

Research Article

Uniform Blow-Up Rates and Asymptotic Estimates of Solutions for Diffusion Systems with Nonlocal Sources

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This paper investigates the local existence of the nonnegative solution and the finite time blow-up of solutions and boundary layer profiles of diffusion equations with nonlocal reaction sources; we also study the global existence and that the rate of blow-up is uniform in all compact subsets of the domain, the blow-up rate of $|u(t)|_\infty$ is precisely determined.

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1. Introduction

In this paper, we study the following reaction-diffusion system with nonlocal nonlinear source:

$$\begin{aligned} u_t &= \Delta u + a(x) |v(t)|_r^p, & x \in \Omega, t > 0, \\ v_t &= \Delta v + b(x) |u(t)|_r^q, & x \in \Omega, t > 0, \\ u(x, t) &= v(x, t) = 0, & x \in \partial\Omega, t > 0, \\ u(x, 0) &= u_0(x), \quad v(x, 0) = v_0(x), & x \in \Omega, \end{aligned} \tag{1.1}$$

where Ω is an open ball of $\mathbb{R}^N$ centered at the origin of radius R , $|u(t)|_r = (\int_\Omega |u(x, t)|^r dx)^{1/r}$, $1 \leq r < \infty$ and $p, q \geq r$. A nonnegative solution of (1.1) is a pair of nonnegative functions $(u(x, t), v(x, t))$ such that $(u(x, t), v(x, t)) \in C(\bar{\Omega} \times [0, T]) \cap C^{2,1}(\Omega \times (0, T))$ and satisfies (1.1). For a solution $(u(x, t), v(x, t))$ of (1.1), we define

$$T^* = T^*(u, v) = \sup \{T > 0 : (u, v) \text{ are bounded and satisfy (1.1)}\}. \tag{1.2}$$

2 Differential Equations and Nonlinear Mechanics

Note that if $T^* < +\infty$, then (u, v) blows up in L^∞ norm, in the sense that $\lim_{t \rightarrow T^*} \|u(t)\|_{L^\infty} = +\infty$ or $\lim_{t \rightarrow T^*} \|v(t)\|_{L^\infty} = +\infty$; in this case, we say that the solution blows up in finite time. If $T^* = \infty$, then (u, v) is a global solution of (1.1).

In the past several decades, many physical phenomena were formulated into nonlocal mathematical models (see [1–6]). It has also been suggested that nonlocal growth terms present a more realistic model of population dynamics (see [7]). System (1.1) is related to some ignition models in physics for compressible reactive gases.

A lot of effort has been devoted in the past few years to the study of blow-up rates and profiles for local semilinear parabolic equations of the type

$$u_t - \Delta u = u^p; \quad (1.3)$$

see [8–11] and the references therein. Several interesting blow-up results which concern the blow-up condition, blow-up set, and blow-up rate are presented; see [12–16] and references therein.

The blow-up property of the solution to a single equation of the form

$$\begin{aligned} u_t &= \Delta u + a(x) |u(t)|_r^p, \quad x \in \Omega, \quad t > 0, \\ u(x, t) &= 0, \quad x \in \partial\Omega, \quad t > 0, \\ u(x, 0) &= u_0(x), \quad x \in \Omega \end{aligned} \quad (1.4)$$

has been discussed by many authors; see [1, 4] and the references therein. In [1], Souplet introduced a new method for investigating the rate and profile of blow-up of solutions to problem (1.4) with $a(x) = \text{constant} = 1$. He proved that if $p > 1$, then uniformly on compact subsets of Ω holds

$$\lim_{t \rightarrow T} (T - t)^{1/(p-1)} u(x, t) = \lim_{t \rightarrow T} (T - t)^{1/(p-1)} |u(t)|_\infty = [(p-1)|\Omega|^{p/r}]^{-1/(p-1)}. \quad (1.5)$$

Very recently, Liu et al. [4] proved the global blow-up and determined the blow-up rate for problem (1.4) with $a(x) \neq \text{constant}$.

Our present work is inspired by [1, 4], mentioned before, and [3, 5, 6, 12–19]. In [17], Escobedo and Herrero studied the system

$$u_t = \Delta u + v^p, \quad v_t = \Delta v + u^q \quad (1.6)$$

with homogeneous Dirichlet boundary conditions. They showed that if $pq \leq 1$, every solution of (1.6) is global, while for $pq > 1$, there are solutions that blow-up and others that are global according to the size of initial data. The blow-up rates of solutions to (1.6) were considered in [3, 5, 6].

In [12], Wang discussed the finite time blow-up of the positive solution to the problem

$$\begin{aligned} u_t &= \Delta u + u^m v^n, \quad x \in \Omega, \quad t > 0, \\ v_t &= \Delta v + u^p v^q, \quad x \in \Omega, \quad t > 0, \\ u(x, t) &= v(x, t) = 0, \quad x \in \partial\Omega, \quad t > 0, \\ u(x, 0) &= u_0(x) \geq 0, \quad v(x, 0) = v_0(x) \geq 0, \quad x \in \Omega. \end{aligned} \quad (1.7)$$

Let λ_1 be the first eigenvalue of $-\Delta$ in Ω with null Dirichlet boundary condition. His results are the following.

(i) Assume that

$$m > 1, n > 0, p = 0, q = 1, \lambda_1 < 1, \quad m \leq 1 + \frac{n(1 - \lambda_1)}{\lambda_1}, \quad (1.8)$$

or

$$q > 1, p > 0, n = 0, m = 1, \lambda_1 < 1, \quad q \leq 1 + \frac{p(1 - \lambda_1)}{\lambda_1}. \quad (1.9)$$

Furthermore, if $m = 1 + n(1 - \lambda_1)/\lambda_1$ in (1.8) or $q = 1 + p(1 - \lambda_1)/\lambda_1$ in (1.9), it is assumed that $\lambda_1 < 2/3$. Then, for any nontrivial initial data, that is, $u_0(x) \not\equiv 0, v_0(x) \not\equiv 0$, the solution of (1.7) blows up in finite time.

(ii) If (1.8), (1.9), and the conditions that $m \leq 1, q \leq 1$, and $np \leq (1 - m)(1 - q)$ do not hold, then the solution of (1.7) blows up in finite time for large initial data.

