INFLUENCE OF TEMPERATURE-DEPENDENT VISCOSITY ON THE MHD COUETTE FLOW OF DUSTY FLUID WITH HEAT TRANSFER

HAZEM A. ATTIA

Received 23 December 2005; Revised 21 February 2006; Accepted 29 May 2006

This paper studies the effect of variable viscosity on the transient Couette flow of dusty fluid with heat transfer between parallel plates. The fluid is acted upon by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below. The upper plate is moving with a uniform velocity while the lower is kept stationary. The governing nonlinear partial differential equations are solved numerically and some important effects for the variable viscosity and the uniform magnetic field on the transient flow and heat transfer of both the fluid and dust particles are indicated.

Copyright © 2006 Hazem A. Attia. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction

The study of the flow of dusty fluids has important applications in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, and fluid droplets sprays.

The hydrodynamic flow of dusty fluids was studied by a number of authors [6–8, 13, 14]. Later, the influence of the magnetic field on the flow of electrically conducting dusty fluids was studied [1, 5, 11, 12, 16]. Most of these studies are based on constant physical properties. More accurate prediction for the flow and heat transfer can be achieved by taking into account the variation of these properties, especially the variation of the fluid viscosity with temperature [9]. Klemp et al. [10] have studied the effect of temperature-dependent viscosity on the entrance flow in a channel in the hydrodynamic case. Attia and Kotb [4] studied the steady MHD fully developed flow and heat transfer between two parallel plates with temperature-dependent viscosity. Later, Attia [3] has extended the problem to the transient state.

Hindawi Publishing Corporation
Differential Equations and Nonlinear Mechanics
Volume 2006, Article ID 75290, Pages 1–14
DOI 10.1155/DENM/2006/75290

In the present work, the effect of variable viscosity on the unsteady flow of an electrically conducting, viscous, incompressible dusty fluid and heat transfer between parallel nonconducting porous plates is studied. The fluid is flowing between two electrically insulating infinite plates maintained at two constants but different temperatures. An external uniform magnetic field is applied perpendicular to the plates. The upper plate is moving with a uniform velocity while the lower is kept stationary. The magnetic Reynolds number is assumed small so that the induced magnetic field is neglected. The fluid is acted upon by a constant pressure gradient and its viscosity is assumed to vary exponentially with temperature. The flow and temperature distributions of both the fluid and dust particles are governed by the coupled set of the momentum and energy equations. The Joule and viscous dissipation terms in the energy equation are taken into consideration. The governing coupled nonlinear partial differential equations are solved numerically using the finite difference approximations. The effects of the external uniform magnetic field and the temperature-dependent viscosity on the time development of both the velocity and temperature distributions are discussed.

2. Description of the problem

The dusty fluid is assumed to be flowing between two infinite horizontal plates located at the $y = \pm h$ planes. The dusty particles are assumed to be uniformly distributed throughout the fluid. The two plates are assumed to be electrically nonconducting and kept at two constant temperatures, T_1 for the lower plate and T_2 for the upper plate with $T_2 > T_1$. The upper plate is moving with a uniform velocity U_o while the lower is kept stationary. A constant pressure gradient is applied in the x-direction and the parallel plates are assumed to be porous and subjected to a uniform suction from above and injection from below. Thus the y-component of the velocity is constant and denoted by v_o. A uniform magnetic field B_o is applied in the positive y-direction. By assuming a very small magnetic Reynolds number the induced magnetic field is neglected [17]. The fluid motion starts from rest at $t = 0$, and the no-slip condition at the plates implies that the fluid and dust particles velocities have neither a z- nor an x-component at $y = \pm h$. The initial temperatures of the fluid and dust particles are assumed to be equal to T_1 and the fluid viscosity is assumed to vary exponentially with temperature. Since the plates are infinite in the x- and z-directions, the physical variables are invariant in these directions. The flow of the fluid is governed by the Navier-Stokes equation [17]

$$\rho\frac{\partial u}{\partial t}+\rho v_o\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\sigma B_o^2u-KN(u-u_p), \tag{2.1}$$

where ρ is the density of clean fluid, μ is the viscosity of clean fluid, u is the velocity of fluid, u_p is the velocity of dust particles, σ is the electric conductivity, p is the pressure acting on the fluid, N is the number of dust particles per unit volume, and K is a constant. The first three terms in the right-hand side are, respectively, the pressure gradient, viscosity, and Lorentz force terms. The last term represents the force term due to the relative motion between fluid and dust particles. It is assumed that the Reynolds number of the relative velocity is small. In such a case the force between dust and fluid is proportional

to the relative velocity [14]. The motion of the dust particles is governed by Newton's second law [14]

$$m_p \frac{\partial u_p}{\partial t} = KN(u - u_p), \tag{2.2}$$

where m_p is the average mass of dust particles. The initial and boundary conditions on the velocity fields are, respectively, given by

$$t = 0 : u = u_p = 0. \tag{2.3}$$

For $t > 0$, the no-slip condition at the plates implies that

$$\begin{aligned} y = -h &: u = 0, \\ y = h &: u = U_o. \end{aligned} \tag{2.4}$$

