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Sustainable Economic Growth and Exhaustible Resources: A Model and Estimation for the US

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This paper studies current models on sustainable economic growth with resource constraints and explores to what extent resource constraints can be overcome by substitution and technological change. We also study the problem of intergenerational equity and the different criteria that have been suggested in the literature. The central part of this paper is the presentation of stylized facts on exhaustible resources and an estimation of a basic model with resource constraints for US time series data. The estimated years left until depletion and the empirical trends of the ratios of capital stock and consumption to resources seem to indicate that there might be a threat to sustainable growth in the future. In our estimation, we obtain parameter values, which help to interpret the extent to which growth with exhaustible resources is sustainable.

INTRODUCTION

Since the early 1970s, there is a growing concern that human activity depletes resources and pollutes the environment. As economic decisions are restricted by the finiteness of natural resources, and by the limited capacity of the nature to absorb pollution, attention is devoted to the question whether it is possible and desirable to continue present patterns of economic growth.

In 1972, economists like Meadows *et al.* (1972) or Daly (1987, 1989) formulated pessimistic predictions about a "sudden and uncontrollable decline in both population and industrial capacity" if no "conditions for ecological and economic stability that is sustainable far into the future" are established.[†] Other economists like Beckerman (1974) have the optimistic view that technological progress and the discovery of new substitutes make continued economic growth possible. A general consensus of the economic growth debate is that there are trade-offs among environmental and economic goals. The agreement is that economic activity, which ignores the biological or social system, is not sustainable. There exist many different definitions of sustainability, but all of them have two points in common. First, they recognize that resource and

environmental constraints affect the patterns of development and consumption in the long-run. Second, they are concerned about equity between generations (intergenerational equity). One of the most famous definitions is stated by the Brundtland Commission in 1987: "sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs." Similarly Solow (1974)[*] defines sustainablity as "an obligation to conduct ourselves so that we leave to future the option or the capacity to be as well off as we are." Pearce *et al.* (1990) point out that "natural capital stock should not decrease over time" whereas Pezzey (1989) defines sustainable economic growth as "non-declining output or consumption over time" and sustainable economic development as "non-declining utility over time."[*]

As to the sustainability of natural resources, one can distinguish between renewable and non-renewable resources. Dynamic models on renewable resources can be found in Clark (1990), Semmler and Sieveking (1994) and Sieveking and Semmler (1997). In those papers, theorems on the sustainability of resource economics, which have been developed by studying one resource, only are evaluated for the case when resources interact as an

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ecological system. For the latter case, resource management problems and policies aiming at conserving resources are studied. Furthermore, in Semmler and Sieveking (1994), credit financed extractions of resources are considered and the fate of resources explored.

The current paper deals with dynamic models with exhaustible resources. We discuss prototype growth models that incorporate and study the consequences of finitely available exhaustible resources. Some of the problems studied here will also arise in the case of renewable resources, so for example, the problem of intergenerational justice. The remainder of the paper is organized as follows. In the second section, we survey growth models with natural resource constraints. The third section discusses the problem of intergenerational justice. The fourth section presents stylized facts on exhaustible resources pertaining to the US. The fifth section presents the estimation of our growth model and the sixth section concludes the paper.

MODERN GROWTH MODELS AND RESOURCE CONSTRAINTS

We here review growth models, which take into account the presence of exhaustible natural resources. The main interest of the analysis is the question to what extent the process of economic growth is restricted by the finiteness of resource stocks and whether sustained consumption and utility levels are feasible.

We first consider a basic model where the exhaustible natural resource is used as an input for the production of a good, which is then either consumed or added to the capital stock to enhance future production. Then, the consequences of different extensions concerning technological progress are analyzed, and, finally, it is assumed that natural resources may provide services in preserved states.

Before turning to the description of the models, it is important to have a clear distinction between renewable and exhaustible resources. The main feature of an exhaustible resource is that its growth rate is nil, and that it is not recyclable. Furthermore, it is used up when used as an input in production. To study a meaningful economic problem, the natural resource must be essential, i.e. production is impossible without it.‡

The Basic Model

Dasgupta and Heal (1974), Heal (1985), Stiglitz (1974; 1993) and Solow (1973) analyze the optimal depletion of exhaustible natural resources in the context of a growth model where the resource is used as an input for the production of a composite commodity.* The production function F depends on the flow of the exhaustible resource

at date t and on the stock of a reproducible good at date t. In order to obtain greatest possible social welfare, the present value of utility U derived from consumption C_t of the produced good is maximized subject to the evolution of the reproducible capital stock K_t and the constraints imposed by the finiteness of the resource stock S_t:

$$\text{Max} \int_0^\infty U(C_t) e^{-\delta t} \, dt \qquad (1)$$

Subject to

$$\dot{K}_t = F(K_t, R_t) - C_t$$

$$S_t = S_0 - \int_0^\infty R_t \, dt \quad \dot{S}_t = -R_t$$

δ denotes the discount rate, and R_t is the flow of the exhaustible resource. The initial capital stocks K_0 and S_0 are strictly positive and given. The production function $F(K_t, R_t)$ is assumed to be increasing, strictly concave, twice continuously differentiable and homogenous of degree unity. The utility function $U(C_t)$ is supposed to be strictly concave, and for $C_t \rightarrow 0$, its first derivative is infinity. Here, the extraction of the resource is assumed to be costless.

Solving the maximization problem and combining the optimality conditions yields the following results (for details see "The Basic Model" section in the Appendix A): first, along an optimal path, the rate of consumption depends on the discount rate δ, on the elasticity of marginal utility of consumption η and on the marginal productivity of reproducible capital F_K:

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - \delta}{\eta} \qquad (2)$$

with $\eta(C_t) = -C_t U''(C_t)/U'(C_t)$ and $F_K = \partial F(K_t, R_t)/\partial K_t$. Equation (2) states that the higher the discount rate, the more the rate of consumption falls over time along an optimal path. Second, along an optimal path, the rates of return of exhaustible and reproducible capital are equal:

$$F_K = \frac{\partial F_R}{\partial t} \frac{1}{F_R} \qquad (3)$$

with $F_R = \partial F(K_t, R_t)/\partial R_t$. If the production function is homogenous of degree one, it is possible to set $x_t = K_t/R_t$ with $f(x_t) = F(K_t/R_t, 1)$. Substituting $F_R = f(x_t) - x_t f'(x_t)$ and $F_K = f'(x_t)$ in Eq. (3) yields the following capital-resource ratio along an optimal path:

$$\frac{\dot{x}_t}{x_t} = \sigma \frac{f(x_t)}{x_t} \qquad (4)$$

with

$$\sigma = \frac{-f'(x_t)(f(x_t) - x_t f'(x_t))}{x_t f(x_t) f''(x_t)}$$

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‡Dasgupta and Heal, 1979.
*see also Davison (1978).

as the elasticity of substitution between reproducible capital and the exhaustible resource. Equation (4) represents the rate at which reproducible capital is substituted for the exhaustible resource. It depends on the elasticity of substitution and on the average product per unit of fixed capital.