In [13], Wang evaluated the blow-up rate of the solution to (1.7) with $\Omega = B_R(0)$. Under some suitable conditions, he obtained that

$$\begin{aligned} c(T - t)^{-\theta} &\leq \max_{0 \leq |x| \leq R} u(\cdot, t) = u(0, t) \leq C(T - t)^{-\theta}, \quad t \in [0, T), \\ c(T - t)^{-\sigma} &\leq \max_{0 \leq |x| \leq R} v(\cdot, t) = v(0, t) \leq C(T - t)^{-\sigma}, \quad t \in [0, T), \end{aligned} \quad (1.10)$$

for some positive constants c and C , here $\theta = (1 + n - q)/(np - (1 - m)(1 - q))$ and $\sigma = (1 + p - m)/(np - (1 - m)(1 - q))$, and T is the blow-up time of (u, v) .

In [14, 15], Galaktionov et al. considered the system

$$u_t = \Delta u^{\gamma+1} + v^p, \quad v_t = \Delta v^{\mu+1} + u^q \quad \text{for } (x, t) \in \Omega \times (0, T) \quad (1.11)$$

with homogeneous Dirichlet boundary conditions, where $p > 1, q > 1, \gamma > 0, \mu > 0$. Several interesting results are established. Their results show that $P_c = pq - (1 + \gamma)(1 + \mu)$ is the critical exponent of (1.11), namely, if $P_c < 0$ solutions are global for all initial data, and if $P_c > 0$ solutions blow-up in finite time for sufficiently large initial data.

In this paper, we will prove that $P_c = pq - 1$ is also the critical exponent of system (1.1).

The purpose of this paper is to determine the critical exponents as well as the estimates for blow-up rates and boundary layer profiles of the reaction-diffusion system (1.1). As for the function $a(x), b(x), u_0(x), v_0(x)$, we assume that

- (A₁) $a(x), b(x) \in C^2(\Omega), u_0(x), v_0(x) \in C^{2+\alpha}(\Omega), \alpha \in (0, 1); a(x), b(x), u_0(x), v_0(x) > 0$ in Ω , and $a(x) = b(x) = u_0(x) = v_0(x) = 0$ on $\partial\Omega$.
- (A₂) $a(x), b(x), u_0(x)$, and $v_0(x)$ are radially symmetric, that is, $a(x) = a(r), b(x) = b(r), u_0(x) = u_0(r)$, and $v_0(x) = v_0(r)$ with $r = |x|$. $a(r), b(r), u_0(r)$, and $v_0(r)$ are nonincreasing for $r \in [0, R]$.

This paper is organized as follows. In Section 2, we investigate the global existence and finite time blow-up of system (1.1). Section 3 is devoted to the blow-up set and blow-up rate of solutions to (1.1). In Section 4, we give the boundary layer estimates.

2. Global existence and finite time blow-up

In this section, we start with the definition of super- and sub-solution of system (1.1).

Definition 2.1. A pair of nonnegative functions $(\bar{u}(x, t), \bar{v}(x, t))$ is called a supersolution of (1.1) if $(\bar{u}(x, t), \bar{v}(x, t)) \in C(\bar{\Omega} \times [0, T]) \cap C^{2,1}(\Omega \times (0, T))$ and satisfy

$$\begin{aligned} \bar{u}_t &\geq \Delta \bar{u} + a(x) |\bar{v}(t)|_r^p, & (x, t) \in \Omega \times (0, T), \\ \bar{v}_t &\geq \Delta \bar{v} + b(x) |\bar{u}(t)|_r^q, & (x, t) \in \Omega \times (0, T), \\ \bar{u}(x, t) &\geq \bar{v}(x, t) \geq 0, & x \in \partial\Omega, t > 0, \\ \bar{u}(x, 0) &\geq u_0(x), \quad \bar{v}(x, 0) \geq v_0(x), & x \in \bar{\Omega}. \end{aligned} \tag{2.1}$$

A pair of nonnegative functions $(\underline{u}(x, t), \underline{v}(x, t))$ is called a subsolution of (1.1) if $(\underline{u}(x, t), \underline{v}(x, t)) \in C(\bar{\Omega} \times [0, T]) \cap C^{2,1}(\Omega \times (0, T))$ and satisfy

$$\begin{aligned} \underline{u}_t &\leq \Delta \underline{u} + a(x) |\underline{v}(t)|_r^p, & (x, t) \in \Omega \times (0, T), \\ \underline{v}_t &\leq \Delta \underline{v} + b(x) |\underline{u}(t)|_r^q, & (x, t) \in \Omega \times (0, T), \\ \underline{u}(x, t) &= \underline{v}(x, t) = 0, & x \in \partial\Omega, t > 0, \\ \underline{u}(x, 0) &\leq u_0(x), \quad \underline{v}(x, 0) \leq v_0(x), & x \in \bar{\Omega}, \end{aligned} \tag{2.2}$$

where $1 \leq r < +\infty$, $p, q \geq r$.

We set $Q_T = \Omega \times (0, T]$ and $S_T = \partial\Omega \times (0, T]$. A weak solution of (1.1) is a vector function which is both a subsolution and a supersolution of (1.1). The following comparison lemma plays a crucial role in our proof which can be obtained by similar arguments as in [16].

LEMMA 2.2. Assume (A_1) – (A_2) hold, $w(x, t), z(x, t) \in C(\bar{Q}_T) \cap C^{2,1}(Q_T)$ and satisfy

$$\begin{aligned} w_t - \Delta w &\geq a(x) d_1(t) \int_{\Omega} c_1(x, t) z(x, t) dx, & (x, t) \in Q_T, \\ z_t - \Delta z &\geq b(x) d_2(t) \int_{\Omega} c_2(x, t) w(x, t) dx, & (x, t) \in Q_T, \\ w(x, t), z(x, t) &\geq 0, & (x, t) \in S_T, \\ w(x, 0), z(x, 0) &\geq 0, & x \in \Omega, \end{aligned} \tag{2.3}$$

where $d_i(t), c_i(x, t) \geq 0$ ($i = 1, 2$) in Q_T , and are bounded continuous functions. Then $w(x, t), z(x, t) \geq 0$ on $\bar{Q}_T$.