Heat transfer takes place from the upper hot plate towards the lower cold plate by conduction through the fluid. Also, there is a heat generation due to both the Joule and viscous dissipations. The dust particles gain heat energy from the fluid by conduction through their spherical surface. Two energy equations are required which describe the temperature distributions for both the fluid and dust particles and are, respectively, given by [15]

$$\rho c \frac{\partial T}{\partial t} + \rho c v_o \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \sigma B_o^2 u^2 + \frac{\rho_p C_s}{\gamma_T} (T_p - T), \tag{2.5}$$

$$\frac{\partial T_p}{\partial t} = -\frac{1}{\gamma_T} (T_p - T), \tag{2.6}$$

where T is the temperature of the fluid, T_p is the temperature of the particles, c is the specific heat capacity of the fluid at constant pressure, C_s is the specific heat capacity of the particles, k is the thermal conductivity of the fluid, γ_T is the temperature relaxation time ($= 3 \Pr \gamma_p C_s / 2c$), γ_p is the velocity relaxation time ($= 2\rho_s D^2 / 9\mu$), ρ_s is the material density of dust particles ($= 3\rho_p / 4\pi D^3 N$), and D is the average radius of dust particles. The last three terms in the right-hand side of (2.5) represent the viscous dissipation, the Joule dissipation, and the heat conduction between the fluid and dust particles. The initial and boundary conditions on the temperature fields are given as

$$\begin{aligned} t \le 0 &: T = T_p = 0, \\ t > 0, \quad y = -h &: T = T_1, \\ t > 0, \quad y = h &: T = T_2. \end{aligned} \tag{2.7}$$

The viscosity of the fluid is assumed to depend on temperature and is defined as $\mu = \mu_o f(T)$. For practical reasons which are shown to be suitable for most kinds of fluids [2, 10], the viscosity is assumed to vary exponentially with temperature. The function

$f(T)$ takes the form [2, 10], $f(T) = e^{-b(T-T_1)}$, where the parameter b has the dimension of $[T]^{-1}$ and such that at $T = T_1$, $f(T_1) = 1$ and then $\mu = \mu_o$. This means that μ_o is the viscosity coefficient at $T = T_1$. The parameter a_1 may take positive values for liquids such as water, benzene, or crude oil. In some gases like air, helium, or methane a_1 it may be negative, that is, the coefficient viscosity increases with temperature [2, 10].

The temperature variations within a convective flow give rise to variations in the properties of the fluid, in the density and viscosity, for example. An analysis including the full effects of these is so complicated that some approximations become essential. The equations are commonly used in a form known as the Boussinesq approximation. In the Boussinesq approximation, variations of all fluid properties other than the density are ignored completely. Variations of the density are ignored except insofar as they give rise to gravitational force [18]. Therefore, a buoyancy force term may be included in the Navier-Stokes equation which equals $-\alpha\rho\Delta T$, where α is the coefficient of expansion of the fluid. Such a buoyancy term may be neglected on the basis of either ΔT small, that is, $T_2 - T_1$ is small, or small α which is a reasonable approximation for liquids and perfect gases [18].

The problem is simplified by writing the equations in the nondimensional form. The characteristic length is taken to be h and the characteristic velocity is U_o. We define the following nondimensional quantities:

$$(\hat{x},\hat{y}) = \frac{(x,y)}{h}, \qquad \hat{t} = \frac{tU_o}{h}, \qquad \hat{P} = \frac{P}{\rho U_o^2}, \qquad \lambda = -\frac{d\hat{p}}{d\hat{x}}, \qquad (\hat{u},\hat{v}) = \frac{(u,v)}{U_o},$$
$$(\hat{u}_p,\hat{v}_p) = \frac{(u_p,v_p)}{U_o}, \qquad \hat{T} = \frac{T-T_1}{T_2-T_1}, \qquad \hat{T}_p = \frac{T_p-T_1}{T_2-T_1}, \tag{2.8}$$

$f(\hat{T}) = e^{-b(T_2-T_1)\hat{T}} = e^{-a\hat{T}}$, a is the viscosity variation parameter,
$\mathrm{Ha}^2 = \sigma B_o^2 h^2/\mu_o$, Ha is the Hartmann number,
$R = KNh^2/\mu_o$ is the particle concentration parameter,
$G = m_p U_o/(hK)$ is the particle mass parameter,
$S = v_o/U_o$ is the suction parameter,
$\mathrm{Pr} = \mu_o c/k$ is the Prandtl number,
$\mathrm{Ec} = U_o^2/(c(T_2 - T_1))$ is the Eckert number,
$L_o = \rho h^2/\mu_o\gamma_T$ is the temperature relaxation time parameter.

