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Research Article

Dynamics of a System of Rational Higher-Order Difference Equation

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We focus on a system of a rational m-order difference equation $x_{n+1} = (x_{n-m+1})/(A + y_n y_{n-1} \cdots y_{n-m+1})$, $y_{n+1} = (y_{n-m+1})/(B + x_n x_{n-1} \cdots x_{n-m+1})$, $n = 0, 1, \ldots$, where $A, B, x_0, x_{-1}, \ldots, x_{-m+1}, y_0, y_{-1}, \ldots, y_{-m+1} \in (0, \infty)$. We investigate the dynamical behavior of positive solution for the system.

1. Introduction

In 2011, Kurbanli et al. [1] studied the behavior of positive solutions of the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} + 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} + 1}, \tag{1}$$

where the initial conditions are arbitrary nonnegative real numbers.

In the same year, Kurbanli [2] studied the behavior of solutions of the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \tag{2}$$
$$z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1} - 1},$$

where the initial conditions are arbitrary real numbers. Moreover, Kurbanli [3] studied the behavior of the solutions of the difference equation system

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \quad z_{n+1} = \frac{1}{y_n z_n}, \tag{3}$$

where $x_0, x_{-1}, y_0, y_{-1}, z_0, z_{-1} \in \mathbb{R}$ such that $y_0 x_{-1} \neq 1$, $x_0 y_{-1} \neq 1$ and $y_0 z_0 \neq 1$.

In [4], Liu et al. gave more results of the solution of the system (2) including a new and simple expression of z_n and the asymptotical behavior of the solution.

In [5], Stević showed that the system of difference equations

$$x_{n+1} = \frac{a x_{n-1}}{b y_n x_{n-1} + c}, \quad y_{n+1} = \frac{\alpha y_{n-1}}{\beta x_n y_{n-1} + \gamma}, \quad n = 0, 1, \ldots, \tag{4}$$

can be solved.

In 2012, Gu and Ding [6] derived two canonical state space forms from multiple-input multiple-output systems described by difference equations.

The system of two nonlinear difference equations

$$x_{n+1} = A + \frac{y_n}{x_{n-p}}, \quad y_{n+1} = A + \frac{x_n}{y_{n-q}}, \quad n = 0, 1, \ldots, \tag{5}$$

was studied by Papaschinopoulos and Schinas [7], where $p, q \in \mathbb{N}$.

Moreover, the system of rational difference equations

$$x_{n+1} = \frac{x_n}{a + cy_n}, \quad y_{n+1} = \frac{y_n}{b + dx_n}, \quad n = 0, 1, \ldots, \tag{6}$$

was studied by Clark et al. [8, 9], where $a, b, c, d \in (0, \infty)$ and $x_0, y_0 \in [0, \infty)$.

Liu et al. [10] studied the behavior of a system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \quad z_{n+1} = \frac{1}{x_n z_{n-1}}, \quad n = 0, 1, \ldots, \tag{7}$$

where the initial conditions are nonzero real numbers.

In 2012, Zhang et al. [11] studied the solutions, stability character, and asymptotic behavior of the system of a rational third-order difference equation

$$x_{n+1} = \frac{x_{n-2}}{A + y_n y_{n-1} y_{n-2}}, \quad y_{n+1} = \frac{y_{n-2}}{B + x_n x_{n-1} x_{n-2}}, \quad n = 0, 1, \ldots, \tag{8}$$

where $A, B, x_0, x_{-1}, x_{-2}, y_0, y_{-1}, y_{-2} \in (0, \infty)$.

In this paper, we studied the solutions, stability character, and asymptotic behavior of the system of a rational m-order difference equation

$$x_{n+1} = \frac{x_{n-m+1}}{A + y_n y_{n-1} \cdots y_{n-m+1}}, \quad y_{n+1} = \frac{y_{n-m+1}}{B + x_n x_{n-1} \cdots x_{n-m+1}}, \quad n = 0, 1, \ldots, \tag{9}$$

where $A, B, x_0, x_{-1}, \ldots, x_{-m+1}, y_0, y_{-1}, \ldots, y_{-m+1} \in (0, \infty)$.

2. Preliminaries

Let $m \in \mathbb{N}$ and let $f : I_x^m \times I_y^m \to I_x$ and $g : I_x^m \times I_y^m \to I_y$ be continuously differentiable functions, where I_x and I_y are intervals in $\mathbb{R}$.