Proof. Let $K = \max\{K_1, K_2\} + 1$, where

$$\begin{aligned} K_1 &= \sup_{t \in (0, T]} a(0) d_1(t) \int_{\Omega} c_1(x, t) dx, \\ K_2 &= \sup_{t \in (0, T]} b(0) d_2(t) \int_{\Omega} c_2(x, t) dx. \end{aligned} \tag{2.4}$$

Since $c_i(x, t)$, $d_i(t)$ are bounded and continuous in Q_T , we know that $K < +\infty$. Let $w_1 = e^{-Kt}w$, $z_1 = e^{-Kt}z$, then we can deduce that $w_1(x, t), z_1(x, t) \geq 0$ on $\overline{Q}_T$. In fact, since $w_1(x, t), z_1(x, t) \geq 0$ for $(x, t) \in S_T$ or $x \in \Omega$, $t = 0$, if $\min\{w_1(x, t), z_1(x, t)\} < 0$ for some $(x, t) \in \overline{Q}_T$, then (w_1, z_1) has a negative minimum in Q_T . Without loss of generality, we can assume that $\min\{w_1(x, t), z_1(x, t)\}$ is taken at $(x_1, t_1) \in Q_T$ and $w_1(x_1, t_1) \leq w_1(x, t)$, $w_1(x_1, t_1) \leq z_1(x, t)$ for all $(x, t) \in \overline{Q}_T$. Using the first inequality in (2.3), we find that

$$w_{1t} - \Delta w_1 \geq -Kw_1(x, t) + a(x)d_1(t) \int_{\Omega} c_1(x, t)z_1(x, t)dx, \quad (x, t) \in Q_T, \quad (2.5)$$

and then it follows from $c_1(x, t) \geq 0$ in Q_T and (A_2) that

$$w_{1t}(x_1, t_1) - \Delta w_1(x_1, t_1) \geq \left(-K + a(x_1)d_1(t_1) \int_{\Omega} c_1(x, t_1)dx\right)w_1(x_1, t_1) \geq -w_1(x_1, t_1) > 0. \quad (2.6)$$

On the contrary, if $w_1(x, t)$ attains negative minimum at (x_1, t_1) , then,

$$w_1(x, t) \leq 0, \quad \Delta w_1(x_1, t_1) \geq 0, \quad w_{1t}(x_1, t_1) \leq 0, \quad (2.7)$$

and hence

$$w_{1t}(x_1, t_1) - \Delta w_1(x_1, t_1) \geq 0, \quad (2.8)$$

which leads to a contradiction to inequality (2.6). Thus $\min\{w_1(x, t), z_1(x, t)\} \geq 0$ on $\overline{Q}_T$, and therefore $w(x, t), z(x, t) \geq 0$ on $\overline{Q}_T$. $\square$

In order to get global existence and blow-up results, we need the following comparison principle which is a direct consequence of Lemma 2.2.

COROLLARY 2.3. *Let (u, v) be the unique nonnegative solution of (1.1). Assume that a pair of nonnegative functions $(w, z) \in C(\overline{Q}_T) \cap C^{2,1}(Q_T)$ and satisfy*

$$\begin{aligned} \omega_t &\geq (\leq) \Delta \omega + a(x) |z(t)|_r^p, \quad (x, t) \in \Omega \times (0, T), \\ z_t &\geq (\leq) \Delta z + b(x) |\omega(t)|_r^q, \quad (x, t) \in \Omega \times (0, T), \\ \omega(x, t) &\geq (=) z(x, t) \geq (=) 0, \quad x \in \partial\Omega, t > 0, \\ \omega(x, 0) &\geq (\leq) u_0(x), \quad z(x, 0) \geq (\leq) v_0(x), \quad x \in \overline{\Omega}. \end{aligned} \quad (2.9)$$

Then $(w(x, t), z(x, t)) \geq (\leq) (u(x, t), v(x, t))$ on $\overline{Q}_T$.

Proof. We only prove $(w(x, t), z(x, t)) \geq (u(x, t), v(x, t)) \geq (0, 0)$. A similar argument can be proved in other case. Let $\varphi_1(x, t) = w(x, t) - u(x, t)$, $\varphi_2(x, t) = z(x, t) - v(x, t)$. By the

mean value theorem,

$$\begin{aligned}
|z(t)|_r^p - |v(t)|_r^p &= \left(\int_{\Omega} |z(x,t)|^r dx \right)^{p/r} - \left(\int_{\Omega} |v(x,t)|^r dx \right)^{p/r} \\
&= \frac{p}{r} (\eta_1(t))^{(p-r)/r} \left[\int_{\Omega} (z^r(x,t) - v^r(x,t)) dx \right] \\
&= p (\eta_1(t))^{(p-r)/r} \left[\int_{\Omega} (\eta_2(x,t))^{r-1} (z(x,t) - v(x,t)) dx \right] \\
&= p (\eta_1(t))^{(p-r)/r} \left[\int_{\Omega} (\eta_2(x,t))^{r-1} \varphi_2(x,t) dx \right], \tag{2.10} \\
|w(t)|_r^q - |u(t)|_r^q &= \frac{q}{r} (\eta_3(t))^{(q-r)/r} \left[\int_{\Omega} (w^r(x,t) - u^r(x,t)) dx \right] \\
&= q (\eta_3(t))^{(q-r)/r} \left[\int_{\Omega} (\eta_4(x,t))^{r-1} (w(x,t) - u(x,t)) dx \right] \\
&= q (\eta_3(t))^{(q-r)/r} \left[\int_{\Omega} (\eta_4(x,t))^{r-1} \varphi_1(x,t) dx \right],
\end{aligned}$$

where $\eta_1, \eta_3 \geq 0$ are some intermediate values between $|z(t)|_r^r = \int_{\Omega} |z|^r dx$ and $|v(t)|_r^r = \int_{\Omega} |v|^r dx$, $|w(t)|_r^r = \int_{\Omega} |w|^r dx$, and $|u(t)|_r^r = \int_{\Omega} |u|^r dx$, respectively, $\eta_2, \eta_4 \geq 0$ are some intermediate values between $z(x,t)$ and $v(x,t)$, $w(x,t)$ and $u(x,t)$, respectively. Then by (2.9)-(2.10), the functions φ_1, φ_2 satisfies the relation

$$\begin{aligned}
\varphi_{1t} &\geq \triangle \varphi_1 + a(x) p (\eta_1(t))^{(p-r)/r} \left[\int_{\Omega} (\eta_2(x,t))^{r-1} \varphi_2(x,t) dx \right], \quad (x,t) \in \Omega \times (0,T), \\
\varphi_{2t} &\geq \triangle \varphi_2 + b(x) q (\eta_3(t))^{(q-r)/r} \left[\int_{\Omega} (\eta_4(x,t))^{r-1} \varphi_1(x,t) dx \right], \quad (x,t) \in \Omega \times (0,T), \\
\varphi_1(x,t), \varphi_2(x,t) &\geq 0, \quad x \in \partial\Omega, t > 0, \\
\varphi_1(x,0), \varphi_2(x,0) &\geq 0, \quad x \in \bar{\Omega}. \tag{2.11}
\end{aligned}$$