In terms of the above nondimensional variables and parameters (2.1) to (2.7) take the form (the hats are dropped for convenience)

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = \lambda + f(T)\frac{\partial^2 u}{\partial y^2} + \frac{\partial f(T)}{\partial y}\frac{\partial u}{\partial y} - \mathrm{Ha}^2 u - R(u-u_p), \tag{2.9}$$

$$G\frac{\partial u_p}{\partial t} = (u-u_p), \tag{2.10}$$

$$t \le 0 : u = u_p = 0,$$
$$\mathrm{t} > 0, \quad y = -1 : u = 0, \tag{2.11}$$
$$\mathrm{t} > 0, \quad y = 1 : u = 1,$$

$$\frac{\partial T}{\partial t}+S\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}+\operatorname{Ec} f(T)\left(\frac{\partial u}{\partial y}\right)^2+\operatorname{Ec}\operatorname{Ha}^2 u^2+\frac{2R}{3\Pr}(T_p-T), \tag{2.12}$$

$$\frac{\partial T_p}{\partial t}=-L_o(T_p-T), \tag{2.13}$$

$$t\le 0: T=T_p=0,$$

$$t>0,\quad y=-1: T=0, \tag{2.14}$$

$$t>0,\quad y=1: T=1.$$

Equations (2.9), (2.10), (2.12), and (2.13) represent a system of coupled and nonlinear partial differential equations which are solved numerically under the initial and boundary conditions (2.11) and (2.14) using the finite difference approximations. A linearization technique is first applied to replace the nonlinear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank-Nicolson implicit method is used at two successive time levels [2]. An iterative scheme is used to solve the linearized system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas algorithm [2]. Finite difference equations relating the variables are obtained by writing the equations at the midpoint of the computational cell and then replacing the different terms by their second-order central difference approximations in the y-direction. The diffusion terms are replaced by the average of the central differences at two successive time levels. The computational domain is divided into meshes each of dimension Δt and Δy in time and space, respectively. We define the variables $v=\partial u/\partial y$ and $H=\partial\theta/\partial y$ to reduce the second-order differential equations (2.9) and (2.12) to first-order differential equations which are

$$\begin{aligned}
&\left(\frac{u_{i+1,j+1}-u_{i,j+1}+u_{i+1,j}-u_{i,j}}{2\Delta t}\right)+S\left(\frac{v_{i+1,j+1}+v_{i,j+1}+v_{i+1,j}+v_{i,j}}{4}\right)\\
&\quad=\alpha+\left(\frac{\overline{f}_1(T)_{i,j+1}+\overline{f}_1(T)_{i,j}}{2}\right)\times\left(\frac{(v_{i+1,j+1}+v_{i,j+1})-(v_{i+1,j}+v_{i,j})}{2\Delta y}\right)\\
&\qquad+\left(\frac{\overline{f}_1(T)_{i,j+1}-\overline{f}_1(T)_{i,j}}{\Delta y}\right)\left(\frac{v_{i+1,j+1}+v_{i,j+1}+v_{i+1,j}+v_{i,j}}{4}\right)\\
&\qquad-\operatorname{Ha}^2\left(\frac{u_{i+1,j+1}+u_{i,j+1}+u_{i+1,j}+u_{i,j}}{4}\right)-R\left(\frac{u_{i+1,j+1}+u_{i,j+1}+u_{i+1,j}+u_{i,j}}{4}\right)\\
&\qquad+R\left(\frac{u_{p_{i+1,j+1}}+u_{p_{i,j+1}}+u_{p_{i+1,j}}+u_{p_{i,j}}}{4}\right),
\end{aligned}$$

