems in $\mathbb{R}^N$* , J. Differential Equations **147** (1998), no. 1, 123–154.
- [10] S. Hu and N. S. Papageorgiou, *Handbook of Multivalued Analysis. Vol. I. Theory*, Mathematics and Its Applications, vol. 419, Kluwer Academic Publishers, Dordrecht, 1997.

- [11] D. A. Kandilakis and N. S. Papageorgiou, *Existence theorems for nonlinear boundary value problems for second order differential inclusions*, J. Differential Equations **132** (1996), no. 1, 107–125.
- [12] S. Kyritsi, N. Matzakos, and N. S. Papageorgiou, *Periodic problems for strongly nonlinear second-order differential inclusions*, J. Differential Equations **183** (2002), no. 2, 279–302.
- [13] R. Manásevich and J. Mawhin, *Boundary value problems for nonlinear perturbations of vector p -Laplacian-like operators*, J. Korean Math. Soc. **37** (2000), no. 5, 665–685.
- [14] J. Mawhin, *Periodic solutions of systems with p -Laplacian-like operators*, Nonlinear Analysis and Its Applications to Differential Equations (Lisbon, 1998), Progr. Nonlinear Differential Equations Appl., vol. 43, Birkhäuser Boston, Massachusetts, 2001, pp. 37–63.
- [15] J. Mawhin and J. R. Ward, *Periodic solutions of some forced Liénard differential equations at resonance*, Arch. Math. (Basel) **41** (1983), no. 4, 337–351.
- [16] D. O'Regan and R. Precup, *Fixed point theorems for set-valued maps and existence principles for integral inclusions*, J. Math. Anal. Appl. **245** (2000), no. 2, 594–612.
- [17] M. Palmucci and F. Papalini, *A nonlinear multivalued problem with nonlinear boundary conditions*, Set Valued Mappings with Applications in Nonlinear Analysis (R. P. Agarwal and D. O'Regan, eds.), Ser. Math. Anal. Appl., vol. 4, Taylor & Francis, London, 2002, pp. 383–402.
- [18] R. Precup, *Fixed point theorems for decomposable multivalued maps and some applications*, to appear in Z. Anal. Anwendungen.
- [19] T. Pruszko, *Some applications of the topological degree theory to multivalued boundary value problems*, Dissertationes Math. (Rozprawy Mat.) **229** (1984), 48 pp.
- [20] M. Zhang, *Nonuniform nonresonance of semilinear differential equations*, J. Differential Equations **166** (2000), no. 1, 33–50.

Michael E. Filippakis: Department of Mathematics, National Technical University, Zografou Campus, 15780 Athens, Greece

E-mail address: mfil@math.ntua.gr

Nikolaos S. Papageorgiou: Department of Mathematics, National Technical University, Zografou Campus, 15780 Athens, Greece

E-mail address: npapg@math.ntua.gr

Special Issue on Intelligent Computational Methods for Financial Engineering

Call for Papers

As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods:** artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning

- **Application fields:** asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects:** decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

Authors should follow the Journal of Applied Mathematics and Decision Sciences manuscript format described at the journal site <http://www.hindawi.com/journals/jamds/>. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at <http://mts.hindawi.com/>, according to the following timetable:

Manuscript Due	December 1, 2008
First Round of Reviews	March 1, 2009
Publication Date	June 1, 2009

Guest Editors

Lean Yu, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; yulean@amss.ac.cn

Shouyang Wang, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; sywang@amss.ac.cn

K. K. Lai, Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; mskkklai@cityu.edu.hk