Lemma 2.2 implies that $\varphi_1, \varphi_2 \geq 0$, that is, $(w(x,t), z(x,t)) \geq (u(x,t), v(x,t))$. $\square$

From Corollary 2.3, we have the following lemma.

LEMMA 2.4. *Let (u, v) be the unique nonnegative solution of (1.1), and suppose that $(\bar{u}, \bar{v})$ and $(\underline{u}, \underline{v})$ are supersolution and subsolution of problem (1.1), respectively, then $(\bar{u}, \bar{v}) \geq (u, v) \geq (\underline{u}, \underline{v})$ on $\bar{Q}_T$.*

THEOREM 2.5. *Assume (A_1) -(A_2) hold, and $pq < 1$, then every nonnegative solution of system (1.1) exists globally.*

Proof. Let $\varphi(x)$ be the unique positive solution of the linear elliptic problem

$$-\Delta \varphi(x) = 1, \quad x \in \Omega; \quad \varphi(x) = 0, \quad x \in \partial\Omega. \tag{2.12}$$

Denote $C = \max_{x \in \Omega} \varphi(x)$. Then, $0 \leq \varphi(x) \leq C$. We define the functions $\bar{u}(x, t)$ and $\bar{v}(x, t)$ as

$$\bar{u} = (K(\varphi + 1))^{l_1}, \quad \bar{v} = (K(\varphi + 1))^{l_2}, \quad (2.13)$$

where $l_1, l_2 < 1$ and $K > 0$ will be fixed later. Clearly, $(\bar{u}, \bar{v})$ is bounded for any $T > 0$ and $\bar{u} \geq K^{l_1}$, $\bar{v} \geq K^{l_2}$.

Then we have

$$\begin{aligned} \bar{u}_t - \Delta \bar{u} &= -K^{l_1} (l_1 (l_1 - 1) (\varphi + 1)^{l_1-2} |\nabla \varphi|^2 + l_1 (\varphi + 1)^{l_1-1} \Delta \varphi) \geq l_1 (C + 1)^{l_1-1} K^{l_1}, \\ a(x) |\bar{v}|_r^p &= a(x) K^{p l_2} |(\varphi + 1)^{l_2}|_r^p \leq a(0) |\Omega|^{p/r} (C + 1)^{p l_2} K^{p l_2}, \\ \bar{v}_t - \Delta \bar{v} &\geq l_2 (C + 1)^{l_2-1} K^{l_2}, \quad b(x) |\bar{u}|_r^q \leq b(0) |\Omega|^{q/r} (C + 1)^{q l_1} K^{q l_1}. \end{aligned} \quad (2.14)$$

Denote

$$K_1 = \left(\frac{a(0) |\Omega|^{p/r}}{l_1} (C + 1)^{p l_2 - l_1 + 1} \right)^{1/(l_1 - p l_2)}, \quad K_2 = \left(\frac{b(0) |\Omega|^{q/r}}{l_2} (C + 1)^{q l_1 - l_2 + 1} \right)^{1/(l_2 - q l_1)}. \quad (2.15)$$

Now, since $pq < 1$, we can choose two positive constants $l_1, l_2 < 1$ such that

$$p < \frac{l_1}{l_2} < \frac{1}{q}, \quad (2.16)$$

hence $p l_2 < l_1$, $q l_1 < l_2$. We can choose K sufficiently large such that

$$K > \max \{K_1, K_2\}, \quad (2.17)$$

$$(K(\varphi + 1))^{l_1} \geq u_0(x), \quad (K(\varphi + 1))^{l_2} \geq v_0(x). \quad (2.18)$$

Now, it follows from (2.14)–(2.18) that $(\bar{u}, \bar{v})$ is a positive supersolution of (1.1). Hence by Lemma 2.4, $(u, v) \leq (\bar{u}, \bar{v})$, which implies that (u, v) exists globally. This completes the proof. $\square$

THEOREM 2.6. *Assume (A_1) – (A_2) hold, and $pq > 1$, then the nonnegative solution of system (1.1) exists globally for “small” initial data.*

Proof. Clearly, there exist positive constants $l_1, l_2 < 1$ such that

$$p > \frac{l_1}{l_2} > \frac{1}{q}, \quad (2.19)$$

hence $p l_2 > l_1$, $q l_1 > l_2$. We can choose K sufficiently small such that

$$K < \min \{K_1, K_2\}. \quad (2.20)$$

Furthermore, assume that u_0, v_0 are small enough to satisfy (2.18). Then it follows from (2.14), (2.18)–(2.20) that $(\bar{u}, \bar{v})$ is a positive supersolution of (1.1). We can also see that the solution is bounded from below. This completes the proof. $\square$

Remark 2.7. Furthermore, denote by $\psi(x)$ the unique positive solution of the linear elliptic problem

$$-\Delta\psi(x) = 1, \quad x \in \Omega_1; \quad \psi(x) = 0, \quad x \in \partial\Omega_1, \quad (2.21)$$

here $\Omega_1 \subset \subset \Omega$. It is obvious that $\psi(x)$ depends on Ω_1 continuously. By the comparison principle for elliptic equation, we have $\psi < \varphi$ on Ω_1 .