$$
\begin{aligned}
&G\left(\frac{u_{p_{i+1,j+1}}-u_{p_{i,j+1}}+u_{p_{i+1,j}}-u_{p_{i,j}}}{2\Delta t}\right)\\
&\quad=\left(\frac{u_{i+1,j+1}+u_{i,j+1}+u_{i+1,j}+u_{i,j}}{4}-\frac{u_{p_{i+1,j+1}}+u_{p_{i,j+1}}+u_{p_{i+1,j}}+u_{p_{i,j}}}{4}\right),\\
&\left(\frac{T_{i+1,j+1}-T_{i,j+1}+T_{i+1,j}-T_{i,j}}{2\Delta t}\right)+S\left(\frac{H_{i+1,j+1}+H_{i,j+1}+H_{i+1,j}+H_{i,j}}{4\,\mathrm{Pr}}\right)\\
&\quad=\left(\frac{\overline{f}_2(T)_{i,j+1}+\overline{f}_2(T)_{i,j}}{2\,\mathrm{Pr}}\right)\times\left(\frac{(H_{i+1,j+1}+H_{i,j+1})-(H_{i+1,j}+H_{i,j})}{2\Delta y}\right)\\
&\qquad+\mathrm{Ec}\left(\frac{\overline{f}_1(T)_{i,j+1}+\overline{f}_1(T)_{i,j}}{2}\right)\left(\frac{\overline{v}_{i+1,j+1}+\overline{v}_{i,j+1}+\overline{v}_{i+1,j}+\overline{v}_{i,j}}{2}\right)\\
&\qquad\times\left(\frac{v_{i+1,j+1}+v_{i,j+1}+v_{i+1,j}+v_{i,j}}{2}\right)+\mathrm{Ec}\,\mathrm{Ha}^2\left(\frac{\overline{u}_{i+1,j+1}+\overline{u}_{i,j+1}+\overline{u}_{i+1,j}+\overline{u}_{i,j}}{2}\right)\\
&\qquad\times\left(\frac{u_{i+1,j+1}+u_{i,j+1}+u_{i+1,j}+u_{i,j}}{2}\right)\\
&\qquad+\frac{2R}{3\,\mathrm{Pr}}\left(\frac{T_{p_{i+1,j+1}}+T_{p_{i,j+1}}+T_{p_{i+1,j}}+T_{p_{i,j}}}{4}-\frac{T_{i+1,j+1}+T_{i,j+1}+T_{i+1,j}+T_{i,j}}{4}\right),\\
&\left(\frac{T_{p_{i+1,j+1}}-T_{p_{i,j+1}}+T_{p_{i+1,j}}-T_{p_{i,j}}}{2\Delta t}\right)\\
&\quad=-L_o\left(\frac{T_{p_{i+1,j+1}}+T_{p_{i,j+1}}+T_{p_{i+1,j}}+T_{p_{i,j}}}{4}-\frac{T_{i+1,j+1}+T_{i,j+1}+T_{i+1,j}+T_{i,j}}{4}\right).
\end{aligned}
\tag{2.15}
$$

The variables with bars are given initial guesses from the previous time steps and an iterative scheme is used at every time to solve the linearized system of difference equations. Computations have been made for $R = 0.5$, $G = 0.8$, $\lambda = 5$, $\mathrm{Pr} = 1$, $\mathrm{Ec} = 0.2$, and $L_o = 0.7$. Grid-independence studies show that the computational domain $0 < t < \infty$ and $-1 < y < 1$ can be divided into intervals with step sizes $\Delta t = 0.0001$ and $\Delta y = 0.005$ for time and space, respectively. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns u, v, u_p, T, H, and T_p for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1 < y < 1$ at every time step. Less than 7 approximations are required to satisfy these convergence criteria for all ranges of the parameters studied here.

3. Results and discussions

The exponential dependence of the viscosity on temperature results in decomposing the viscous force term in the momentum equation into two terms. The variations of these

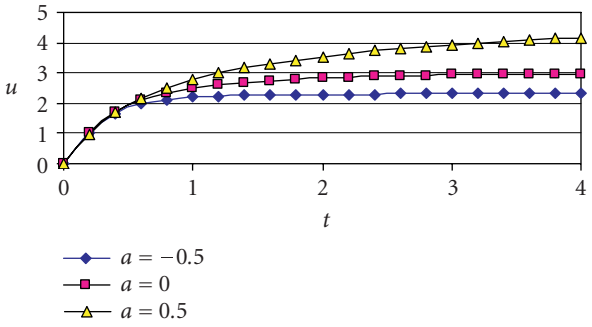


Figure 3.1. The evolution of u for different values of a (Ha = 0, $S = 0$).

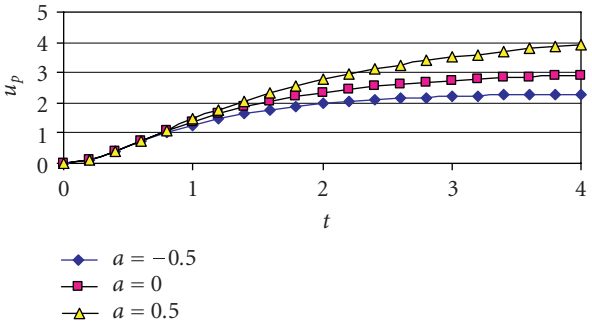


Figure 3.2. The evolution of u_p for different values of a (Ha = 0, $S = 0$).

resulting terms with the viscosity variation parameter a and their relative magnitudes have an important effect on the flow and temperature fields in the absence or presence of the applied uniform magnetic field.