For any $(x_0, y_0), (x_{-1}, y_{-1}), \ldots, (x_{-m+1}, y_{-m+1}) \in I_x \times I_y$, the system of difference equations

$$x_{n+1} = f(x_n, x_{n-1}, \ldots, x_{n-m+1}, y_n, y_{n-1}, \ldots, y_{n-m+1}), \quad y_{n+1} = g(x_n, x_{n-1}, \ldots, x_{n-m+1}, y_n, y_{n-1}, \ldots, y_{n-m+1}), \quad n = 0, 1, \ldots, \tag{10}$$

has a unique solution $\{(x_n, y_n)\}_{n=-m+1}^{\infty}$.

Definition 1. A point $(\overline{x}, \overline{y}) \in I_x \times I_y$ is called an equilibrium point of the system (10) if $\overline{x} = f(\overline{x}, \overline{x}, \ldots, \overline{x}, \overline{y}, \overline{y}, \ldots, \overline{y})$ and $\overline{y} = g(\overline{x}, \overline{x}, \overline{x}, \ldots, \overline{y}, \overline{y}, \ldots, \overline{y})$.

Definition 2. The linearized system of the system (10) about the equilibrium $(\overline{x}, \overline{y})$ is the system of linear difference equations

$$x_{n+1} = \sum_{i=0}^{m-1} \left(\frac{\partial f(\overline{x}, \overline{x}, \ldots, \overline{x}, \overline{y}, \overline{y}, \ldots, \overline{y})}{\partial x_{n-i}} x_{n-i} + \frac{\partial f(\overline{x}, \overline{x}, \ldots, \overline{x}, \overline{y}, \overline{y}, \ldots, \overline{y})}{\partial y_{n-i}} y_{n-i} \right),$$
$$y_{n+1} = \sum_{i=0}^{m-1} \left(\frac{\partial g(\overline{x}, \overline{x}, \ldots, \overline{x}, \overline{y}, \overline{y}, \ldots, \overline{y})}{\partial x_{n-i}} x_{n-i} + \frac{\partial g(\overline{x}, \overline{x}, \ldots, \overline{x}, \overline{y}, \overline{y}, \ldots, \overline{y})}{\partial y_{n-i}} y_{n-i} \right). \tag{11}$$

Definition 3. An equilibrium point $(\overline{x}, \overline{y})$ of the system (10) is said to be stable relative to $I_x \times I_y$ if for every $\epsilon > 0$, there exists $\delta > 0$ such that for any $(x_0, y_0), (x_{-1}, y_{-1}), \ldots, (x_{-m+1}, y_{-m+1}) \in I_x \times I_y$, with

$$\max \left\{ \sum_{i=-m+1}^{0} |x_i - \overline{x}|, \sum_{i=-m+1}^{0} |y_i - \overline{y}| \right\} < \delta. \tag{12}$$

One has $\max\{|x_n - \overline{x}|, |y_n - \overline{y}|\} < \epsilon$ for all $n \geq -m + 1$.

Definition 4. An equilibrium point $(\overline{x}, \overline{y})$ of the system (10) is called an attractor relative to $I_x \times I_y$ if for all $(x_0, y_0), (x_{-1}, y_{-1}), \ldots, (x_{-m+1}, y_{-m+1}) \in I_x \times I_y$, one has $\lim_{n\to\infty} x_n = \overline{x}$ and $\lim_{n\to\infty} y_n = \overline{y}$.

Definition 5. An equilibrium point $(\overline{x}, \overline{y})$ of the system (10) is said to be asymptotically stable relative to $I_x \times I_y$ if it is stable, and it is also an attractor.

Definition 6. An equilibrium point $(\overline{x}, \overline{y})$ of the system (10) is said to be unstable if it is not stable.

Theorem 7 (see [12]). *Let $X(n+1) = F(X(n))$, $n = 0, 1, \ldots$, be a system of difference equations and let $\overline{X}$ be the equilibrium point of the system. If all eigenvalues of the Jacobian matrix evaluated at $\overline{X}$ lie inside the open unit disk, then $\overline{X}$ is asymptotically stable. If one of them has a modulus greater than one, then $\overline{X}$ is unstable.*

Theorem 8 (see [13]). *Let $X(n+1) = F(X(n))$, $n = 0, 1, \ldots$, be a system of difference equations and let $\overline{X}$ be the equilibrium point of the system. Assume that the characteristic polynomial of the system about $\overline{X}$ is $a_0\lambda^n + a_1\lambda^{n-1} + \cdots + a_{n-1}\lambda + a_n$ where $a_i \in \mathbb{R}$ for all i and $a_0 > 0$. Then all roots of the characteristic equation lie inside the open unit disk if and only if $\Delta_k > 0$ for* *all positive integer $k \le n$, where Δ_k is the principal minor of order k of the $n \times n$ matrix*

$$\Delta_n = \begin{pmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{pmatrix}. \quad (13)$$