THEOREM 2.8. *Assume (A_1) – (A_2) hold, if $pq = 1$, then the nonnegative solution of (1.1) is global if the domain $(|\Omega|)$ is sufficiently small.*

Proof. If $pq = 1$, there exist positive constants $l_1, l_2 < 1$ such that

$$p = \frac{l_1}{l_2} = \frac{1}{q}, \quad (2.22)$$

hence $pl_2 = l_1$, $ql_1 = l_2$. Without loss of generality, we may assume that every domain under consideration is in a sufficiently large ball B . Denote by $\varphi_B(x)$ the unique positive solution of the following linear elliptic problem:

$$-\Delta\varphi(x) = 1, \quad x \in B; \quad \varphi(x) = 0, \quad x \in \partial B. \quad (2.23)$$

Let $C_0 = \max_{x \in B} \varphi_B(x)$. From Remark 2.7, we have $C \leq C_0$. Then we may assume that $|\Omega|$ is sufficiently small such that

$$|\Omega| < \min \left\{ \left(\frac{l_1}{a(0)(C_0 + 1)} \right)^{r/p}, \left(\frac{l_2}{b(0)(C_0 + 1)} \right)^{r/q} \right\}. \quad (2.24)$$

Furthermore, choose K large enough to satisfy (2.18). Then, it follows from (2.14), (2.18), and (2.24) that $(\bar{u}, \bar{v})$ is a positive supersolution of (1.1). By Lemma 2.4, we achieve the desired result. $\square$

THEOREM 2.9. *Assume (A_1) – (A_2) hold, and $pq > 1$, then the nonnegative solution of system (1.1) blows up if initial data is sufficiently large.*

Proof. Let $\varphi(x)$ be the first eigenfunction of $-\Delta$ in $H_0^1(\Omega)$ and let λ_1 be the corresponding eigenvalue. We choose $\varphi(x)$ such that $\varphi(x) > 0$ in Ω and $\max_{x \in \bar{\Omega}} \varphi(x) = 1$.

Since $pq > 1$, there exist two positive constants m, n such that $p > m/n$, $q > n/m$. Set $\gamma = \min\{np - m + 1, mq - n + 1\}$, $L = \min\{a(x)m^{-1}(\int_{\Omega} |\varphi|^{mr} dy)^{p/r}, b(x)n^{-1}(\int_{\Omega} |\varphi|^{mr} dy)^{q/r}\}$. Let $s(t)$ be the solution of the Cauchy problem: $s' = -\lambda_1 s + Ls^\gamma$, $s(0) = s_0 > 0$. Since $\gamma > 1$, then $s(t)$ blows up in finite time for sufficiently large datum s_0 .

Set $\underline{u}(x, t) = s^m(t)\varphi^m(x)$, $\underline{v}(x, t) = s^n(t)\varphi^n(x)$. We can assert that $(\underline{u}, \underline{v})$ is a subsolution of system (1.1). A direct computation yields

$$\begin{aligned}
 \Delta \underline{u} + a(x) \left(\int_{\Omega} |\underline{v}|^r dy \right)^{p/r} &= s^m (m\varphi^{m-1} \Delta \varphi + m(m-1)\varphi^{m-2} |\nabla \varphi|^2) + a(x) s^{np} \left(\int_{\Omega} |\varphi|^{nr} dy \right)^{p/r} \\
 &\geq ms^m \varphi^m \left(-\lambda_1 + a(x) s^{np-m} m^{-1} \left(\int_{\Omega} |\varphi|^{nr} dy \right)^{p/r} \right) \\
 &\geq ms^{m-1} \varphi^m s' = \underline{u}_t, \\
 \Delta \underline{v} + b(x) \left(\int_{\Omega} |\underline{u}|^r dy \right)^{q/r} &= s^n (n\varphi^{n-1} \Delta \varphi + n(n-1)\varphi^{n-2} |\nabla \varphi|^2) + b(x) s^{mq} \left(\int_{\Omega} |\varphi|^{mr} dy \right)^{q/r} \\
 &\geq ns^n \varphi^n \left(-\lambda_1 + b(x) s^{mq-n} n^{-1} \left(\int_{\Omega} |\varphi|^{mr} dy \right)^{q/r} \right) \\
 &\geq ns^{n-1} \varphi^n s' = \underline{v}_t.
 \end{aligned} \tag{2.25}$$

Therefore, $(\underline{u}, \underline{v})$ is a subsolution of problem (1.1) provided that the initial data are sufficiently large such that $u_0 \geq \underline{u}(x, 0)$, $v_0 \geq \underline{v}(x, 0)$. By Lemma 2.4, we get that $(\underline{u}, \underline{v}) \leq (u, v)$ and (u, v) blows up in finite time. $\square$

From Theorems 2.5-2.6 and Theorems 2.8-2.9, we see that the critical exponent of the system is $pq = 1$.

Remark 2.10. If $a(x) = \text{constant}$, $b(x) = \text{constant}$, then the conclusions of Theorems 2.5-2.6 and Theorems 2.8-2.9 still hold for $\Omega \subset \mathbb{R}^N$ being a bounded domain with smooth boundary.

3. Uniform blow-up profiles

In this section, we assume that the nonnegative solution (u, v) of (1.1) blows up in finite time, we denote the blow-up time of the solution (u, v) by T^* . Throughout this section, we investigate the blow-up profile of the system (1.1). At first, we cite an important result which belongs to Liu et al. for uncouple diffusion equations with nonlocal nonlinear source (see [4]) as the basic lemma of our discussion. In the proof, the authors make use of the maximum principle (see [20, 21]) and sub-supersolution method (see [16]).

From [4, Theorem 3.1], we give the following lemma.

LEMMA 3.1. *Let $u \in C^{2,1}(\overline{\Omega} \times (0, T))$ be the solution of the problem*

$$\begin{aligned}
 u_t &= \Delta u + a(x)g(t), \quad x \in \Omega, \quad t > 0, \\
 u(x, t) &= 0, \quad x \in \partial\Omega, \quad t > 0, \\
 u(x, 0) &= u_0(x), \quad x \in \Omega,
 \end{aligned} \tag{3.1}$$

where the function $g(t) \geq 0$ will depend on the solution u , and $G(t) = \int_0^t g(s)ds$. Assume

that (A_1) , (A_2) hold, and $g(t)$ is nonnegative, continuous, and nondecreasing on $(0, T^*)$, $\lim_{t \rightarrow T^*} G(t) = +\infty$, then

$$\lim_{t \rightarrow T^*} \frac{u(x, t)}{G(t)} = a(x), \quad (3.2)$$

uniformly in all compact subsets of Ω .

In this section, we sometimes use the notation $u \sim v$ for $\lim_{t \rightarrow T^*} u(t)/v(t) = 1$. Denote

$$\begin{aligned} g_1(t) &= |v(t)|_r^p, \quad g_2(t) = |u(t)|_r^q, \\ G_1(t) &= \int_0^t g_1(s) ds, \quad G_2(t) = \int_0^t g_2(s) ds, \quad t \in (0, T^*), \end{aligned} \quad (3.3)$$

and set

$$\begin{aligned} U(t) &= \max_{x \in \bar{\Omega}} u(x, t), \quad V(t) = \max_{x \in \bar{\Omega}} v(x, t), \quad t \in [0, T^*), \\ a_0 &= \max_{x \in \bar{\Omega}} a(x), \quad b_0 = \max_{x \in \bar{\Omega}} b(x), \end{aligned} \quad (3.4)$$

then we have the following lemma.

LEMMA 3.2. *Let (u, v) be a nonnegative solution of (1.1). Assume that the initial data u_0 and v_0 satisfy (A_1) – (A_2) , and*

- (i) *(u, v) has blow-up time $T^* < \infty$,*
- (ii) *$u_t, v_t \geq 0$ for $(x, t) \in \Omega \times (0, T^*)$.*

Then, we have

$$\lim_{t \rightarrow T^*} G_1(t) = \lim_{t \rightarrow T^*} G_2(t) = +\infty, \quad (3.5)$$

and there exist two positive constants C_1 and C_2 such that

$$u(x, t) \leq a_0 G_1(t) + C_1, \quad v(x, t) \leq b_0 G_2(t) + C_2, \quad (x, t) \in \bar{\Omega} \times [0, T^*). \quad (3.6)$$

Proof. Rewrite system (1.1) as follows:

$$\begin{aligned} u_t &= \Delta u(x, t) + a(x)g_1(t), \quad (x, t) \in \Omega \times (0, T^*), \\ v_t &= \Delta v(x, t) + b(x)g_2(t), \quad (x, t) \in \Omega \times (0, T^*). \end{aligned} \quad (3.7)$$

Using similar arguments as in [22], we give the proof of this lemma. Let

$$U(t) = \max_{x \in \bar{\Omega}} u(x, t) = u(x_0, t), \quad V(t) = \max_{x \in \bar{\Omega}} v(x, t) = v(x_1, t). \quad (3.8)$$

Then functions $U(t)$, $V(t)$ satisfy

$$U'(t) = u_t(x_0, t) = \Delta u(x_0, t) + a(x_0)g_1(t), \quad V'(t) = v_t(x_1, t) = \Delta v(x_1, t) + b(x_1)g_2(t) \quad (3.9)$$

since $\Delta u(x_0, t) \leq 0$, $\Delta v(x_1, t) \leq 0$, we get

$$0 \leq U'(t) \leq a_0 g_1(t), \quad 0 \leq V'(t) \leq b_0 g_2(t), \quad \text{a.e. } (0, T^*). \quad (3.10)$$

Integrating the above inequalities over $(0, t)$ for $t \in (0, T^*)$, we get

$$0 \leq U(t) \leq U(0) + a_0 G_1(t), \quad 0 \leq V(t) \leq V(0) + b_0 G_2(t). \quad (3.11)$$

Since the nonnegative solution (u, v) of (1.1) blows up in finite time T^* , we know that

$$\lim_{t \rightarrow T^*} U(t) = \lim_{t \rightarrow T^*} \max_{x \in \bar{\Omega}} u(x, t) = +\infty, \quad \lim_{t \rightarrow T^*} V(t) = \lim_{t \rightarrow T^*} \max_{x \in \bar{\Omega}} v(x, t) = +\infty. \quad (3.12)$$

Then (3.5) follows from (3.11), (3.12), and the facts that $U(0) = \max_{x \in \bar{\Omega}} u_0 < +\infty$ and $V(0) = \max_{x \in \bar{\Omega}} v_0 < +\infty$. Moreover, inequality (3.6) follows from (3.11), (3.12), and nonnegativity of $U(t)$ and $V(t)$, where $C_1 = U(0) = \max_{x \in \bar{\Omega}} u_0(x)$ and $C_2 = V(0) = \max_{x \in \bar{\Omega}} v_0(x)$. $\square$

Remark 3.3. Lemma 3.2 implies that if u and v have a finite blow-up time T^* , then $G_1(t)$ and $G_2(t)$ blow-up in the same time T^* also.

From Lemmas 3.1 and 3.2, we get the following theorem immediately.

THEOREM 3.4. *Let (u, v) be a classical solution of (1.1) with blow-up time T^* , then*

$$\lim_{t \rightarrow T^*} \frac{u(x, t)}{G_1(t)} = a(x), \quad \lim_{t \rightarrow T^*} \frac{v(x, t)}{G_2(t)} = b(x), \quad (3.13)$$

uniformly in all compact subsets of Ω .

As a straightforward result of Theorem 3.4, we have the following theorem on the blow-up set.

THEOREM 3.5. *Let (u, v) be blow-up solution of (1.1), then the blow-up set of (1.1) is the whole domain Ω , that is to say, the blow-up solution (u, v) has a global blow-up.*

THEOREM 3.6. *Assume $pq > 1$, let (u, v) be a solution of (1.1) with blow-up time T^* , then*

$$\lim_{t \rightarrow T^*} (T^* - t)^\alpha u(x, t) = a(x) C_1 \left(\int_{\Omega} a^r(x) dx \right)^{pq/r(1-pq)} \left(\int_{\Omega} b^r(x) dx \right)^{p/r(1-pq)}, \quad (3.14)$$

$$\lim_{t \rightarrow T^*} (T^* - t)^\beta v(x, t) = b(x) C_2 \left(\int_{\Omega} a^r(x) dx \right)^{q/r(1-pq)} \left(\int_{\Omega} b^r(x) dx \right)^{pq/r(1-pq)}, \quad (3.15)$$

in which $\alpha = (p+1)/(pq-1)$, $\beta = (q+1)/(pq-1)$, $C_1 = ((p+1)/(pq-1))^\alpha ((q+1)/(p+1))^{\alpha p/(p+1)}$, $C_2 = ((p+1)/(q+1))^{\beta q/(q+1)} ((q+1)/(pq-1))^\beta$.

Proof. By (3.13) in Theorem 3.4, it follows that

$$\forall x \in \Omega, \quad \lim_{t \rightarrow T^*} \frac{|u(x, t)^r|}{G_1^r(t)} = a^r(x), \quad \lim_{t \rightarrow T^*} \frac{|v(x, t)^r|}{G_2^r(t)} = b^r(x). \quad (3.16)$$

Moreover, (3.6) in Lemma 3.2 implies that for all $\varepsilon > 0$, $0 \leq |u(x, t)^r|/G_1^r(t) \leq a^r(x) + \varepsilon$, $0 \leq |v(x, t)^r|/G_2^r(t) \leq b^r(x) + \varepsilon$ in Ω for t close enough to T^* . By the Lebesgue's dominated convergence theorem, we infer that $\int_{\Omega} |u(y, t)|^r dy \sim \int_{\Omega} a^r(x) dx G_1^r(t)$, $\int_{\Omega} |v(y, t)|^r dy \sim \int_{\Omega} b^r(x) dx G_2^r(t)$ as $t \rightarrow T^*$, then we have

$$\begin{aligned} G_1'(t) &= g_1(t) = |v(t)|_r^p = \left(\int_{\Omega} |v(y, t)|^r dy \right)^{p/r} \sim \left(\int_{\Omega} b^r(x) dx \right)^{p/r} G_2^p(t), \\ G_2'(t) &= g_2(t) = |u(t)|_r^q = \left(\int_{\Omega} |u(y, t)|^r dy \right)^{q/r} \sim \left(\int_{\Omega} a^r(x) dx \right)^{q/r} G_1^q(t), \end{aligned} \quad (3.17)$$

which implies that

$$\left(\int_{\Omega} a^r(x) dx \right)^{q/r} G_1^q G_1' \sim \left(\int_{\Omega} b^r(x) dx \right)^{p/r} G_2^p G_2' \quad \text{as } t \rightarrow T^*. \quad (3.18)$$

Because $G_1(t), G_2(t) \rightarrow \infty$ as $t \rightarrow T^*$, it follows from (3.18) that

$$\left(\int_{\Omega} a^r(x) dx \right)^{q/r} \frac{G_1^{q+1}(t)}{q+1} \sim \left(\int_{\Omega} b^r(x) dx \right)^{p/r} \frac{G_2^{p+1}(t)}{p+1}. \quad (3.19)$$

From (3.17) and (3.19), we have

$$\begin{aligned} G_1'(t) &\sim \left(\int_{\Omega} b^r(x) dx \right)^{p/r} G_2^p(t) \sim \left(\frac{p+1}{q+1} \right)^{p/(p+1)} \left(\int_{\Omega} a^r(x) dx \right)^{pq/r(p+1)} \\ &\quad \times \left(\int_{\Omega} b^r(x) dx \right)^{p/r(p+1)} G_1^{p(q+1)/(p+1)}, \end{aligned} \quad (3.20)$$

it follows that

$$\frac{p+1}{1-pq} \left(G_1^{(1-pq)/(p+1)} \right)' \sim \left(\frac{p+1}{q+1} \right)^{p/(p+1)} \left(\int_{\Omega} a^r(x) dx \right)^{pq/r(p+1)} \left(\int_{\Omega} b^r(x) dx \right)^{p/r(p+1)}, \quad (3.21)$$

that is,

$$\frac{p+1}{1-pq} \left(G_1^{(1-pq)/(p+1)} \right)' = \left(\frac{p+1}{q+1} \right)^{p/(p+1)} \left(\int_{\Omega} a^r(x) dx \right)^{pq/r(p+1)} \left(\int_{\Omega} b^r(x) dx \right)^{p/r(p+1)} + \alpha(t), \quad (3.22)$$

where $\alpha(t) \rightarrow 0$ as $t \rightarrow T^*$. Integrating over (t, T^*) , we have

$$G_1(T^* - t)^{\alpha} \sim C_1 \left(\int_{\Omega} a^r(x) dx \right)^{pq/r(1-pq)} \left(\int_{\Omega} b^r(x) dx \right)^{p/r(1-pq)}. \quad (3.23)$$

From (3.23) and Theorem 3.4, we have

$$\begin{aligned} (T^* - t)^{\alpha} u(x, t) &\sim G_1(t) a(x) (T^* - t)^{\alpha} \\ &\sim a(x) C_1 \left(\int_{\Omega} a^r(x) dx \right)^{pq/r(1-pq)} \left(\int_{\Omega} b^r(x) dx \right)^{p/r(1-pq)}. \end{aligned} \quad (3.24)$$

Then we get (3.14). The second equality (3.15) can be proved analogously. This completes the proof. $\square$

Remark 3.7. From Theorem 3.6, we have

$$\begin{aligned} G_1(t) &\sim C_1 \left(\int_{\Omega} a^r(x) dx \right)^{pq/r(1-pq)} \left(\int_{\Omega} b^r(x) dx \right)^{p/r(1-pq)} (T^* - t)^{-\alpha}, \\ G_2(t) &\sim C_2 \left(\int_{\Omega} a^r(x) dx \right)^{q/r(1-pq)} \left(\int_{\Omega} b^r(x) dx \right)^{pq/r(1-pq)} (T^* - t)^{-\beta} \end{aligned} \quad (3.25)$$

as $t \rightarrow T^*$, C_1, C_2 defined as in Theorem 3.6.

Remark 3.8. If $a(x) = \text{constant}$, $b(x) = \text{constant}$, then the conclusions of Theorem 3.6 and Remark 3.7 still hold for $\Omega \subset \mathbb{R}^N$ being a bounded domain with smooth boundary.

4. Boundary layer estimates

Throughout this section, we deal with boundary layer estimate of (1.1) with $a(x) = a$, $b(x) = b$ in which a, b are constants. At first we cite some conclusions belonging to Souplet (see [1]) for the uncoupled equation (3.1) with $a(x) = 1$.

Definition 4.1. Say that g is standard if it satisfies the following power-like conditions

$$k_1(T-t)^{-1} \leq \frac{g(t)}{G(t)} \leq k_2(T-t)^{-1} \quad \text{as } t \rightarrow T \quad (4.1)$$

for some constant $k_2 \geq k_1 \geq 0$.

Remark 4.2. According to the note after [1, Definition 4.1], we note that if g is standard, then $C_1(T-t)^{-(k_1+1)} \leq g(t) \leq C_2(T-t)^{-(k_1+1)}$ as $t \rightarrow T$. Conversely, g is standard whenever $c_1(T-t)^{-\gamma} \leq g(t) \leq c_2(T-t)^{-\gamma}$. Therefore, $g(t)$ is standard, if and only if $c'_1(T-t)^{-\gamma+1} \leq G(t) \leq c'_2(T-t)^{-\gamma+1}$ as $t \rightarrow T$ for some $\gamma > 1$ and $c_2 \geq c_1 > 0$, $c'_2 \geq c'_1 > 0$.

LEMMA 4.3 [1, Theorem 4.5]. *Let $g(t)$ be standard and let $\omega(x, t)$ be a solution of (3.1) in which $a(x) = 1$ with blow-up time T . Denote by $d(x) = \text{dist}(x, \partial\Omega)$. Then for all $K > 0$, there exist constants $m_k, m'_k > 0$, and some $t_0 \in (0, T)$ such that*

$$m_k \frac{d(x)}{\sqrt{T-t}} G(t) \leq \omega(x, t) \leq m'_k \frac{d(x)}{\sqrt{T-t}} G(t) \quad (4.2)$$

for $(x, t) \in \{(x, t) \in \Omega \times [t_0, T) : d(x) \leq K\sqrt{T-t}\}$.

LEMMA 4.4 [1, Theorem 4.6]. *Let $g(t)$ and $G(t)$ be standard, and let $\omega(x, t)$ be a solution of (3.1) in which $a(x) = 1$ with blow-up time T . Then $|\omega(x, t)|_{\infty} (1 - C(T-t)/d^2(x)) \leq \omega(x, t)$ in $\Omega \times [t_0, T)$ for some $C > 0$ and some $t_0 \in (0, T)$.*

The above lemmas will be used to determine the boundary layer estimates of solutions to problem (1.1). By using the conclusions of blow-up rates for problem (1.1) in Section 3 together with Lemmas 4.3 and 4.4, we have the following results.

LEMMA 4.5. For system (1.1) with $a(x) = a$, $b(x) = b$, the same conclusions of Lemmas 4.3 and 4.4 still hold.

THEOREM 4.6. Under the assumptions of Theorem 3.6, let (u, v) be a solution of (1.1) with blow-up time T . Then for all $K > 0$, there exist some constants $C_2 \geq C_1 > 0$, $C_4 \geq C_3 > 0$ and some $t_0 \in (0, T)$, such that (u, v) satisfies

$$\begin{aligned} C_1 \frac{d(x)}{\sqrt{T-t}} |u(t)|_\infty &\leq u(x, t) \leq C_2 \frac{d(x)}{\sqrt{T-t}} |u(t)|_\infty, \\ C_3 \frac{d(x)}{\sqrt{T-t}} |v(t)|_\infty &\leq v(x, t) \leq C_4 \frac{d(x)}{\sqrt{T-t}} |v(t)|_\infty \end{aligned} \quad (4.3)$$

for $(x, t) \in \{(x, t) \in \Omega \times [t_0, T) : d(x) \leq K\sqrt{T-t}\}$.

Proof. From (3.25), we have $G_1(t) \sim d_1(T-t)^{-\alpha}$, $G_2(t) \sim d_2(T-t)^{-\beta}$ as $t \rightarrow T$, in which $d_1, d_2 > 0$, $\alpha, \beta > 0$. For some $t_0 \in [0, T)$, there exist four positive constants m_i ($1 \leq i \leq 4$) such that

$$\begin{aligned} m_1(T-t)^{-\alpha} &\leq G_1(t) \leq m_2(T-t)^{-\alpha}, \\ m_3(T-t)^{-\beta} &\leq G_2(t) \leq m_4(T-t)^{-\beta} \quad \text{for } t \in [t_0, T). \end{aligned} \quad (4.4)$$

It follows that

$$\begin{aligned} m_1(T-t)^{-\delta_1+1} &\leq G_1(t) \leq m_2(T-t)^{-\delta_1+1}, \\ m_3(T-t)^{-\delta_2+1} &\leq G_2(t) \leq m_4(T-t)^{-\delta_2+1} \quad \text{for } t \in [t_0, T), \end{aligned} \quad (4.5)$$

where $\delta_1 = \alpha + 1 > 1$, $\delta_2 = \beta + 1 > 1$. Hence by Remark 4.2, it follows that $g_1(t)$, $g_2(t)$ are standard. By using Lemma 4.3 and (3.13), we get the results immediately. $\square$

THEOREM 4.7. Under the assumptions of Theorem 3.6, let (u, v) be a solution of (1.1) with blow-up time T . Then for all $K > 0$, there exist some constants $C_5, C_6 > 0$ and some $t_0 \in (0, T)$, such that (u, v)

$$\begin{aligned} |u(t)|_\infty \left(1 - \frac{C_5(T-t)}{d^2(x)}\right) &\leq u(x, t), \\ |v(t)|_\infty \left(1 - \frac{C_6(T-t)}{d^2(x)}\right) &\leq v(x, t) \end{aligned} \quad (4.6)$$

for all $(x, t) \in \Omega \times [t_0, T)$.

Proof of this theorem is similar to the above theorem, so we omit it here.

Remark 4.8. Theorem 4.6 implies some boundary layer estimates that

$$\lim_{t \rightarrow T} \frac{u(x, t)}{|u(t)|_\infty} = 0, \quad \lim_{t \rightarrow T} \frac{v(x, t)}{|v(t)|_\infty} = 0 \quad (4.7)$$

for $x \in \{x \in \Omega : d(x) \leq K\sqrt{T-t}\}$ satisfying $d(x)/\sqrt{T-t} \rightarrow 0$ as $t \rightarrow T$. Similarly, it follows from Theorem 4.7 that

$$\lim_{t \rightarrow T} \frac{u(x, t)}{|u(t)|_\infty} = 1, \quad \lim_{t \rightarrow T} \frac{v(x, t)}{|v(t)|_\infty} = 1 \quad (4.8)$$

for $x \in \Omega$ satisfying $d(x)/\sqrt{T-t} \rightarrow \infty$ as $t \rightarrow T$.

Due to the above discussion, we know that the size of boundary layer of (1.1) decays like $\sqrt{T-t}$.

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