Figures 3.1 and 3.2 indicate the variations of the velocities u and u_p at the center of the channel ($y = 0$) with time for different values of the viscosity variation parameter a and for Ha = 0 and $S = 0$. The figures show that increasing a increases the velocity and the time required to approach the steady-state. The effect of the parameter a on the steady-state time is more pronounced for positive values of a than for negative values. Notice that u reaches the steady state faster than u_p. This is because the fluid velocity is the source for the dust particles velocity. Figure 3.1 shows also that the influence of a on u_p is negligible for some time and then increases as the time develops.

Figures 3.3 and 3.4 present the variations of the temperatures T and T_p at the center of the channel ($y = 0$) with time for different values of the viscosity variation parameter a for Ha = 0 and $S = 0$. The figures show that increasing a increases the temperatures and the steady-state times. Increasing the positive values of a decreases the temperature for some time and then the temperature increases with the increment in a as the time develops. Thus, increasing a increases the steady-state value of the temperature with the appearance of the cross-over of the temperature curves corresponding to different values of a. The time at which the curves intersect increases with the increment in a and is longer

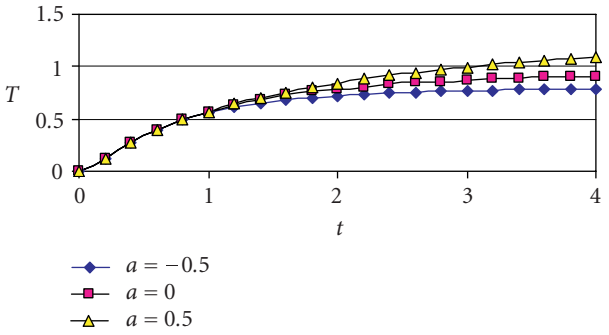


Figure 3.3. The evolution of T for different values of a (Ha = 0, $S = 0$).

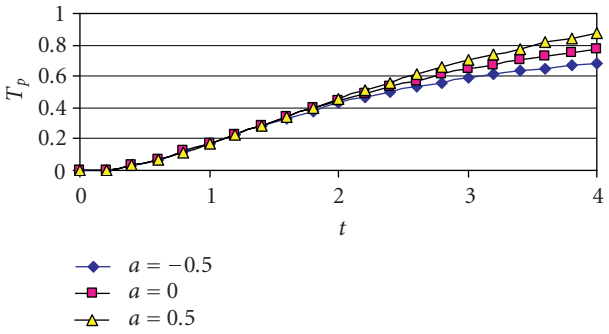


Figure 3.4. The evolution of T_p for different values of a (Ha = 0, $S = 0$).

for T than for T_p, as T_p always follows T. It is noticed that the steady-state values of T_p coincide with the corresponding steady-state values of T, and the time required for T_p to reach the steady state, which depends on a, is longer than that for T.

The application of the uniform magnetic field adds one resistive term to the momentum equation and the Joule dissipation term to the energy equation. Figures 3.5 and 3.6 present the influence of the viscosity variation parameter a on the evolution of both the velocities u and u_p at the center of the channel, respectively, for Ha = 1 and $S = 0$. The magnetic field results in a reduction in the velocities and the steady-state time for all values of a due to its damping effect.

Figures 3.7 and 3.8 present the influence of the viscosity variation parameter a on the evolution of the temperatures T and T_p at the center of the channel, respectively, for Ha = 1 and $S = 0$. Increasing the magnetic field increases the temperatures for all positive values of a except for very small time. This is because the magnetic field has a resistive effect which becomes more pronounced as time develops especially with the case of negative a which has the same resistive effect.

Figures 3.9 and 3.10 indicate the variations of the velocities u and u_p at the center of the channel ($y = 0$) with time for different values of the viscosity variation parameter a and for Ha = 0 and $S = 1$. It is clear that the suction velocity decreases both u and u_p and

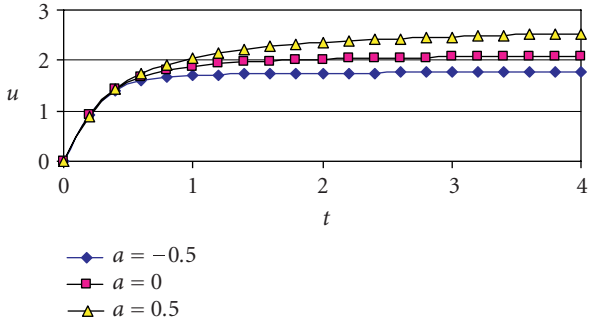


Figure 3.5. The evolution of u for different values of a (Ha = 1, $S = 0$).