In order to conclude whether a positive level of consumption is sustainable over time, Dasgupta and Heal (1974) analyze an economy where output is produced by a CES–production function, i.e. the case of a constant elasticity of substitution. There are three cases to mention:

1. $\sigma = 1$ (i.e. the Cobb–Douglas-production function). The exhaustible resource is essential and infinitely valuable at the margin, whereas the asymptotic value of marginal productivity of capital is zero. Solow (1973) concludes that sustained per capita consumption is feasible if the share of capital exceeds that of natural resources.
2. $0 \leq \sigma < 1$. The exhaustible resource is essential, but finitely valuable at the margin. Thus, a positive and non-decreasing level of consumption over an infinite time horizon is not sustainable.
3. $\infty > \sigma > 1$. Sustained consumption is feasible because in this case, the exhaustible resource is inessential.

Technology

The basic model can be augmented by introducing technical change, which makes it easier to find new substitutes in order to render an essential natural resource inessential. Dasgupta and Heal (1974) assume that technical progress is uncertain: the exact date of discovering a substitute and its detailed characteristics and usefulness are unknown. The new technique is supposed to occur at an unknown date T which is a random number with an exogenously given probability density function ω_t:

$$\text{Probability}\,(T = t) = \omega_t, \quad \int_0^\infty \omega_t\,dt = 1, \quad \omega_t > 0$$

In order to express the situation of uncertainty, the objective is to maximize the expected present value of utility. After some manipulation, one obtains the following maximization problem¶:

$$\text{Max}\int_0^\infty [U(C_t)\Omega_t + \omega_t W(K_t, S_t)]e^{-\delta t}\,dt \qquad (5)$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t, \quad S_t = S_0 - \int_0^\infty R_t\,dt \quad \dot{S}_t = -R_t$$

¶Dasgupta, 1994 pp. 20.
§See also Sieveking and Semmler, 1997.

with $\Omega_t = \int_t^\infty \omega_t\,dt$ and

$$W(K_t, S_t) = \text{Max}\int_T^\infty U(C_t)e^{-\delta[t-T]}\,dt.$$

K_t, C_t, R_t and S_t are all non-negative and the initial values K_0 and S_0 are given.

Solving the maximization problem, combining the first order conditions, and arguing that at the discovery date of the substitute, the then existing stocks of reproducible and natural capital have no economic value anymore because the new technology is more efficient (i.e. $W_K = W_S = 0$), allows for the following conclusions (for details, see "Technology" section in Appendix A): first, along an optimal path, the rate of consumption depends on a modified discount rate:

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - (\delta + \psi_t)}{\eta} \qquad (6)$$

with $\psi_t = \omega_t/\Omega_t$ as the conditional probability of the technological breakthrough at date t, given the substitute has not been discovered earlier. The discount rate is modified by the addition of the factor ψ_t showing the probability of the essential resource becoming inessential as a result of technical progress. Thus, in a situation of uncertainty, the discount rate is higher than in a situation of certainty.§ Obviously, the equation describing the ratio of capital-resource input is the same as before:

$$\frac{\dot{x}_t}{x_t} = \sigma\frac{f(x_t)}{x_t} \qquad (7)$$

Stiglitz (1974, 1993) examines an economy where output is produced by a Cobb–Douglas-production function. He concludes that sustained per capita consumption is feasible, if there is a resource augmenting technical change at any positive rate (for $\sigma \geq 1$). Toman et al. (1993) point out that for the case $\sigma < 1$, sustained *per capita* consumption is possible if technological progress is high enough.

Backstop Technology

So far, it is assumed that the natural resource is exhaustible, i.e. once it is used up, it is impossible to find more, and that the extraction is costless. As an extension, it is now supposed that the resource is available in unlimited quantities, but at various grades and various costs. For example, the ores of a number of metals can be extracted from the deposits currently used, which are exhaustible. If they are used up, the metals themselves can be extracted from the sea or from rock formations, which is much more expensive. Thus, at higher prices, the natural resource may be of unlimited availability. Heal (1993)

calls this a *backstop technology*.* We can incorporate it into the basic model described in "The Basic Model." The total amount of the resource used at date t is denoted as follows:

$$z(t) = \int_0^\infty R_t \, dt$$

It is assumed that at date T, the conventional deposits are exhausted and a backstop technology takes over. Up to the level z_T, the extraction costs rise with cumulative extraction, then the backstop technology is available at a constant cost per unit b. $g(z_t)$ denotes the extraction costs per unit at date t with $\partial g/\partial z_t = g'(z_t) > 0$ for $0 \le z_t \le z_T$ and $g(z_T) = b > 0$ for $z_t \ge z_T$.

The maximization problem is solved in two steps (see "Backstop Technology" section in Appendix A): first, the situation is examined before current deposits are exhausted (maximization problem (8)), second, the situation is examined after the backstop technology has taken over (maximization problem (9)):

$$\text{Max} \int_0^\infty U(C_t) e^{-\delta t} \, dt \tag{8}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t - g(z_t) R_t$$

$$S_t = S_0 - \int_0^\infty R_t \, dt, \quad \dot{S}_t = -R_t$$

where $g(z_t) R_t$ represents the total extraction costs.

$$\text{Max} \int_0^\infty U(C_t) e^{-\delta t} \, dt \tag{9}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t - b R_t$$

The initial capital stocks K_0 and S_0 are strictly positive and given. Computing the conditions along an optimal path of problem (8) yields:

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - \delta}{\eta} \tag{10}$$

and

$$F_K = \frac{\partial F_R}{\partial t} \frac{1}{F_R} + \frac{F_K g(z_t)}{F_R} \tag{11}$$

Substituting $F_K = f'(x_t)$ and $F_R = f(x_t) - x_t f'(x_t)$ with $f(x_t) = F(K_t/R_t, 1)$ results in the following capital-resource ratio along an optimal path:

$$\frac{\dot{x}_t}{x_t} = \frac{\sigma f(x_t)}{x_t} + \frac{f'(x_t)}{x_t f''(x_t)} \frac{g(z_t)}{x_t} \tag{12}$$

Condition (12) is a generalization of condition (4). Here, the equation is augmented by the term $(f'(x_t)/x_t f''(x_t))(g(z_t)/x_t)$ which reflects the cumulative costs of extraction.