3. Results

We note that

(i) if $A < 1$ and $B < 1$ then the system (9) has equilibrium $(0,0)$ and $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$;

(ii) if $A = 1$ and $B < 1$ then the system (9) has equilibrium $(0,0)$ and $(\sqrt[m]{1-B}, 0)$;

(iii) if $A < 1$ and $B = 1$ then the system (9) has equilibrium $(0,0)$ and $(0, \sqrt[m]{1-A})$;

(iv) if $A > 1$ and $B > 1$ then $(0,0)$ is the unique equilibrium point of the system (9).

Theorem 9. *Let (x_n, y_n) be positive solution of the system (9). For all nonnegative integer k, one has*

$$0 \le x_n \le \begin{cases} \dfrac{x_{-m+1}}{A^{k+1}}, & n = mk+1; \\ \dfrac{x_{-m+2}}{A^{k+1}}, & n = mk+2; \\ \vdots & \\ \dfrac{x_0}{A^{k+1}}, & n = mk+m, \end{cases}$$

$$0 \le y_n \le \begin{cases} \dfrac{y_{-m+1}}{B^{k+1}}, & n = mk+1; \\ \dfrac{y_{-m+2}}{B^{k+1}}, & n = mk+2; \\ \vdots & \\ \dfrac{y_0}{B^{k+1}}, & n = mk+m. \end{cases} \quad (14)$$

Proof. Obviously, they are true for $k = 0$. Suppose that they are true for $k = l$. Then

$$x_n = \begin{cases} x_{m(l+1)+1} \le \dfrac{x_{m(l+1)-m+1}}{A} = \dfrac{1}{A} x_{ml+1} \le \dfrac{1}{A}\left(\dfrac{x_{-m+1}}{A^{l+1}}\right), & n = m(l+1)+1; \\ x_{m(m+1)+2} \le \dfrac{x_{m(l+1)-m+2}}{A} = \dfrac{1}{A} x_{ml+2} \le \dfrac{1}{A}\left(\dfrac{x_{-m+2}}{A^{l+1}}\right), & n = m(l+1)+2; \\ \vdots & \\ x_{m(l+1)+m} \le \dfrac{x_{m(l+1)}}{A} = \dfrac{1}{A} x_{ml+m} \le \dfrac{1}{A}\left(\dfrac{x_0}{A^{l+1}}\right), & n = m(l+1)+m, \end{cases}$$

$$y_n = \begin{cases} y_{m(l+1)+1} \le \dfrac{y_{m(l+1)-m+1}}{B} = \dfrac{1}{B} y_{ml+1} \le \dfrac{1}{B}\left(\dfrac{y_{-m+1}}{B^{l+1}}\right), & n = m(l+1)+1; \\ y_{4(l+1)+2} \le \dfrac{y_{m(l+1)-m+2}}{B} = \dfrac{1}{B} y_{ml+2} \le \dfrac{1}{B}\left(\dfrac{y_{-m+2}}{B^{l+1}}\right), & n = 4(m+1)+2; \\ \vdots & \\ y_{m(l+1)+m} \le \dfrac{y_{m(l+1)}}{B} = \dfrac{1}{B} y_{ml+m} \le \dfrac{1}{B}\left(\dfrac{y_0}{B^{l+1}}\right), & n = m(l+1)+m. \end{cases} \quad (15)$$

Thus, they are true for $k = l + 1$.

By the mathematical induction, this proof is completed. □

Corollary 10. *Let (x_n, y_n) be positive solution of the system (9). If $A > 1$ and $B > 1$, then the sequence $\{(x_n, y_n)\}$ converges exponentially to the equilibrium point $(0,0)$.*

Theorem 11. *Let $A > 1$ and $B > 1$. Then the equilibrium point $(0,0)$ of the system (9) is asymptotically stable.*

Proof. The linearized system of the system (9) about the equilibrium $(0,0)$ is

$$\Phi_{n+1} = D\Phi_n, \quad (16)$$

where

$$\Phi_n = \begin{pmatrix} x_n \\ x_{n-1} \\ x_{n-2} \\ \vdots \\ x_{n-m+1} \\ y_n \\ y_{n-1} \\ y_{n-2} \\ \vdots \\ y_{n-m+1} \end{pmatrix},$$

$$D = \begin{pmatrix} 0 & 0 & \cdots & 0 & \frac{1}{A} & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{B} \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \quad (17)$$

The characteristic equation of the system (16) is

$$\left(\lambda^m - \frac{1}{A}\right)\left(\lambda^m - \frac{1}{B}\right) = 0. \quad (18)$$

Thus, $|\lambda| < 1$. By Theorem 7, the equilibrium point $(0,0)$ is asymptotically stable. □

Theorem 12. *Let $A < 1$ and $B < 1$. Then both the equilibrium points $(0,0)$ and $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$ of the system (9) are unstable.*

Proof. We note by the characteristic equation (18) that $|\lambda| > 1$ and then, by Theorem 7, the equilibrium point $(0,0)$ is unstable.

Next, we consider the equilibrium point $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$. The linearized system of the system (9) about the equilibrium $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$ is

$$\Phi_{n+1} = G\Phi_n, \quad (19)$$

where

$$\Phi_n = \begin{pmatrix} x_n \\ x_{n-1} \\ x_{n-2} \\ \vdots \\ x_{n-m+1} \\ y_n \\ y_{n-1} \\ y_{n-2} \\ \vdots \\ y_{n-m+1} \end{pmatrix},$$

$$G = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 & \alpha & \alpha & \cdots & \alpha & \alpha \\ 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta & \beta & \cdots & \beta & \beta & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad (20)$$

in which

$$\alpha = -\sqrt[m]{(1-A)^{m-1}(1-B)}, \quad \beta = -\sqrt[m]{(1-A)(1-B)^{m-1}}. \quad (21)$$

The characteristic polynomial of the system (19) is

$$1 - \alpha\beta - \sum_{i=1}^{m}\left(i\alpha\beta\lambda^{i-1}\right) - 2\lambda^m - \sum_{i=1}^{m-1}\left(i\alpha\beta\lambda^{2m-(i+1)}\right) + \lambda^{2m}. \quad (22)$$

We note the characteristic polynomial $a_0\lambda^{2m} + a_1\lambda^{2m-1} + \cdots + a_{2m-1}\lambda + a_{2m}$ that $a_1 = 0$. Thus, we obtain that not all of $\Delta_k > 0$, $k = 1, 2, \ldots, 2m$. By Theorems 7 and 8, the equilibrium point $(\sqrt[m]{1-B}, \sqrt[m]{1-A})$ is unstable. □

Theorem 13. *Let $A, B < 1$ and $\Omega_1 = (0, \sqrt[m]{1-B}) \times (\sqrt[m]{1-A}, \infty)$, $\Omega_2 = (\sqrt[m]{1-B}, \infty) \times (0, \sqrt[m]{1-A})$. Assume that $\{(x_n, y_n)\}_{n=-m+1}^{\infty}$ satisfies the system (9). Then*

(i) *if $\{(x_n, y_n)\}_{n=-m+1}^{0} \subseteq \Omega_1$, then $\{(x_n, y_n)\}_{n=-m+1}^{\infty} \subseteq \Omega_1$;*

(ii) *if $\{(x_n, y_n)\}_{n=-m+1}^{0} \subseteq \Omega_2$, then $\{(x_n, y_n)\}_{n=-m+1}^{\infty} \subseteq \Omega_2$.*

Proof. (i) Assume that $\{(x_n, y_n)\}_{n=-m+1}^{0} \subseteq \Omega_1$. Then, for any $i \in \{0, 1, \ldots, m-1\}$,

$$x_{i+1} = \frac{x_{i-m+1}}{A + y_i y_{i-1} \cdots y_{i-m+1}} < \frac{x_{i-m+1}}{A + \left(\sqrt[m]{1-A}\right)^m} = x_{i-m+1},$$
$$y_{i+1} = \frac{y_{i-m+1}}{B + x_i x_{i-1} \cdots x_{i-m+1}} > \frac{y_{i-m+1}}{B + \left(\sqrt[m]{1-B}\right)^m} = y_{i-m+1}. \quad (23)$$

Then $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m) \in \Omega_1$.

Next, we suppose that $(x_k, y_k), (x_{k-1}, y_{k-1}), \ldots, (x_{k-m+1}, y_{k-m+1}) \in \Omega_1$ where k is a positive integer. Then

$$x_{k+1} = \frac{x_{k-m+1}}{A + y_k y_{k-1} \cdots y_{k-m+1}} < \frac{x_{k-m+1}}{A + \left(\sqrt[m]{1-A}\right)^m} = x_{k-m+1},$$
$$y_{k+1} = \frac{y_{k-m+1}}{B + x_k x_{k-1} \cdots x_{k-m+1}} > \frac{y_{k-m+1}}{B + \left(\sqrt[m]{1-B}\right)^m} = y_{k-m+1}. \tag{24}$$

Then $(x_{k+1}, y_{k+1}) \in \Omega_1$.

By the mathematical induction, $\{(x_n, y_n)\}_{n=-m+1}^{\infty} \subseteq \Omega_1$.

(ii) This is similar to the proof of (i). □

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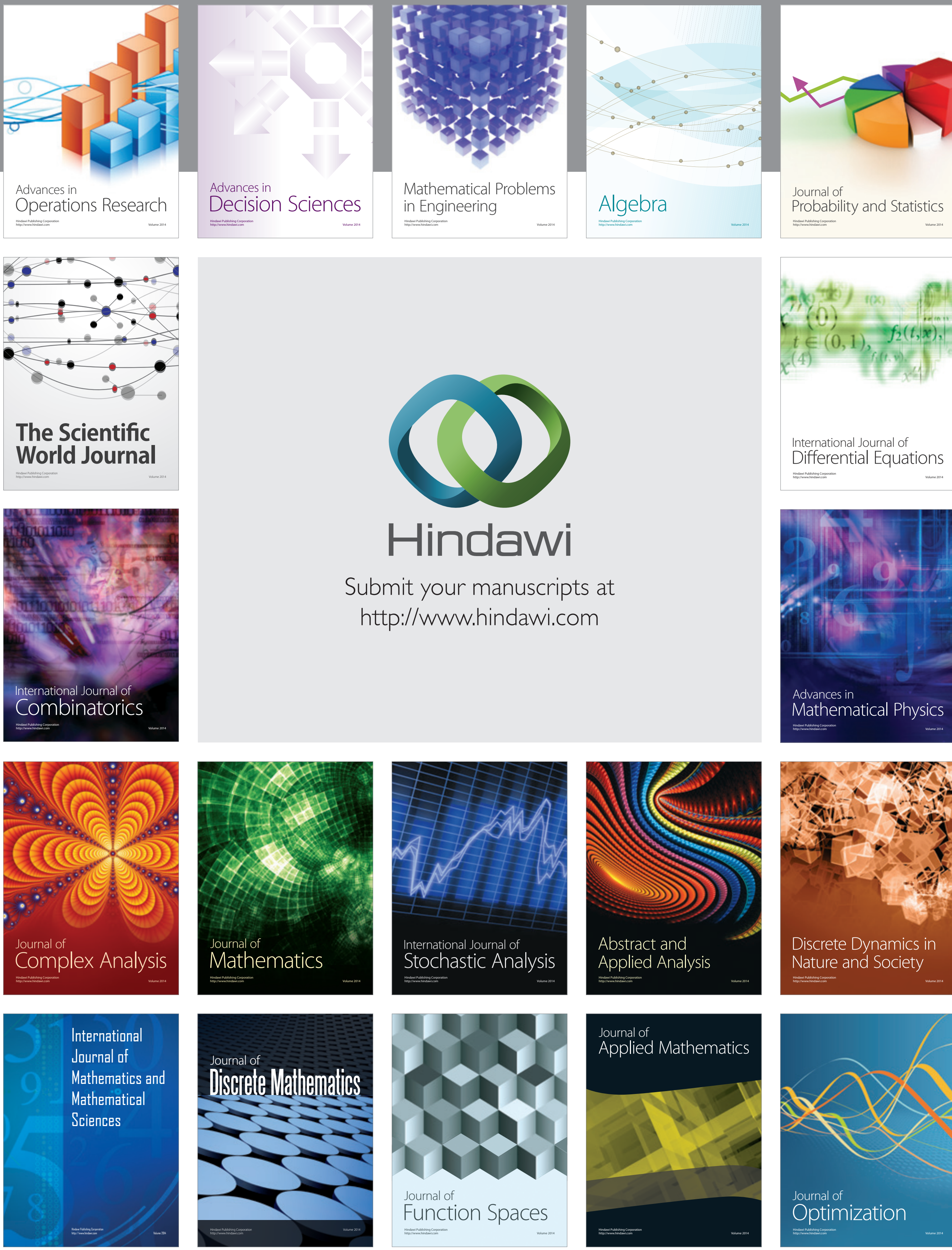
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