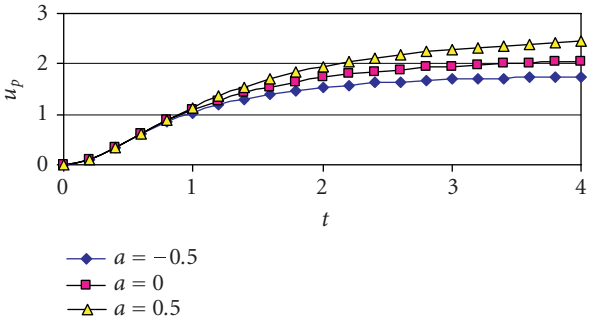


Figure 3.6. The evolution of u_p for different values of a (Ha = 1, $S = 0$).

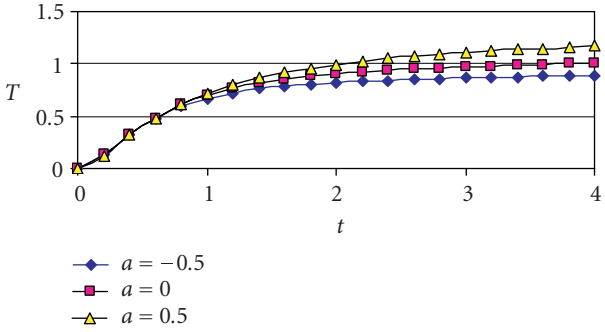


Figure 3.7. The evolution of T for different values of a (Ha = 1, $S = 0$).

their steady-state times as a result of pumping the fluid from the slower lower half region to the center of the channel. The influence of suction on u and u_p is more pronounced for higher values of the parameter a.

Figures 3.11 and 3.12 present the influence of the viscosity variation parameter a on the evolution of the temperatures T and T_p at the center of the channel, respectively, for

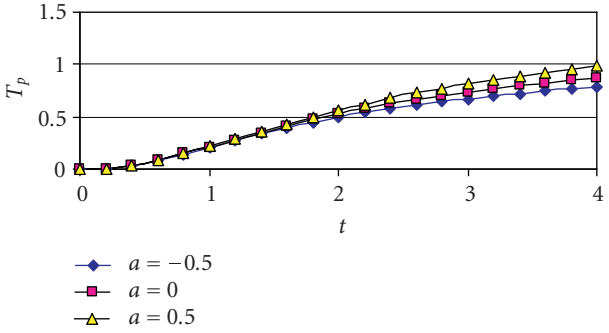


Figure 3.8. The evolution of T_p for different values of a (Ha = 1, $S = 0$).

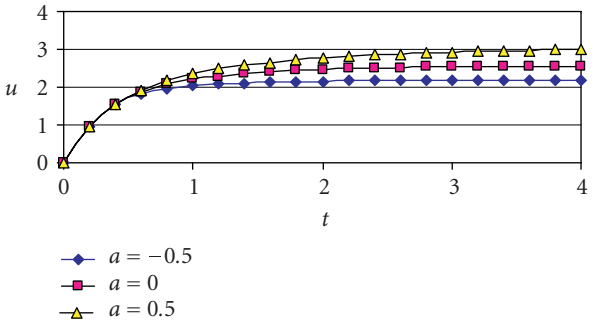


Figure 3.9. The evolution of u for different values of a (Ha = 0, $S = 1$).

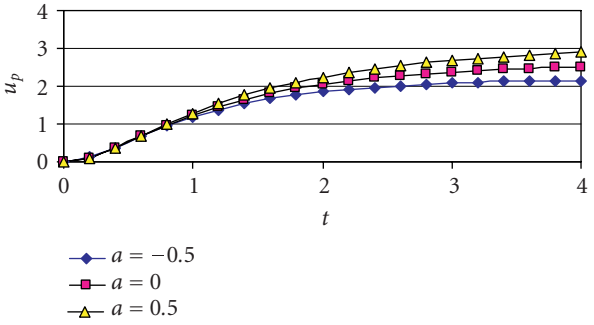


Figure 3.10. The evolution of u_p for different values of a (Ha = 0, $S = 1$).

Ha = 0 and $S = 1$. It is shown that increasing suction velocity decreases both T and T_p and their steady-state times. This results from pumping the fluid from colder lower half region to the center of the channel. The effect of suction on T and T_p is more apparent for higher values of a.

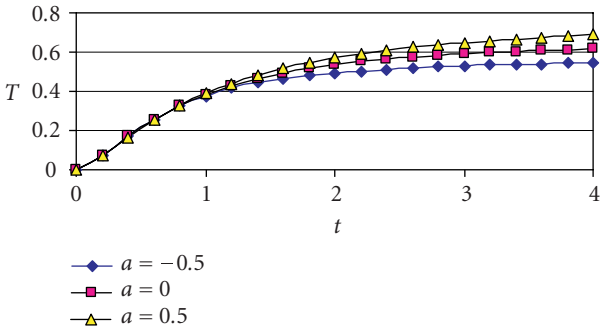


Figure 3.11. The evolution of T for different values of a (Ha = 0, S = 1).