Following Heal (1993), we can draw the following conclusions. During the initial period, the lower-cost stocks of the natural resource are exhausted, and the path of the economy is described by problem (8) and conditions (10) and (11). The difference between prices and extraction costs, i.e. the user costs, decline until they reach zero at date T, when the backstop technology takes over because the lower-cost stocks are totally used up. From then on, the economy behaves according to problem (9) thus, the extraction costs of the natural resource always equal its price.

INTERGENERATIONAL EQUITY

As apparent from the above models, the depletion of resources generates externalities for future generations. Therefore, the problem of intergenerational equity arises. In order to study this problem, we will first show how natural resources may affect the welfare of the society.

The Amenity Value of a natural Resource

There are two ways of how a natural resource contributes to society's welfare. The models described so far refer to the first way: the resource is utilized as an input factor for the production of a composite commodity. The second way of a natural resource serving for the well-being of the society is that it may provide valuable services in preserved states, that is scientific, recreational, and aesthetic values. To take into account these so-called *amenity values of natural resources*, the resource stock S_t is included in the utility function (Krautkraemer, 1985). The objective is to maximize present value of utility (see "The Amenity Value of a Natural Resource" section in Appendix A):

$$\text{Max} \int_0^\infty U(C_t, S_t) e^{-\delta t} \, dt \tag{13}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t \quad S_t = S_0 - \int_0^\infty R_t \, dt \quad \dot{S}_t = -R_t$$

where C_t, K_t, R_t, S_t are all non-negative. $U(C_t, S_t)$ is assumed to be twice continuously differentiable with $U_C = \partial U(C_t, S_t)/\partial C_t > 0$ and $U_S = \partial U(C_t, S_t)/\partial S_t > 0$, $U_{CC} = (\partial^2 U(C_t, S_t)/\partial C_t^2) < 0$ and $U_{SS} = (\partial^2 U(C_t, S_t)/$

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*see also Oren *et al.* (1993).

$\partial S_t) < 0$ and

$$\lim_{C_t \to 0} U_C = \infty.$$

The production function $F(K_t, R_t)$ has the same properties as before. Solving the maximization problem and combining the first order conditions yield the following conditions for the rate of consumption and for the capital-resource ratio along an optimal path:

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - \delta - (U_{CS}/U_C)R_t}{\eta} \qquad (14)$$

$$\frac{\dot{x}_t}{x_t} = \sigma\frac{f(x_t)}{x_t} + \frac{U_S/U_C}{x_t^2 f''(x_t)} \qquad (15)$$

The amenity services of a natural resource affect the extraction and the consumption mainly through their contribution to the rate of return to the resource stock. Resource amenities enhance the value of the resource stock. Therefore, the initial price of the resource must be higher than in the previous models. Furthermore, since $f''(x_t) < 0$, the amenity services lower the rate of change in the capital-resource ratio by the term $((U_S/U_C)/x_t^2 f''(x_t))$. The higher the marginal rate of substitution between amenities and consumption, the higher the reduction of the rate of change of the input ratio.

In this framework where natural environments are valued in their own rights, the so-called *Green Golden Rule*[||] can be introduced which is motivated by the *Golden Rule of Economic Growth*.[#] The Golden Rule of Economic Growth gives the growth path with the highest indefinitely maintainable level of consumption, whereas the Green Golden Rule focuses on the highest indefinitely maintainable level of instantaneous utility. Thus, the Green Golden Rule incorporates the aim of sustainability. Formally, the rule can be written as[**]:

$$\max_{\text{feasible paths}} \lim_{t \to \infty} U(C_t, S_t)$$

If the resource is used as an input factor for the production of a composite commodity, in the long-run, the only constant level of resource input is zero. Since the resource is essential, no output can be produced and consumption is zero. Hence, the highest indefinitely maintainable utility level is feasible, if the total initial stock is conserved.

Intergenerational Equity

Standard growth models, which incorporate the concept of sustainability, focus on the consequences of natural resource constraints on the long-run pattern of economic development and consumption. In order to determine inter-temporal welfare, recent growth theory has used the concept of discounted utilitarism that is the future is discounted in comparison with the present. Ramsey (1928) states that discounting is "ethically indefensible and arises merely from the weakness of the imagination" because a positive discount rate results in an asymmetric treatment of present and future generations. Thus, discounted future utility neglects intergenerational equity as the second important point of the concept of sustainability. In the following, we want to give a brief review of some alternative concepts, which try to meet the objective of a fair treatment of different generations.

A simple way to account for intergenerational equity is to assume the case of a zero utility discount rate, that is present and future generations are given the same weight. Another alternative is to apply the "overtaking criterion" as proposed by Weizäcker (1967), which states that one consumption path is better than another, if from some date on total utility of that path is greater. Formally, if

$$\int_0^T U(C_t^1)\mathrm{d}t \geq \int_0^T U(C_t^2)\mathrm{d}t$$

But applying these approaches gives rise to technical problems. For a zero discount rate, the set of attainable values of the integral may be open, and the way of ranking consumption paths according to the overtaking criterion is incomplete.

According to the Rawlsian Criterion[††] intergenerational equity means: maximize the welfare of the less advantaged generation. Formally,

$$\max_{\text{feasible paths}} \min_{\text{generations } t} (\text{Welfare}_t)$$

The consequence of this decision rule is that the welfare level should be the same for all generations. If a later generation enjoys higher welfare, an earlier generation should increase its own welfare at the expense of the later generation and vice versa. Solow (1973) points out that in comparison with the discounted solution based on utilitarism, the Rawlsian Criterion will use up the natural resource stock faster. Since the utilitarian rule demands higher savings, earlier generations will have a lower standard of living than the constant max–min rule would generate.[*] The Rawlsian Criterion has two main difficulties: first, a society needs an initial capital stock high enough to make a decent standard of living possible, but the explanation of its existence is missing. Second, the rule does not yield a reasonable result, if ongoing technical progress is assumed.[##]

[||]Chichilnisky, 1996; Beltratti *et al.* 1994, 1993, 1995, 1996.
[#]Phelps, 1961.
[**]Heal, 1998, p. 43.
[††]Rawls, 1972.
[*]See also Dixit *et al.* (1980), Hartwick (1977).
[##]A more elaborate version of the Rawls criterion is proposed in Semmler and Sieveking, 2000.

More recently, Chichilnisky (1996) defines two axioms for sustainability, which deal with the problem of intergenerational equity. The first axiom states that the present generation should not dictate the outcome in disregard for the future. The second axiom states that the long-run future should not dictate the present. Welfare criteria, which do satisfy the two axioms, are called sustainable preferences. In order to formulate a criterion, which does belong to the class of sustainable preferences, positive weight is placed on the present and on the very long-run properties of a growth path. Formally,

$$\alpha \int_0^\infty U(C_t, S_t)\Delta(t)\mathrm{d}t + (1 - \alpha)\lim_{t\to\infty} U(C_t, S_t)$$

where $\alpha \in (0, 1)$. $\Delta(t)$ is any measure with $\int_0^\infty \Delta(t)\mathrm{d}t = 1$. If $\Delta(t) = e^{-\delta t}$, the first term is just the discounted integrals of utilities. The second term reflects the limiting properties of the utility stream, and it has already been mentioned as the Green Golden Rule solution. The Chichilnisky Criterion places more weight on the future than the standard approach of discounting utility, but less than the Green Golden Rule. It is possible to apply the Chichilnisky Criterion to neoclassical growth models at the aforementioned type[11] but finding the solution is quite complicated. We therefore leave aside detailed discussions.

This very short review of different welfare criteria has shown that it is very difficult to find approaches, which do meet the objective of permitting intergenerational equity and which are technically operable at the same time. For that reason, discounted utility is still dominant, as it is the technically most convincing approach, though it favors the present over the future.

STYLIZED FACTS

Economic theory states that substitution possibilities, technological progress and the value of the resource in preserved states may prevent the total depletion of natural capital. In this section, the patterns of some selected non-renewable resources of the US economy are analyzed from 1960 to 1995. This then serves as background for our estimations in the next section. Here, extraction rates, resource quantities available today and in the future are examined with respect to the question whether there are reasons to argue that the resources will be soon exhausted or that improved technology and the development of reproducible substitutes make a sustainable economic development possible.

Two different kinds of natural resources fulfill the main characteristics of depletion: fuel minerals such as energy resources, and non-fuel minerals such as metals and industrial minerals. When talking about the limited availability of natural resources, it is important to have

RESOURCE STOCK

nonrecoverable resource | recoverable resource

undiscovered resource | discovered resource

cumulative production | reserves

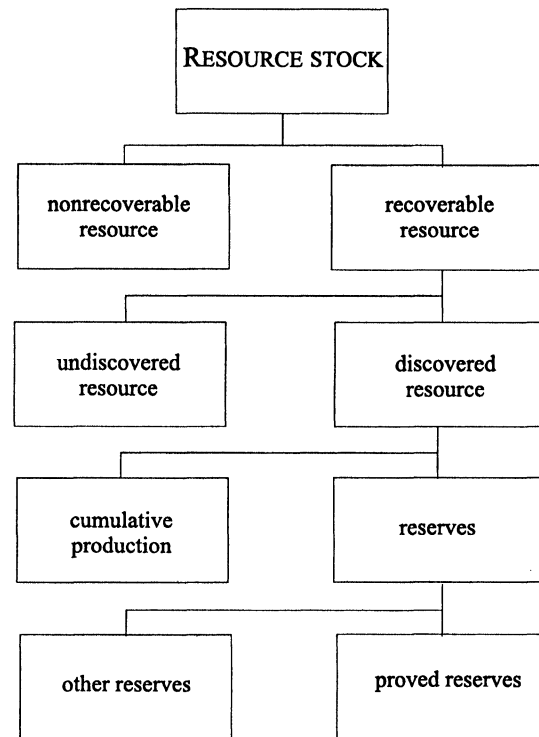other reserves | proved reserves

FIGURE 1 The components of resource stock.

clear definitions of the different components of which the total resource stock consists. Figure 1 explains the different components.

The reserves of the discovered resources consist of proved reserves and other reserves such as inferred reserves (field growth), measured reserves and indicated reserves. Proved reserves are those amounts of the resource that geological and engineering data demonstrate with reasonable certainty to be recoverable in the future from known reservoirs under existing economic and technological conditions. The other reserves consist on one hand of that part of the identified economically recoverable resource that will be added to proved reserves in the future through extensions, revisions and the discovery of new fields in already discovered regions, and on the other hand, on those quantities of the resource that may become economically recoverable in the future from existing production reservoirs through the application of currently available, but as-yet uninstalled recovery technology.[§§] For details on the data sources for the subsequent summaries, see Appendix B.

As noted before, fuel minerals are energy sources such as crude petroleum, coal and natural gas. Since during the last 30 years, the US economy has experienced continued economic growth, that is a rising level of real GDP, total energy consumption has increased by roughly 35%. To satisfy increasing energy demand, the production of

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[11]For a detailed analysis see Heal, 1995.
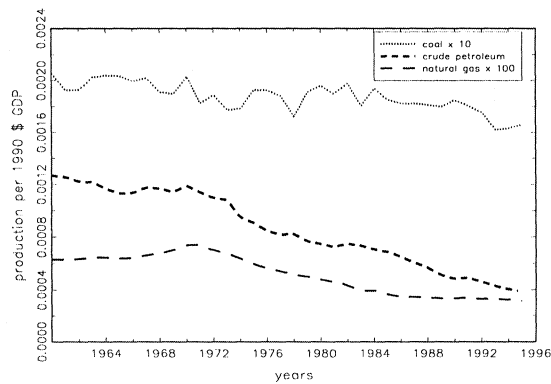[§§]For definitions see Energy Information Administration, (1996; 1997).

FIGURE 2   Fuel: mineral production per 1990 $ GDP (coal production measured short tons, crude petroleum production measured in barrels, natural gas production measured in millions of cubic feet).

FIGURE 3   Metal: production per GDP (metal production in short tons).

especially coal and natural gas have risen. In order to draw conclusions whether the resources are used more efficiently over time, it is interesting to analyze the patterns of production rates per dollar of real GDP, see Fig. 2. Table I summarizes the results.

For every energy resource, the production rates per GDP are falling, i.e. crude petroleum, coal and natural gas are used more efficiently today, which may be the result of improved technology. Non-fuel minerals are on one-hand metals and on the other hand, industrial minerals. Here, only some selected metals such as copper, iron ore, lead and zinc are analyzed. The production rates of copper, iron ore, lead and zinc behave quite irregularly from 1960 to 1995 and no trends can be determined.

Figure 3 and Table II show that the production rates per dollar real GDP are decreasing. Exhaustible energy resources can be substituted by renewable energy sources like for example, wind and sun power.

Figure 4 shows the shares of the different energy sources. Today, nuclear power and renewable energy together are just 20% of total energy production. But the trend is that the production of crude petroleum, coal and natural gas is decreasing, while nuclear and renewable energy production is increasing.
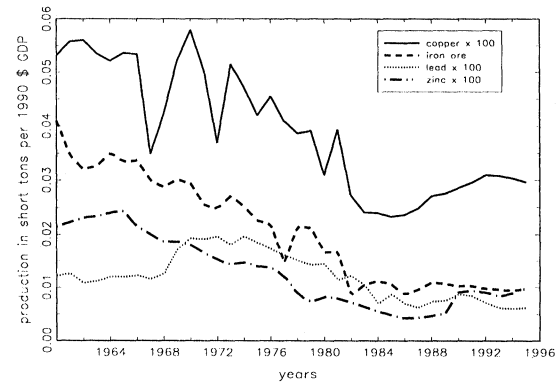
Copper, lead and zinc are metals, which can be substituted by aluminum and plastic. For iron ore, there do not exist any substitutes, but as it will be shown later, the reserves of iron ore are very high and will last for the next centuries.

In order to draw conclusions about future availability of exhaustible resources, it is necessary to have a closer look on the amount of proved and other reserves of a resource in comparison with its production rates.

For the exhaustible energy resources crude petroleum and natural gas, it is possible to plot the reserve-production ratio for the observed time period, see Fig. 5. The smaller the ratio the scarcer is the natural resource. For petroleum and natural gas, the trend is declining, thus, proved reserves will be depleted soon if for example, extensions/discoveries of new fields in already discovered regions or new discoveries do not make reserve additions possible.

Every year, the Energy Information Administration and the US Geological Survey estimate quantities of technically recoverable resource amounts that could be added to the already proved reserves of the US. It is interesting to ask about the number of years it will take to exhaust the today estimated technical recoverable resources quantities. As the production rates of the energy resources, coal and natural gas are steadily

TABLE I   Fuel mineral production per dollar real GDP

	Crude petroleum barrels	Coal in short tons	Natural gas in millions of cubic feet
1960	1.2680×10^{-3}	2.0526×10^{-4}	6.3155×10^{-6}
1995	3.8447×10^{-4}	1.6589×10^{-4}	3.1316×10^{-6}
Change (%)	-70	-19	-48

TABLE II   Non-fuel mineral production per dollar real GDP

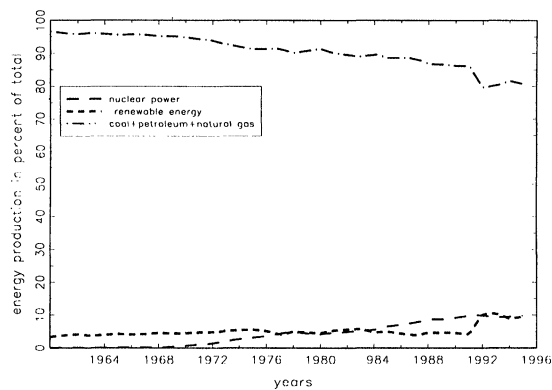|	Copper in short tons	Iron ore in short tons	Lead in short tons	Zinc in short tons
1960	5.3287×10^{-4}	4.0952×10^{-2}	1.2187×10^{-4}	2.1463×10^{-4}
1995	2.9700×10^{-3}	9.8124×10^{-3}	6.1990×10^{-5}	9.8605×10^{-5}
Change (%)	-44	-76	-49	-54

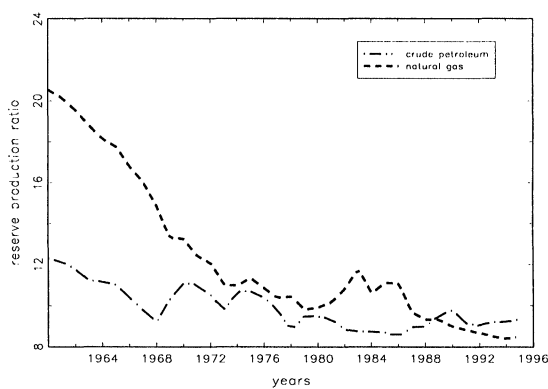FIGURE 4   Energy production sources in percent of total.

FIGURE 5   Reserve–production ratio.

increasing during the observed time period, it is assumed that they continue to increase with an average production growth rate. The other resources do not show any clear trend in their production rates and, therefore, it is supposed that production will continue to follow a stable pattern during the next years. Tables III–V summarize the results.

Although the results of the empirical analysis are restricted by the length of the examined time period and there is a clear decline and finiteness of resources, yet, there may be technological progress that makes a further development of renewable substitutes for exhaustible resources possible. As the US economy has experienced continued economic growth during the second half of this century, the production rates of exhaustible resources have risen and for all analyzed minerals, the production per dollar real GDP has declined. Increased efficiency, as the consequence of advanced technology and the use of close substitutes are likely explanations for that fact.

Assuming no changes in production trends yields predictions of the number of years left until present estimated resource reserves are depleted. The estimated reserves of coal and natural gas will last for the next two or three centuries. As the present estimated reserves of crude petroleum would be depleted in 40 years, the US economy is dependent on oil imports mainly from Arab countries. Substitutes of coal, natural gas and crude petroleum are nuclear and renewable power. Since the share of these alternative energy production sources has increased from 1960 to 1995, it seems likely that further

TABLE III   Estimated reserves of petroleum and natural gas 1995

	Crude petroleum in bill bbl	Natural gas in trill (cu. ft)
Proved reserves	20.2	135
Field growth	60.0	322
Undiscovered resources	30.3	259
Total reserves	110.5	716
Average annual production/average production growth rate	3.0076	0.012
Years left until exhaustion	40	300

TABLE IV   Estimated reserves of coal 1996

Proved reserves	Other reserves	Total reserves	Average production growth rate	Year left until exhaustion
19,428	507,740	527,168	0.026	250

TABLE V   Estimated reserves of some selected metals 1997

	Copper in 1000 metric tons	Iron ore in 1000 metric tons	Lead in 1000 metric tons	Zinc in 1000 metric tons
Proved reserves	45 000	10 000 000	6 500	25 000
Other reserves	90 000	23 000 000	20 000	80 000
Total reserves	135 000	33 000 000	26 500	105 000
Average annual production	1414.25	68844.44	439.72	437.14
Years left until exhaustion	95	480	60	240

research and development will enhance this trend. The reserves of iron ore and zinc are very high while lead and copper seem to be rather scarce. Yet, an increased use of particularly plastics as a renewable substitute may relax the constraints that a sustainable development faces.

These predictions are all made under the assumption that only currently available technology is applied. It seems to be likely that technological progress improves for example, mining and refining methods or makes discoveries of new resources fields possible, and, therefore, enhances present estimations of available reserves. In order to ensure sustainable growth in the sense of minimal degradation of natural capital stocks and intergenerational equity, it appears to be important to develop further renewable substitutes.

ESTIMATION OF THE MODEL

In order to estimate the growth model as described in "Modern Growth Models and Resource Constraints," first, consumer preferences and the technology of producing goods have to be specified, and, second, a data set has to be constructed. In our estimation, we focus on the standard model as presented in "The Basic Model."

It is assumed that the natural resource contributes to economic activity only in one way: it is used as an input factor for the production of a commodity, which is either consumed or added to the capital stock. The present value of utility received from consumption is given by

$$\int_0^\infty \frac{C_t^{1-\eta} - 1}{1 - \eta} e^{-\delta t} \, dt$$

where, η is the elasticity of marginal utility. The technology of goods is described by a Cobb–Douglas-production function, which depends on reproducible capital K_t and the exhaustible resource flow R_t. The elasticity of substitution between reproducible capital and the exhaustible resource σ equals 1. The evolution of capital is determined by

$$\dot{K}_t = K_t^\beta R_t^{1-\beta} - C_t$$

β denotes the share of reproducible capital in production. Maximizing present value of utility and setting $y_t = C_t/R_t$ yields the following estimable system: for a detailed study of the solution, see "Estimation Section" in the Appendix A.

$$\frac{\dot{y}_t}{y_t} = \frac{\beta x_t^{\beta-1} - \delta}{\eta} - \phi \qquad (16)$$

$$\frac{\dot{x}_t}{x_t} = x_t^{\beta-1} \qquad (17)$$

$$\frac{\dot{R}_t}{R_t} = \phi \qquad (18)$$

TABLE VI Estimation results

Parameter	Value	Standard error
β	0.32	3.4668
$\bar{\delta}$	0.03	0.0369
$\bar{\eta}$	0.5	0.2488
ϕ	0.002	0.1856

with ϕ as the growth rate of the exhaustible resource flow.

As time series data, we need consumption, reproducible capital stock[||||] and the exhaustible resource flow.[##] The reproducible capital stock, K_t, is gross real private fixed capital stock and C_t is private consumption. The time series for the resource, R, is based on our own computation. Since the total mineral production value is the amount of extracted exhaustible resources times average prices, it is used to measure the exhaustible resource flow. All time series are deflated by the GDP-price index 1990 $= 100$.

Equations (16)–(18) are estimated directly by using non-linear least squares techniques (NLLS).[***] In the estimation, we have prefixed the discount rate, $\bar{\delta}$, and the parameter of relative risk aversion, $\bar{\eta}$. The reason for this procedure is that the model we are considering—leaving aside substitution, technological change and the role of other inputs—is in its current form, rather incomplete and reliable estimates for the discount rate as well as for relative risk aversion, cannot be expected. Therefore, we prefix them at levels that have been obtained by other recent studies (see Semmler and Gong, 1997). This procedure can be understood as a calibration exercise rather than estimation. The results are summarized in Table VI.

The estimated capital share in income, β, and the estimated growth rate of the resource flow, ϕ, are reasonable.

Although there is a very irregular behavior of the total mineral production value over the observed time period, the data show an enormous increase of the value of resources in the years 1972–1981 caused by the price effects of the oil crisis.

Figures 6 and 7 show the estimated (fitted) and actual time series for the ratio of capital stock to resources and the ratio of consumption to resources. Both figures show that there is, in particular, since 1980, a strong trend toward the depletion of resources relative to capital stock and consumption.

As the figures show, the model with our estimated parameters matches the data well. As already noted, the very simple structure of the model may explain the observable slight correlation of the error terms. We also have supposed that population remains constant which implies that labor as an input factor in production is

[||||]Data on consumption and capital stock are obtained from Citibase (1998).
[##]Source: US Department of Commerce, Economics and Statistics, Bureau of the Census 1965–1997.
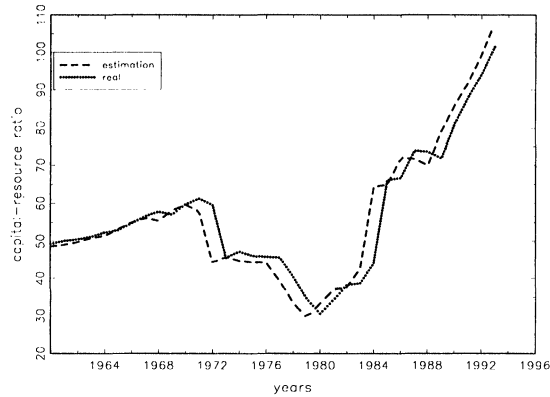[***]Computed with GAUSS–OPTMUM version 3.00.
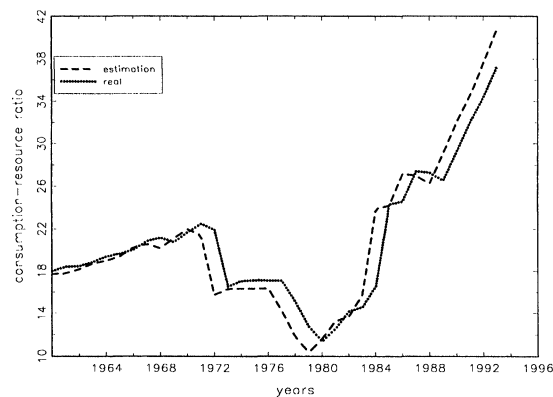
FIGURE 6   Capital−resource ratio.

FIGURE 7   Consumption−resource ratio.

factor labor and technological progress and substitution, and to estimate such a model over a longer time period or with data with a higher frequency.

CONCLUSIONS

This paper attempts to study in a formal model, the growing concern that human activity and economic growth depletes natural resources. Although both, renewable as well as exhaustible resources are threatened by extinction in the process of economic growth[****] this paper in particular focuses on exhaustible economic resources. We pursue the question of whether the current rate of extraction of exhaustible resources is sustainable given the present pattern of economic growth. We present time series data and give rough estimates of depletion time for exhaustible resources. We also study the problem of intergenerational equity and the different criteria that have been suggested in the literature to conserve resources and do justice to future generations. In a particular growth model, we study to what extent resource constraints can be overcome by technological change and substitution. Although it would be advisable to consider worldwide trends in the exhaustion of resources, we because of data problems, restrain our study to empirical trends and stylized facts on exhaustible resources of the US economy. We estimate a standard growth model with resource constraints for US time series data. We can observe from our time series data that there is, disruption by the strong increase of the value of resources due to the oil crisis in the 1970s, a strong decrease of the use of resources relative to capital stock and consumption since 1980. Our econometric estimations concerning the capital share point into the opposite direction. Yet, our estimates, in particular our estimate of the capital share, which indicates—if one follows Solow (1973)—non-sustainable growth, has to be interpreted with some care since the estimated growth model lacks other variables, uses limited time series and low frequency data. Moreover, the lack of data on technological change and substitutes for exhaustible resources prevents us from drawing the strong conclusion that there is a threat to sustainable growth in the future. Yet, the ratio of capital stock or consumption to resources has tripled, possibly indicating future problems concerning the availability of exhaustible resources or this also may indicate that technical change and substitution has reduced the use of resources.

constant, too. This assumption seems justified when analyzing the very short-run, but not appropriate when examining a time period of 35 years. It seems to be likely that incorporating the factor labor into the model would improve the estimation results. Such an improvement is particularly critical for our estimation result on β, since our estimate gives a value of $\beta = 0.32$. This would mean, according to Solow (1973), that sustained per capita growth is not feasible (see "The Basic Model"). Improvement of our estimations by including further factors would most likely give us the result that exhaustible resource share in production, $1 - \beta$, is likely to be smaller. Furthermore, as the stylized facts support, it would be reasonable to allow technological progress and substitution. Finally, the quality of non-linear estimations depends strongly on the number of observations. Since the analyzed time period consist of only 35 data points, it is difficult to achieve sufficient robustness in the estimations. This problem was clearly observable, when we attempted to estimate the prefixed parameters $\bar{\eta}$ and $\bar{\delta}$. Their estimation in fact turned out to be non-robust with respect to the algorithm used. We therefore kept them prefixed.

In summary, although our preliminary results may be illuminating, future research should take into account the

[****]For studies on the extinction of renewable resources, see Clark, 1990; Semmler and Sieveking 1994, 1997, 2000.

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APPENDIX A

The Basic Model

The following maximization problem must be solved:

$$\text{Max} \int_0^{\infty} U(C_t)e^{-\delta t}\, \mathrm{d}t \qquad (A1)$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t, \quad S_t = S_0 - \int_0^\infty R_t \, dt \quad \dot{S}_t = -R_t$$

The current value Hamiltonian is defined as

$$H_c = U(C_t) + p_t(-R_t) + q_t(F(K_t, R_t) - C_t) \qquad \text{(A2)}$$

where p_t denotes the shadow price of the resource constraint and q_t denotes the shadow price of capital accumulation. Computing the first order conditions yields the following equations, which must be fulfilled along an optimal path:

$$\dot{p}_t = \delta p_t \qquad \text{(A3)}$$

$$U'(C_t) = q_t \qquad \text{(A4)}$$

$$p_t = q_t F_R \qquad \text{(A5)}$$

$$\dot{q}_t - \delta q_t = -q_t F_K \qquad \text{(A6)}$$

with $F_R = \partial F(K_t, R_t)/\partial R_t$ and $F_K = \partial F(K_t, R_t)/\partial K_t$. Differentiating Eq. (A4) with respect to time and substituting Eq. (A6) yields the consumption rate along an optimal path:

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - \delta}{\eta} \qquad \text{(A7)}$$

Differentiating Eq. (A5) with respect to time and using Eqs. (A3) and (A6) results in

$$F_K = \frac{\partial F_R}{\partial t} \frac{1}{F_R} \qquad \text{(A8)}$$

Substituting $F_R = f(x_t) - x_t f'(x_t)$ and $F_K = f'(x_t)$ with $f(x_t) = F((K_t/R_t), 1)$ and $x_t = K_t/R_t$ in Eq. (A8) yields the capital–resource ratio along an optimal path:

$$\frac{\dot{x}_t}{x_t} = \sigma \frac{f(x_t)}{x_t} \qquad \text{(A9)}$$

Technology

The following maximization problem must be solved:

$$\text{Max} \int_0^\infty [U(C_t)\Omega_t + \omega_t W(K_t, S_t)] e^{-\delta t} \, dt \qquad \text{(A10)}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t \quad S_t = S_0 - \int_0^\infty R_t \, dt \quad \dot{S}_t = -R_t$$

The current value Hamiltonian is

$$H_c = U(C_t)\Omega_t + \omega_t W(K_t, S_t)$$
$$+ p_t(-R_t) + q_t(F(K_t, R_t) - C_t) \qquad \text{(A11)}$$

Computing the first order conditions yields

$$\dot{p}_t = \delta p_t \qquad \text{(A12)}$$

$$\Omega_t U'(C_t) = q_t \qquad \text{(A13)}$$

$$p_t = \omega_t W_S + q_t F_R \qquad \text{(A14)}$$

$$\dot{q}_t - \delta q_t = -\omega_t W_K - q_t F_K \qquad \text{(A15)}$$

Differentiating Eq. (A13) with respect to time and substituting Eq. (A15) results in

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - \delta + \psi_t(W_K - U'(C_t))/U'(C_t)}{\eta} \qquad \text{(A16)}$$

with $\psi_t = \omega_t/\Omega_t$. If $W_k = W_S = 0$, one gets the following consumption rate along an optimal path:

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - (\delta + \psi_t)}{\eta} \qquad \text{(A17)}$$

Backstop Technology

The following maximization problem must be solved

$$\text{Max} \int_0^\infty U(C_t) e^{-\delta t} \, dt \qquad \text{(36)}$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t - g(z_t)R_t, \quad S_t = S_0 - \int_0^\infty R_t \, dt,$$

$$\dot{S}_t = -R_t$$

where $g(z_t)R_t$ represents the total extraction costs. The current value Hamiltonian is

$$H_c = U(C_t) + p_t(-R_t) + q_t(F(K_t, R_t) - C_t$$
$$- g(z_t)R_t) \qquad \text{(A19)}$$

Computing the first order conditions yields

$$\dot{p}_t = \delta p_t + q_t g'(z_t)R_t = \delta p_t q_t g'(z_t)\dot{z}_t$$

$$= \delta p_t q_t \frac{\partial g(z_t)}{\partial t} \qquad \text{(A20)}$$

$$U'(C_t) = q_t \qquad \text{(A21)}$$

$$-p_t = q_t(F_R - g(z_t)) \qquad \text{(A22)}$$

$$\dot{q}_t - \delta q_t = -q_t F_K \qquad \text{(A23)}$$

Differentiating Eq. (A22) with respect to time and using Eqs. (A20) and (A23) results in

$$F_K = \frac{\partial F_R}{\partial t} \frac{1}{F_R} + \frac{F_K g(z_t)}{F_R} \qquad \text{(A24)}$$

Substituting $F_R = f(x_t) - x_t f'(x_t)$ and $F_K = f'(x_t)$ in Eq. (A24) results in the following capital–resource ratio

along an optimal path:

$$\frac{\dot{x}_t}{x_t} = \frac{\sigma f(x_t)}{x_t} + \frac{f'(x_t)}{x_t f''(x_t)} \frac{g(z_t)}{x_t} \qquad (A25)$$

The Amenity Value of a Natural Resource

The following maximization problem must be solved

$$\text{Max} \int_0^\infty U(C_t, S_t) e^{-\delta t} \, dt \qquad (A26)$$

subject to

$$\dot{K}_t = F(K_t, R_t) - C_t \quad S_t = S_0 - \int_0^\infty R_t \, dt \quad \dot{S}_t = -R_t$$

The current value Hamiltonian is defined as

$$H_c = U(C_t, S_t) + p_t(-C_t) + q_t(F(K_t, R_t) - C_t) \quad (A27)$$

Computing the first order conditions yields

$$\dot{p}_t - \delta p_t = -U_S \qquad (A28)$$

$$U_C = q_t \qquad (A29)$$

$$p_t = q_t F_R \qquad (A30)$$

$$\dot{q}_t - \delta q_t = -q_t F_K \qquad (A31)$$

Differentiating Eq. (A29) with respect to time and substituting Eq. (A31) results in the following consumption rate along an optimal path:

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - \delta - (U_{CS}/U_C)R_t}{\eta} \qquad (A32)$$

Differentiating Eq. (A30) with respect to time and using Eqs. (A28) and (A31) yields

$$F_K = \frac{\partial F_R}{\partial t}\frac{1}{F_R} + \frac{U_S/U_C}{F_R} \qquad (A33)$$

Substituting $F_R = f(x_t) - x_t f'(x_t)$ and $F_K = f'(x_t)$ in Eq. (A33) yields the following capital–resource ratio

$$\frac{\dot{x}_t}{x_t} = \sigma\frac{f(x_t)}{x_t} + \frac{U_S/U_C}{x_t^2 f''(x_t)} \qquad (A34)$$

Estimation

The following maximization problem has to be solved:

$$\text{Max} \int_0^\infty \frac{C_t^{1-\eta}}{1-\eta} e^{-\delta t} \, dt \qquad (A35)$$

subject to

$$\dot{K}_t = K_t^\beta R_t^{1-\beta} - C_t \qquad (A36)$$

$$\dot{S}_t = -R_t \qquad (A37)$$

The optimality conditions are

$$\frac{\dot{C}_t}{C_t} = \frac{F_K - \delta}{\eta} \qquad (A38)$$

$$\frac{\dot{x}_t}{x_t} = \frac{f(x_t)}{x_t} \qquad (A39)$$

With $F_K = \beta K_t^{\beta-1} R_t^{1-\beta} = \beta(K_t/R_t)^{\beta-1} = \beta x_t^{\beta-1}$ and $F(K_t/R_t, 1) = f(x_t) = x^\beta$ one gets:

$$\frac{\dot{C}_t}{C_t} = \frac{\beta x_t^{\beta-1} - \delta}{\eta} \qquad (A40)$$

$$\frac{\dot{x}_t}{x_t} = x_t^{\beta-1} \qquad (A41)$$

Setting $y_t = C_t/R_t$ yields $\dot{y}_t/y_t = (\dot{C}_t/C_t) - (\dot{R}_t/R_t)$ and the following estimable system is obtained:

$$\frac{\dot{y}_t}{y_t} = \frac{\beta x_t^{\beta-1} - \delta}{\eta} - \phi \qquad (A42)$$

$$\frac{\dot{x}_t}{x_t} = x_t^{\beta-1} \qquad (A43)$$

$$\frac{\dot{R}_t}{R_t} = \phi \qquad (A44)$$

where ϕ denotes the growth rate of the resource flow.

APPENDIX B

Data Sources

● Annual data from 1960 to 1995 for the following time series are, taken from the US Department of Commerce, Economics and Statistics, Bureau of the Census (1965–1997):

crude petroleum production;
coal production;
natural gas production;
copper production;
iron ore production;
zinc production;
lead production;
share of nuclear power, renewable energy and fuel minerals in total energy production;
total mineral production value;
proved reserves of crude petroleum and natural gas;
gross private consumption; and
accumulated gross fixed capital formation.

● Estimated reserves of crude petroleum and natural gas are from US Geological Survey (1995a):

proved reserves;
field growth;
undiscovered resources; and
total reserves.
- Estimated reserves of coal are from Energy Information Administration (1996):
proved reserves;

other reserves; and
total reserves.
- Estimated reserves of copper, iron ore, lead and zinc are from US Geological Survey (1999):
proved reserves;
other reserves; and
total reserves.