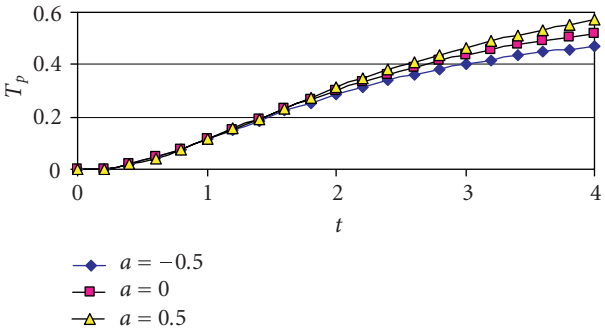


Figure 3.12. The evolution of T_p for different values of a (Ha = 0, S = 1).

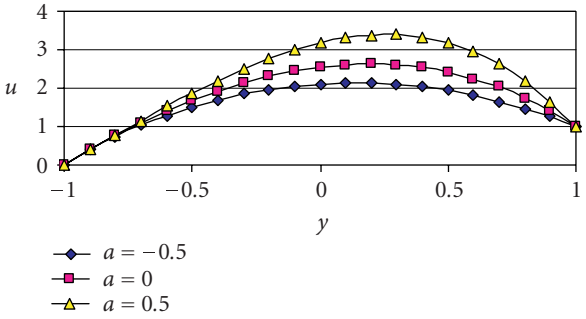


Figure 3.13. Steady-state profile of u for various values of a (Ha = 0.5, S = 0.5).

Figures 3.13 and 3.14 present the influence of the viscosity variation parameter a on the steady-state profile of the velocities u and u_p, respectively, for Ha = 0.5 and S = 0.5. It is clear that increasing a increases u and u_p for all values of y due to the increase in viscosity. It is clear also that the steady-state velocity attains more than three times the wall velocity due to the effect of the applied pressure gradient. Figures 3.15 and 3.16

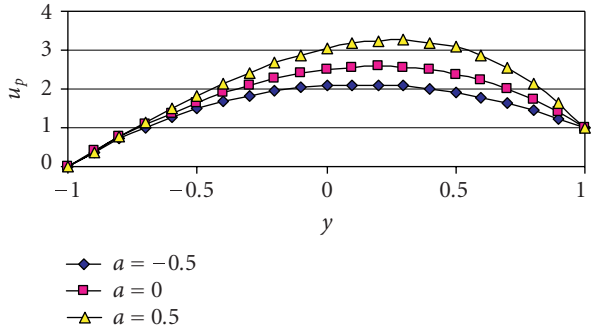


Figure 3.14. Steady-state profile of u_p for various values of a (Ha = 0.5, S = 0.5).

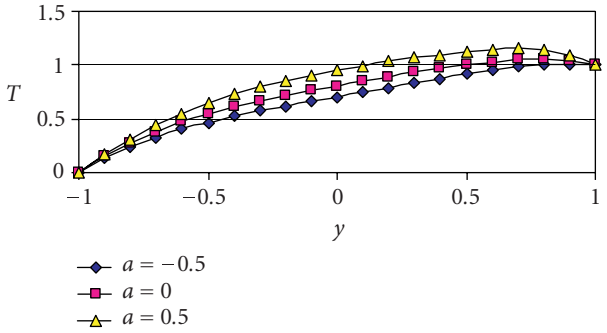


Figure 3.15. Steady-state profile of T for various values of a (Ha = 0.5, S = 0.5).

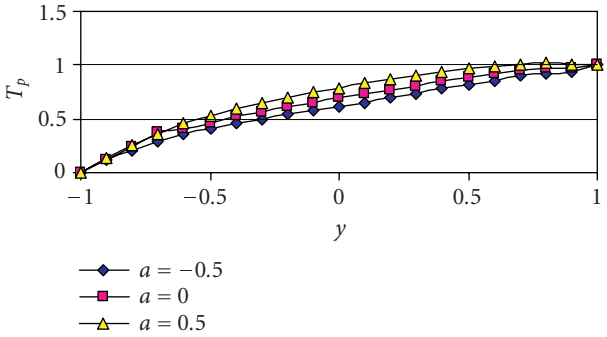


Figure 3.16. Steady-state profile of T_p for various values of a (Ha = 0.5, S = 0.5).

present the influence of the viscosity variation parameter a on the steady-state profile of the temperatures T and T_p, respectively, for Ha = 0.5 and S = 0.5. Increasing a increases both T and T_p as a result of increasing the velocities and their gradients which increase the viscous and Joule dissipations. Also, it is shown that the temperatures exceed unity

for some locations (i.e., the temperature of the upper plate) due to the heating effect of the dissipations.

4. Conclusions

In this paper the effect of a temperature-dependent viscosity, suction and injection velocity, and an external uniform magnetic field on the unsteady flow and temperature distributions of an electrically conducting viscous incompressible dusty fluid between two parallel porous plates has been studied. The viscosity was assumed to vary exponentially with temperature and the Joule and viscous dissipations were taken into consideration. The most interesting result was the cross-over of the temperature curves due to the variation of the parameter a and the influence of the magnetic field in the suppression of such cross-over. On the other hand, changing the magnetic field results in the appearance of cross-over in the temperature curves for a given negative value of a. Also, changing the viscosity variation parameter a leads to asymmetric velocity profiles about the central plane of the channel ($y = 0$) which is similar to the effect of variable percolation perpendicular to the plates.

References

[1] A. L. Aboul-Hassan, H. Sharaf El-Din, and A. A. Megahed, *Temperature distribution in a dusty conducting fluid flowing through two parallel infinite plates due to the motion of one of them*, Proceedings of 1st International Conference of Engineering Mathematics and Physics, Cairo, 1991, pp. 723–734.

[2] W. F. Ames, *Numerical Solutions of Partial Differential Equations*, 2nd ed., Academic Press, New York, 1977.

[3] H. A. Attia, *Transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity*, Mechanics Research Communications **26** (1999), no. 1, 115–121.

[4] H. A. Attia and N. A. Kotb, *MHD flow between two parallel plates with heat transfer*, Acta Mechanica **117** (1996), no. 1–4, 215–220.

[5] A. K. Borkakoti and A. Bharali, *Hydromagnetic flow and heat transfer between two horizontal plates, the lower plate being a stretching sheet*, Quarterly of Applied Mathematics **40** (1983), no. 4, 461–467.

[6] L. A. Dixit, *Unsteady flow of a dusty viscous fluid through rectangular ducts*, Indian Journal of Theoretical Physics **28** (1980), no. 2, 129.

[7] A. K. Ghosh and D. K. Mitra, *Flow of a dusty fluid through horizontal pipes*, Revue Roumaine de Physique **29** (1984), 631–646.

[8] R. K. Gupta and S. C. Gupta, *Flow of a dusty gas through a channel with arbitrary time varying pressure gradient*, Journal of Applied Mathematics and Physics **27** (1976), 119.

[9] H. Herwig and G. Wicken, *The effect of variable properties on laminar boundary layer flow*, Warme-und Stoffubertragung **20** (1986), 47–57.

[10] K. Klemp, H. Herwig, and M. Selmann, *Entrance flow in channel with temperature dependent viscosity including viscous dissipation effects*, Proceedings of the 3rd International Congress of Fluid Mechanics, vol. 3, Cairo, 1990, pp. 1257–1266.

[11] A. A. Megahed, A. L. Aboul-Hassan, and H. Sharaf El-Din, *Effect of Joule and viscous dissipation on temperature distributions through electrically conducting dusty fluid*, Proceedings of 5th Miami International Symposium on Multi-Phase Transport and Particulate Phenomena, vol. 3, Florida, 1988, p. 111.

[12] P. Mitra and P. Bhattacharyya, *Unsteady hydromagnetic laminar flow of a conducting dusty fluid between two parallel plates started impulsively from rest*, Acta Mechanica **39** (1981), no. 3-4, 171–182.
[13] V. R. Prasad and N. C. P. Ramacharyulu, *Unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient*, Defense Science Journal **30** (1979), 125–130.
[14] P. G. Saffman, *On the stability of laminar flow of a dusty gas*, Journal of Fluid Mechanics **13** (1962), 120–128.
[15] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, New York, 1968.
[16] K. K. Singh, *Unsteady flow of a conducting dusty fluid through a rectangular channel with time dependent pressure gradient*, Indian Journal of Pure and Applied Mathematics **8** (1976), no. 9, 1124–1131.
[17] G. W. Sutton and A. Sherman, *Engineering Magnetohydrodynamics*, McGraw-Hill, New York, 1965.
[18] D. J. Tritton, *Physical Fluid Dynamics*, ELBS & Van Nostrand Reinhold, London, 1979.

Hazem A. Attia: Department of Mathematics, College of Science, Al-Qasseem University,
P.O. Box 237, Buraidah 81999, Saudi Arabia
E-mail address: ah1113@yahoo.com

Mathematical Problems in Engineering

Special Issue on Time-Dependent Billiards

Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

Manuscript Due	December 1, 2008
First Round of Reviews	March 1, 2009
Publication Date	June 1, 2009

Guest Editors

Edson Denis Leonel, Departamento de Estatística, Matemática Aplicada e Computação, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil ; edleonel@rc.unesp.br

Alexander Loskutov, Physics Faculty, Moscow State University, Vorob'evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru