

Research Article

Generalized Projective Synchronization between Two Different Neural Networks with Mixed Time Delays

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The generalized projective synchronization (GPS) between two different neural networks with nonlinear coupling and mixed time delays is considered. Several kinds of nonlinear feedback controllers are designed to achieve GPS between two different such neural networks. Some results for GPS of these neural networks are proved theoretically by using the Lyapunov stability theory and the LaSalle invariance principle. Moreover, by comparison, we determine an optimal nonlinear controller from several ones and provide an adaptive update law for it. Computer simulations are provided to show the effectiveness and feasibility of the proposed methods.

1. Introduction

Over the past two decades, the investigation on the synchronization of complex networks has attracted a great deal of attention due to its potential applications in various fields, such as physics, mathematics, secure communication, engineering, automatic control, biology, and sociology [1–9]. In the literature, there are many widely studied synchronization patterns, which define the correlated in-time behaviors among the nodes in a dynamical network, for example, complete synchronization [10–12], lag synchronization [13–15], anti-synchronization [16–18], phase synchronization [19–21], projective synchronization [22–32], and so on. Projective synchronization reflects a kind of proportionality between

the synchronized states, so it is an interesting research topic and has many applications. For instance, if this proportional feature is applied to M-nary digital communication, the communication speed can be accelerated substantially. In view of this merit, many researchers throw themselves into generalized projective synchronization.

Recent years have witnessed many written achievements on projective synchronization between two identical complex dynamical networks [22–30]. We introduce three typical references here. In [28], Chen et al. studied projective synchronization of time-delayed chaotic systems in a driven-response complex network, where the nodes are not partially linear and the scale factors are different from each other. In [29], Feng et al. investigated projective-anticipating and projective-lag synchronization on complex dynamical networks composed of a large number of interconnected components, in which the dynamics of the nodes of the complex networks were time-delayed chaotic systems without the limitation of the partial linearity. In [30], Wang et al. explored the problem of outer synchronization between two complex networks with the same topological structure and time-varying coupling delay with a new mixed outer synchronization behavior. In addition, a novel nonfragile linear state feedback controller is designed to realize the mixed outer synchronization between two networks and proved analytically by using the Lyapunov-Krasovskii stability theory.

However, in real world, studying the phenomena of synchronizing two different complex networks is closer to reality. The so-called different implies that the drive and response networks have different node dynamics, different number of nodes, or different topological structures. Recently, some related works have come out, such as [31, 32]. In [31], Zheng et al. probed into adaptive projective synchronization between two complex networks with different topological structures, although its systems contained time-varying delays. Some results on topology identification were obtained which can be also seen as a highlight of this paper. In [32], the generalized projective synchronization with the above three differences was investigated based on the LaSalle invariance principle. However, the model of complex network only has linear coupling and coupling time delay.

Due to the finite information transmission and processing speeds among the units, the connection delays in realistic modeling of many large networks with communication must be taken into account. Therefore, it is important to study the effect of time delay in synchronization of coupled systems. Usually, time delay involves two parts. One is the delay inside the systems, called internal delay. The other is caused by the exchange of information between systems referred to as coupling delay. Moreover, nonlinear functions can display more nature phenomena. Hence, the internal delay and nonlinear functions are introduced into the considered neural networks in this writing. What is more, the nonlinear functions in the drive and response networks are also different. In particular, three comparable nonlinear controllers are presented to realize the GPS based on the LaSalle invariance principle and some basic inequalities. On the contrary, an optimal nonlinear controller is produced eventually. In the last theorem, we use an adaptive control technique for the optimal nonlinear controller in order to make the feedback control gain small enough.

Notation. Throughout this paper, R^n and $R^{m \times n}$ denote n -dimensional Euclidean space and the set of $m \times n$ real matrices, respectively. $\lambda_{\min}(A)$ represents the smallest eigenvalue of a symmetric matrix A . \otimes is the Kronecker product. The superscript T of x^T or A^T denotes the transpose of the vector $x \in R^n$ or the matrix $A \in R^{m \times n}$. I_n is identity matrix with n nodes.

2. Model Description and Preliminaries

Consider a general neural network with mixed time delays consisting of N_1 nodes and nonlinear couplings, which is described as follows:

$$\begin{aligned} \dot{x}_i(t) = & -C_1 x_i(t) + A_1 f_1(x_i(t)) + B_1 g_1(x_i(t - \tau_1)) + \sum_{j=1}^{N_1} G_{ij} \Gamma_1 h_1(x_j(t)) \\ & + \sum_{j=1}^{N_1} G_{ij}^T \Gamma_2 h_1(x_j(t - \tau_2)), \quad i \in \mathcal{O}_1, \end{aligned} \quad (2.1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in R^n$, ($i \in \mathcal{O}_1 = \{1, 2, \dots, N_1\}$) are the state variables of the i th node at time t ; $C_1 = \text{diag}\{c_{11}, c_{12}, \dots, c_{1n}\}$ is the decay constant matrix with $c_{1m} > 0$ ($m \in \{1, 2, \dots, n\}$); $A_1 = (a_{ij}^1)_{n \times n}$ and $B_1 = (b_{ij}^1)_{n \times n}$ are system matrices; $f_1(x_i(t)) = [f_1^1(x_{i1}(t)), f_2^1(x_{i2}(t)), \dots, f_n^1(x_{in}(t))]^T$, $g_1(x_i(t)) = [g_1^1(x_{i1}(t)), g_2^1(x_{i2}(t)), \dots, g_n^1(x_{in}(t))]^T$ and $h_1(x_i(t)) = [h_1^1(x_{i1}(t)), h_2^1(x_{i2}(t)), \dots, h_n^1(x_{in}(t))]^T$ are the continuous functions of the neurons; the positive constants τ_1 and τ_2 are internal delay and coupling delay, respectively; Γ_1 and Γ_2 are the inner coupling matrices at time t and $t - \tau_2$, respectively, which describe the way of linking the components in each pair of connected two nodes; $G = (G_{ij})_{N_1 \times N_1}$ and $G^T = (G_{ij}^T)_{N_1 \times N_1}$ are coupling configuration matrices which are not necessarily irreducible and symmetric.

In this paper, the neural network (2.1) is used as the drive network, and the response neural network consisting of N_2 nodes is expressed by

$$\begin{aligned} \dot{y}_i(t) = & -C_2 y_i(t) + A_2 f_2(y_i(t)) + B_2 g_2(y_i(t - \tau_1)) + \sum_{j=1}^{N_2} H_{ij} \Gamma_1 h_2(y_j(t)) \\ & + \sum_{j=1}^{N_2} H_{ij}^T \Gamma_2 h_2(y_j(t - \tau_2)) + u_i(t), \quad i \in \mathcal{O}_2, \end{aligned} \quad (2.2)$$

where $y_i(t) = (y_{i1}(t), y_{i2}(t), \dots, y_{in}(t))^T \in R^n$, ($i \in \mathcal{O}_2 = \{1, 2, \dots, N_2\}$) are the state variables of the i th node at time t . Without loss of generality, we suppose $N_1 \geq N_2 > 0$. $C_2 = \text{diag}\{c_{21}, c_{22}, \dots, c_{2n}\}$ is the decay constant matrix with $c_{2m} > 0$ ($m \in \{1, 2, \dots, n\}$); $A_2 = (a_{ij}^2)_{n \times n}$ and $B_2 = (b_{ij}^2)_{n \times n}$ are system matrices; $f_2(y_i(t)) = [f_1^2(y_{i1}(t)), f_2^2(y_{i2}(t)), \dots, f_n^2(y_{in}(t))]^T$, $g_2(y_i(t)) = [g_1^2(y_{i1}(t)), g_2^2(y_{i2}(t)), \dots, g_n^2(y_{in}(t))]^T$ and $h_2(y_i(t)) = [h_1^2(y_{i1}(t)), h_2^2(y_{i2}(t)), \dots, h_n^2(y_{in}(t))]^T$ are the continuous functions of the neurons; τ_1 , τ_2 , Γ_1 , and Γ_2 have the same meaning as those in (2.1). $H = (H_{ij})_{N_2 \times N_2}$ and $H^T = (H_{ij}^T)_{N_2 \times N_2}$ are coupling configuration matrices which are not necessarily irreducible and symmetric either.

Now, two mathematical definitions for the generalized projective synchronization are introduced as follows.

Definition 2.1. If there is a nonzero constant σ such that

$$\lim_{t \rightarrow +\infty} \|y_i(t) - \sigma x_i(t)\| = 0, \quad i \in \mathcal{O}_2, \quad (2.3)$$

the GPS between neural networks (2.1) and (2.2) is said to be achieved. The parameter σ is called a scaling factor.

Definition 2.2. A continuous function $f(\cdot) : R \rightarrow R$ is said to be the *nonnegative-bound function class*, denoted as $f \in \text{NBF}(\xi)$, if there exists a positive scalar ξ , such that

$$0 \leq \frac{f(x) - f(y)}{x - y} \leq \xi \quad (2.4)$$

holds for any $x, y \in R$.

The following hypothesis is used throughout the paper.

Assumption 2.3. For activation functions $f_i^1, f_i^2, g_i^1, g_i^2, h_i^1, h_i^2$ ($i \in \{1, 2, \dots, n\}$), there exist positive constants $\theta_i, \beta_i, \mu_i, \gamma_i, \pi_i, \delta_i$ ($i \in \{1, 2, \dots, n\}$), such that $f_i^1(\cdot) \in \text{NBF}(\theta_i)$, $f_i^2(\cdot) \in \text{NBF}(\beta_i)$, $g_i^1(\cdot) \in \text{NBF}(\mu_i)$, $g_i^2(\cdot) \in \text{NBF}(\gamma_i)$, $h_i^1(\cdot) \in \text{NBF}(\pi_i)$, $h_i^2(\cdot) \in \text{NBF}(\delta_i)$.

For better convenience, we denote that

$$\begin{aligned} \theta &= \max_{1 \leq i \leq n} \{\theta_i\}; & \mu &= \max_{1 \leq i \leq n} \{\mu_i\}; & \pi &= \max_{1 \leq i \leq n} \{\pi_i\}; \\ \beta &= \max_{1 \leq i \leq n} \{\beta_i\}; & \gamma &= \max_{1 \leq i \leq n} \{\gamma_i\}; & \delta &= \max_{1 \leq i \leq n} \{\delta_i\}. \end{aligned} \quad (2.5)$$

Lemma 2.4 (see [33]). For a matrix $B = (b_{ij}) \in R^{p \times q}$, denote $\alpha(B) = (1/2) \max[p, q] \max_{i,j} |b_{ij}|$, then

$$x^T B y \leq \alpha(B) (x^T x + y^T y) \quad (2.6)$$

holds for all $x \in R^p$, $y \in R^q$.

3. GPS between Two Different Neural Networks with Mixed Time Delays

In this section, we will make a study of GPS between two different neural networks with mixed time delays by means of the LaSalle invariance principle; the Lyapunov direct method, and nonlinear feedback control technique.

Define the synchronization errors between the drive network (2.1) and the response network (2.2) as $e_i(t) = y_i(t) - \sigma x_i(t)$, $i \in \mathcal{O}_2$, then we have the following error system:

$$\begin{aligned} \dot{e}_i(t) &= -C_2 e_i(t) + (C_1 - C_2) \sigma x_i(t) + A_2 f_2(y_i(t)) - \sigma A_1 f_1(x_i(t)) + B_2 g_2(y_i(t - \tau_1)) \\ &\quad - \sigma B_1 g_1(x_i(t - \tau_1)) + \sum_{j=1}^{N_2} H_{ij} \Gamma_1 h_2(y_j(t)) - \sigma \sum_{j=1}^{N_1} G_{ij} \Gamma_1 h_1(x_j(t)) \\ &\quad + \sum_{j=1}^{N_2} H_{ij}^T \Gamma_2 h_2(y_j(t - \tau_2)) - \sigma \sum_{j=1}^{N_1} G_{ij}^T \Gamma_2 h_1(x_j(t - \tau_2)) + u_i(t), \quad i \in \mathcal{O}_2. \end{aligned} \quad (3.1)$$

Theorem 3.1. Suppose Assumption 2.3 holds; if the nonlinear controllers are chosen as follows:

$$\begin{aligned} u_i(t) &= \sigma C_2 x_i(t) - C_1 y_i(t) - k_i e_i(t) - A_2 f_2(\sigma x_i(t)) + \sigma A_1 f_1\left(\frac{y_i(t)}{\sigma}\right) - B_2 g_2(\sigma x_i(t - \tau_1)) \\ &\quad + \sigma B_1 g_1\left(\frac{y_i(t - \tau_1)}{\sigma}\right) - \sum_{j=1}^{N_2} \left[H_{ij} \Gamma_1 h_2(\sigma x_j(t)) + H_{ij}^T \Gamma_2 h_2(\sigma x_j(t - \tau_2)) \right] \end{aligned}$$

$$\begin{aligned}
& + \sigma \sum_{j=1}^{N_2} G_{ij} \Gamma_1 h_1 \left(\frac{y_j(t)}{\sigma} \right) + \sigma \sum_{j=1}^{N_2} G_{ij}^r \Gamma_2 h_1 \left(\frac{y_j(t - \tau_2)}{\sigma} \right) \\
& + \sigma \sum_{j=N_2+1}^{N_1} \left[G_{ij} \Gamma_1 h_1(x_j(t)) + G_{ij}^r \Gamma_2 h_1(x_j(t - \tau_2)) \right], \quad i \in \mathcal{O}_2,
\end{aligned} \tag{3.2}$$

where k_i are the feedback control gains, let $k = \min_{i \in \mathcal{O}_2} \{k_i\}$, and when

$$\begin{aligned}
k & \geq -\lambda_{\min}(C_1 + C_2) + \alpha(A_2)(\beta^2 + 1) + \alpha(B_2)(1 + \gamma^2) + \alpha(A_1) \left(|\sigma| + \frac{\theta^2}{|\sigma|} \right) \\
& + \alpha(B_1)|\sigma| + \alpha(H \otimes \Gamma_1)(\delta^2 + 1) + \alpha(H^r \otimes \Gamma_2)(1 + \delta^2) \\
& + \alpha(G \otimes \Gamma_1) \left(|\sigma| + \frac{\pi^2}{|\sigma|} \right) + \alpha(G^r \otimes \Gamma_2)|\sigma| + \frac{\alpha(B_1)\mu^2}{|\sigma|} + \frac{\alpha(G^r \otimes \Gamma_2)\pi^2}{|\sigma|} + \epsilon_1,
\end{aligned} \tag{3.3}$$

where ϵ_1 is a positive constant, then the GPS between the two neural networks (2.1) and (2.2) can be achieved.

Proof. Consider the Lyapunov functional candidate

$$\begin{aligned}
V(t) & = \frac{1}{2} \sum_{i=1}^{N_2} e_i^T(t) e_i(t) + \left(\frac{\alpha(B_1)\mu^2}{|\sigma|} + \alpha(B_2)\gamma^2 \right) \sum_{i=1}^{N_2} \int_{t-\tau_1}^t e_i^T(s) e_i(s) ds \\
& + \left(\frac{\alpha(G^r \otimes \Gamma_2)\pi^2}{|\sigma|} + \alpha(H^r \otimes \Gamma_2)\delta^2 \right) \sum_{i=1}^{N_2} \int_{t-\tau_2}^t e_i^T(s) e_i(s) ds.
\end{aligned} \tag{3.4}$$

Calculating \dot{V} with respect to t along the solution of (3.1), and noticing the nonlinear feedback controllers (3.2), one has

$$\begin{aligned}
\dot{V}(t)|_{(3)} & = \sum_{i=1}^{N_2} e_i^T(t) \dot{e}_i(t) + \left(\frac{\alpha(B_1)\mu^2}{|\sigma|} + \alpha(B_2)\gamma^2 \right) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) - e_i^T(t - \tau_1) e_i(t - \tau_1) \right] \\
& + \left(\frac{\alpha(G^r \otimes \Gamma_2)\pi^2}{|\sigma|} + \alpha(H^r \otimes \Gamma_2)\delta^2 \right) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) - e_i^T(t - \tau_2) e_i(t - \tau_2) \right] \\
& = \sum_{i=1}^{N_2} e_i^T(t) \left[- (C_1 + C_2) e_i(t) - k_i e_i(t) + A_2 [f_2(y_i(t)) - f_2(\sigma x_i(t))] \right. \\
& \quad \left. + B_2 [g_2(y_i(t - \tau_1)) - g_2(\sigma x_i(t - \tau_1))] \right]
\end{aligned}$$

$$\begin{aligned}
& + \sum_{j=1}^{N_2} H_{ij} \Gamma_1 [h_2(y_j(t)) - h_2(\sigma x_j(t))] \\
& + \sum_{j=1}^{N_2} H_{ij}^T \Gamma_2 [h_2(y_j(t - \tau_2)) - h_2(\sigma x_j(t - \tau_2))] \\
& + \sigma A_1 \left[f_1 \left(\frac{y_i(t)}{\sigma} \right) - f_1(x_i(t)) \right] + \sigma B_1 \left[g_1 \left(\frac{y_i(t - \tau_1)}{\sigma} \right) - g_1(x_i(t - \tau_1)) \right] \\
& + \sigma \sum_{j=1}^{N_2} G_{ij} \Gamma_1 \left[h_1 \left(\frac{y_j(t)}{\sigma} \right) - h_1(x_j(t)) \right] \\
& + \sigma \sum_{j=1}^{N_2} G_{ij}^T \Gamma_2 \left[h_1 \left(\frac{y_j(t - \tau_2)}{\sigma} \right) - h_1(x_j(t - \tau_2)) \right] \\
& + \left(\frac{\alpha(B_1)\mu^2}{|\sigma|} + \alpha(B_2)\gamma^2 \right) \sum_{i=1}^{N_2} [e_i^T(t)e_i(t) - e_i^T(t - \tau_1)e_i(t - \tau_1)] \\
& + \left(\frac{\alpha(G^T \otimes \Gamma_2)\pi^2}{|\sigma|} + \alpha(H^T \otimes \Gamma_2)\delta^2 \right) \sum_{i=1}^{N_2} [e_i^T(t)e_i(t) - e_i^T(t - \tau_2)e_i(t - \tau_2)].
\end{aligned} \tag{3.5}$$

By utilizing Lemma 2.4 and Assumption 2.3, we have the following four inequalities: the first one is

$$\begin{aligned}
& \sum_{i=1}^{N_2} e_i^T(t) A_2 (f_2(y_i(t)) - f_2(\sigma x_i(t))) \\
& \leq \alpha(A_2) \sum_{i=1}^{N_2} \left[e_i^T(t)e_i(t) + (f_2(y_i(t)) - f_2(\sigma x_i(t)))^T (f_2(y_i(t)) - f_2(\sigma x_i(t))) \right] \\
& \leq \alpha(A_2) (\beta^2 + 1) \sum_{i=1}^{N_2} e_i^T(t)e_i(t).
\end{aligned} \tag{3.6}$$

Denote $H_2(e(t)) = [(h_2(y_1(t)) - h_2(\sigma x_1(t)))^T, (h_2(y_2(t)) - h_2(\sigma x_2(t)))^T, \dots, (h_2(y_{N_2}(t)) - h_2(\sigma x_{N_2}(t)))^T]^T$ and $e(t) = (e_1^T(t), e_2^T(t), \dots, e_{N_2}^T(t))^T$, and then we can get the second inequality:

$$\begin{aligned}
& \sum_{i=1}^{N_2} \sum_{j=1}^{N_2} e_i^T(t) H_{ij} \Gamma_1 (h_2(y_j(t)) - h_2(\sigma x_j(t))) \\
& = e^T(t) (H \otimes \Gamma_1) H_2(e(t)) \\
& \leq \alpha(H \otimes \Gamma_1) [e^T(t)e(t) + H_2^T(e(t)) H_2(e(t))]
\end{aligned}$$

$$\begin{aligned}
&= \alpha(H \otimes \Gamma_1) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + (h_2(y_i(t)) - h_2(\sigma x_i(t)))^T (h_2(y_i(t)) - h_2(\sigma x_i(t))) \right] \\
&\leq \alpha(H \otimes \Gamma_1) (\delta^2 + 1) \sum_{i=1}^{N_2} e_i^T(t) e_i(t),
\end{aligned} \tag{3.7}$$

and the third one is as follows:

$$\begin{aligned}
&\sum_{i=1}^{N_2} e_i^T(t) (\sigma A_1) \left[f_1 \left(\frac{y_i(t)}{\sigma} \right) - f_1(x_i(t)) \right] \\
&\leq |\sigma| \alpha(A_1) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \left(f_1 \left(\frac{y_i(t)}{\sigma} \right) - f_1(x_i(t)) \right)^T \left(f_1 \left(\frac{y_i(t)}{\sigma} \right) - f_1(x_i(t)) \right) \right] \\
&\leq |\sigma| \alpha(A_1) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \frac{\theta^2}{\sigma^2} e_i^T(t) e_i(t) \right] \\
&= \left(|\sigma| + \frac{\theta^2}{|\sigma|} \right) \alpha(A_1) \sum_{i=1}^{N_2} e_i^T(t) e_i(t).
\end{aligned} \tag{3.8}$$

Let $H_1(e(t)) = [(h_1(y_1(t)/\sigma) - h_1(x_1(t)))^T, (h_1(y_2(t)/\sigma) - h_1(x_2(t)))^T, \dots, (h_1(y_{N_2}(t)/\sigma) - h_1(x_{N_2}(t)))^T]^T$; thus, we have the last one:

$$\begin{aligned}
&\sigma \sum_{i=1}^{N_2} \sum_{j=1}^{N_2} e_i^T(t) G_{ij} \Gamma_1 \left[h_1 \left(\frac{y_j(t)}{\sigma} \right) - h_1(x_j(t)) \right] \\
&= \sigma e^T(t) (G \otimes \Gamma_1) H_1(e(t)) \\
&\leq |\sigma| \alpha(G \otimes \Gamma_1) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \left(h_1 \left(\frac{y_i(t)}{\sigma} \right) - h_1(x_i(t)) \right)^T \left(h_1 \left(\frac{y_i(t)}{\sigma} \right) - h_1(x_i(t)) \right) \right] \\
&\leq |\sigma| \alpha(G \otimes \Gamma_1) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \frac{\pi^2}{\sigma^2} e_i^T(t) e_i(t) \right] \\
&= \left(|\sigma| + \frac{\pi^2}{|\sigma|} \right) \alpha(G \otimes \Gamma_1) \sum_{i=1}^{N_2} e_i^T(t) e_i(t).
\end{aligned} \tag{3.9}$$

Similarly, we can obtain the following four inequalities:

$$\begin{aligned}
&\sum_{i=1}^{N_2} e_i^T(t) B_2 [g_2(y_i(t - \tau_1)) - g_2(\sigma x_i(t - \tau_1))] \\
&\leq \alpha(B_2) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \gamma^2 e_i^T(t - \tau_1) e_i(t - \tau_1) \right],
\end{aligned} \tag{3.10}$$

$$\sum_{i=1}^{N_2} \sum_{j=1}^{N_2} e_i^T(t) H_{ij}^T \Gamma_2 (h_2(y_j(t-\tau_2)) - h_2(\sigma x_j(t-\tau_2))) \quad (3.11)$$

$$\leq \alpha(H^T \otimes \Gamma_2) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \delta^2 e_i^T(t-\tau_2) e_i(t-\tau_2) \right],$$

$$\sum_{i=1}^{N_2} e_i^T(t) (\sigma B_1) \left[g_1 \left(\frac{y_i(t-\tau_1)}{\sigma} \right) - g_1(x_i(t-\tau_1)) \right] \quad (3.12)$$

$$\leq |\sigma| \alpha(B_1) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \frac{\mu^2}{\sigma^2} e_i^T(t-\tau_1) e_i(t-\tau_1) \right],$$

$$\sigma \sum_{i=1}^{N_2} \sum_{j=1}^{N_2} e_i^T(t) G_{ij}^T \Gamma_2 \left[h_1 \left(\frac{y_j(t-\tau_2)}{\sigma} \right) - h_1(x_j(t-\tau_2)) \right] \quad (3.13)$$

$$\leq |\sigma| \alpha(G^T \otimes \Gamma_2) \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) + \frac{\pi^2}{\sigma^2} e_i^T(t-\tau_2) e_i(t-\tau_2) \right].$$

Substituting (3.6)–(3.13) into (3.5), we can obtain

$$\begin{aligned} \dot{V}(t) \leq & \left[-\lambda_{\min}(C_1 + C_2) + \alpha(A_2)(\beta^2 + 1) + \alpha(B_2)(1 + \gamma^2) + \alpha(A_1) \left(|\sigma| + \frac{\theta^2}{|\sigma|} \right) + \alpha(B_1)|\sigma| \right. \\ & + \alpha(H \otimes \Gamma_1)(\delta^2 + 1) + \alpha(H^T \otimes \Gamma_2)(1 + \delta^2) + \alpha(G \otimes \Gamma_1) \left(|\sigma| + \frac{\pi^2}{|\sigma|} \right) + \alpha(G^T \otimes \Gamma_2)|\sigma| - k \\ & \left. + \frac{\alpha(B_1)\mu^2}{|\sigma|} + \frac{\alpha(G^T \otimes \Gamma_2)\pi^2}{|\sigma|} \right] e^T(t) e(t), \end{aligned} \quad (3.14)$$

Taking account of condition (3.3), we have $\dot{V}(t) \leq -e_1 e^T(t) e(t) \leq 0$.

Clearly, $S = \{e_i(t) = 0, k_i = k, i \in \mathcal{O}_2\}$ is the largest invariant set contained in $\{\dot{V}(t) = 0\} = \{e_i(t) = 0, i \in \mathcal{O}_2\}$. In terms of the LaSalle invariant principle, the trajectory asymptotically converges to the largest invariant set S with any initial value of (3.1), namely, $\lim_{t \rightarrow +\infty} \|e_i(t)\| = 0, i \in \mathcal{O}_2$. Hence, the GPS between neural networks (2.1) and (2.2) is realized. The proof is completed. \square

In Theorem 3.1, the generalized projective synchronization of two different neural networks with time delay has been investigated by choosing suitable nonlinear feedback controllers. However, it requires feedback control gains to be large in u_i given by (3.2), which is not practical. Hence, it is desirable to improve the scheme for reducing the feedback control gains to be as small as possible. Now, we give the following improvement scheme.

Theorem 3.2. *Suppose Assumption 2.3 holds; if the nonlinear controllers are chosen as follows:*

$$\begin{aligned}
u_i(t) = & \sigma(C_2 - C_1)x_i(t) - A_2f_2(\sigma x_i(t)) + \sigma A_1f_1(x_i(t)) - B_2g_2(\sigma x_i(t - \tau_1)) \\
& + \sigma B_1g_1(x_i(t - \tau_1)) - \sum_{j=1}^{N_2} \left[H_{ij}\Gamma_1h_2(\sigma x_j(t)) + H_{ij}^T\Gamma_2h_2(\sigma x_j(t - \tau_2)) \right] \\
& + \sigma \sum_{j=1}^{N_1} \left[G_{ij}\Gamma_1h_1(x_j(t)) + G_{ij}^T\Gamma_2h_1(x_j(t - \tau_2)) \right] - k_i e_i(t), \quad i \in \mathcal{O}_2,
\end{aligned} \tag{3.15}$$

where k_i are the feedback gains, denote $k = \min_{i \in \mathcal{O}_2} \{k_i\}$, and if

$$\begin{aligned}
k \geq & -\lambda_{\min}(C_2) + \alpha(A_2)(\beta^2 + 1) + \alpha(B_2)(1 + \gamma^2) \\
& + \alpha(H \otimes \Gamma_1)(\delta^2 + 1) + \alpha(H^T \otimes \Gamma_2)(1 + \delta^2) + e_2,
\end{aligned} \tag{3.16}$$

where e_2 is a positive constant, then the GPS between neural networks (2.1) and (2.2) can be achieved.

Proof. Select the following Lyapunov functional candidate:

$$V(t) = \frac{1}{2} \sum_{i=1}^{N_2} e_i^T(t) e_i(t) + \alpha(B_2)\gamma^2 \sum_{i=1}^{N_2} \int_{t-\tau_1}^t e_i^T(s) e_i(s) ds + \alpha(H^T \otimes \Gamma_2)\delta^2 \sum_{i=1}^{N_2} \int_{t-\tau_2}^t e_i^T(s) e_i(s) ds. \tag{3.17}$$

Differentiating V with respect to time along (3.1), we have

$$\begin{aligned}
\frac{dV}{dt} \Big|_{(3)} = & \sum_{i=1}^{N_2} e_i^T(t) \dot{e}_i(t) + \alpha(B_2)\gamma^2 \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) - e_i^T(t - \tau_1) e_i(t - \tau_1) \right] \\
& + \alpha(H^T \otimes \Gamma_2)\delta^2 \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) - e_i^T(t - \tau_2) e_i(t - \tau_2) \right] \\
= & \sum_{i=1}^{N_2} e_i^T(t) \left[-C_2 e_i(t) - k_i e_i(t) + A_2(f_2(y_i(t)) - f_2(\sigma x_i(t))) \right. \\
& + B_2(g_2(y_i(t - \tau_1)) - g_2(\sigma x_i(t - \tau_1))) + \sum_{j=1}^{N_2} H_{ij}\Gamma_1(h_2(y_j(t)) - h_2(\sigma x_j(t))) \\
& \left. + \sum_{j=1}^{N_2} H_{ij}^T\Gamma_2(h_2(y_j(t - \tau_2)) - h_2(\sigma x_j(t - \tau_2))) \right]
\end{aligned}$$

$$\begin{aligned}
& + \alpha(B_2)\gamma^2 \sum_{i=1}^{N_2} \left[e_i^T(t)e_i(t) - e_i^T(t - \tau_1)e_i(t - \tau_1) \right] \\
& + \alpha(H^T \otimes \Gamma_2)\delta^2 \sum_{i=1}^{N_2} \left[e_i^T(t)e_i(t) - e_i^T(t - \tau_2)e_i(t - \tau_2) \right].
\end{aligned} \tag{3.18}$$

Substituting (3.6), (3.7), (3.10), and (3.11) into (3.18), we can obtain

$$\begin{aligned}
\dot{V}(t) \leq & \left[-\lambda_{\min}(C_2) + \alpha(A_2)(\beta^2 + 1) + \alpha(B_2)(1 + \gamma^2) \right. \\
& \left. + \alpha(H \otimes \Gamma_1)(\delta^2 + 1) + \alpha(H^T \otimes \Gamma_2)(1 + \delta^2) - k \right] e^T(t)e(t).
\end{aligned} \tag{3.19}$$

By condition (3.16), we have $\dot{V}(t) \leq -e_2 e^T(t)e(t) \leq 0$. Similarly, in light of the proof of Theorem 3.1, the GPS between neural networks (2.1) and (2.2) can be achieved under nonlinear controllers (3.15) too. \square

Remark 3.3. Comparing the infimum of the feedback control gain k in Theorem 3.2 with that in Theorem 3.1 implies that $k_{\text{res}} = -\lambda_{\min}(C_1) + \alpha(A_1)(|\sigma| + \theta^2/|\sigma|) + \alpha(B_1)|\sigma| + \alpha(G \otimes \Gamma_1)(|\sigma| + \pi^2/|\sigma|) + \alpha(G^T \otimes \Gamma_2)|\sigma| + \alpha(B_1)\mu^2/|\sigma| + \alpha(G^T \otimes \Gamma_2)\pi^2/|\sigma|$ is the extra part compared with the needed feedback gain in Theorem 3.2. Usually, it can be considered that $k_{\text{res}} > 0$, which will be demonstrated in simulation.

Furthermore, in order to obtain much smaller feedback control gains, we choose more suitable controllers as follows.

Theorem 3.4. *Suppose Assumption 2.3 holds. Under the nonlinear controllers*

$$\begin{aligned}
u_i(t) = & \sigma C_2 x_i(t) - C_1 y_i(t) - A_2 f_2(\sigma x_i(t)) + \sigma A_1 f_1(x_i(t)) - B_2 g_2(\sigma x_i(t - \tau_1)) \\
& + \sigma B_1 g_1(x_i(t - \tau_1)) - \sum_{j=1}^{N_2} \left[H_{ij} \Gamma_1 h_2(\sigma x_j(t)) + H_{ij}^T \Gamma_2 h_2(\sigma x_j(t - \tau_2)) \right] \\
& + \sigma \sum_{j=1}^{N_1} \left[G_{ij} \Gamma_1 h_1(x_j(t)) + G_{ij}^T \Gamma_2 h_1(x_j(t - \tau_2)) \right] - k_i e_i(t), \quad i \in \mathcal{O}_2,
\end{aligned} \tag{3.20}$$

where k_i are the feedback control gains, denote $k = \min_{i \in \mathcal{O}_2} \{k_i\}$, and if

$$\begin{aligned}
k \geq & -\lambda_{\min}(C_1 + C_2) + \alpha(A_2)(\beta^2 + 1) + \alpha(B_2)(1 + \gamma^2) + \alpha(H \otimes \Gamma_1)(\delta^2 + 1) \\
& + \alpha(H^T \otimes \Gamma_2)(1 + \delta^2) + \epsilon_3,
\end{aligned} \tag{3.21}$$

where ϵ_3 is some positive constant, then the GPS between the two neural networks (2.1) and (2.2) can be achieved.

Proof. Choose the same Lyapunov functional as (3.17) in the proof of Theorem 3.2, and then we can get

$$\begin{aligned}
\frac{dV}{dt} \mid (3) &= \sum_{i=1}^{N_2} e_i^T(t) \left[- (C_1 + C_2)e_i(t) - k_i e_i(t) + A_2(f_2(y_i(t)) - f_2(\sigma x_i(t))) \right. \\
&\quad + B_2(g_2(y_i(t - \tau_1)) - g_2(\sigma x_i(t - \tau_1))) + \sum_{j=1}^{N_2} H_{ij} \Gamma_1 (h_2(y_j(t)) - h_2(\sigma x_j(t))) \\
&\quad \left. + \sum_{j=1}^{N_2} H_{ij}^T \Gamma_2 (h_2(y_j(t - \tau_2)) - h_2(\sigma x_j(t - \tau_2))) \right] \\
&\quad + \alpha(B_2) \gamma^2 \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) - e_i^T(t - \tau_1) e_i(t - \tau_1) \right] \\
&\quad + \alpha(H^T \otimes \Gamma_2) \delta^2 \sum_{i=1}^{N_2} \left[e_i^T(t) e_i(t) - e_i^T(t - \tau_2) e_i(t - \tau_2) \right].
\end{aligned} \tag{3.22}$$

Combining (3.6), (3.7), (3.10), (3.11), and (3.22), we could change inequality (3.19) into

$$\begin{aligned}
\dot{V}(t) &\leq \left[-\lambda_{\min}(C_1 + C_2) + \alpha(A_2)(\beta^2 + 1) + \alpha(B_2)(1 + \gamma^2) \right. \\
&\quad \left. + \alpha(H \otimes \Gamma_1)(\delta^2 + 1) + \alpha(H^T \otimes \Gamma_2)(1 + \delta^2) - k \right] e^T(t) e(t).
\end{aligned} \tag{3.23}$$

Taking into account condition (3.21), the GPS can be realized under nonlinear controllers (3.18). \square

Remark 3.5. It is easy to find that the infimum of k in (3.21) has one more term $-\lambda_{\min}(C_1)$ than (3.16). Because C_1 is diagonally positive definite, $-\lambda_{\min}(C_1) < 0$ demonstrates the required k in Theorem 3.4 is smaller than the one in Theorem 3.2.

If the two neural networks (2.1) and (2.2) have identical number of nodes, node dynamics, and topological structure, that is, $N_1 = N_2$, $C_1 = C_2$, $A_1 = A_2$, $B_1 = B_2$, $f_1 = f_2$, $g_1 = g_2$, $G_{ij} = H_{ij}$, $G_{ij}^T = H_{ij}^T$, and $\Gamma_1 = \Gamma_2$, the error system (3.1) can be rewritten as follows:

$$\begin{aligned}
\dot{e}_i(t) &= -C_1 e_i(t) + A_1 [f_1(y_i(t)) - \sigma f_1(x_i(t))] + B_1 [g_1(y_i(t - \tau_1)) - \sigma g_1(x_i(t - \tau_1))] \\
&\quad + u_i(t) + \sum_{j=1}^{N_1} G_{ij} \Gamma_1 [h_1(y_j(t)) - \sigma h_1(x_j(t))] \\
&\quad + \sum_{j=1}^{N_1} G_{ij}^T \Gamma_2 [h_2(y_j(t - \tau_2)) - \sigma h_2(x_j(t - \tau_2))], \quad i \in \mathcal{J}_2.
\end{aligned} \tag{3.24}$$

Thus, we can obtain the following corollary for synchronizing the error system (3.24).

Corollary 3.6. *Suppose the two neural networks (2.1) and (2.2) have identical number of nodes, node dynamics, and topological structure. If the following nonlinear controllers:*

$$\begin{aligned} u_i(t) = & -A_1 [f_1(\sigma x_i(t)) - \sigma f_1(x_i(t))] - B_1 [g_1(\sigma x_i(t - \tau_1)) - \sigma g_1(x_i(t - \tau_1))] \\ & - k_i e_i(t) - \sum_{j=1}^{N_1} G_{ij} \Gamma_1 [h_1(y_j(t)) - \sigma h_1(x_j(t))] \\ & - \sum_{j=1}^{N_1} G_{ij}^T \Gamma_2 [h_2(y_j(t - \tau_2)) - \sigma h_2(x_j(t - \tau_2))], \quad i \in \mathcal{O}_1, \end{aligned} \quad (3.25)$$

are employed, where k_i are the feedback control gains, denote $k = \min_{i \in \mathcal{O}_2} \{k_i\}$, and when $k \geq -C_1 + \alpha(A_1)(\beta^2 + 1) + \alpha(B_1)(1 + \gamma^2)$, the error system (3.24) can be synchronized.

Proof. We construct the Lyapunov function as follows:

$$V(t) = \frac{1}{2} \sum_{i=1}^{N_2} e_i^T(t) e_i(t) + \alpha(B_1) \gamma^2 \sum_{i=1}^{N_2} \int_{t-\tau_1}^t e_i^T(s) e_i(s) ds. \quad (3.26)$$

We can conclude that $k \geq -C_1 + \alpha(A_1)(\beta^2 + 1) + \alpha(B_1)(1 + \gamma^2)$ by using the same method as that in the above theorems.

Furthermore, let us assume that system (3.24) is without time-delay terms; then, system (3.24) reduces to

$$\begin{aligned} \dot{e}_i(t) = & -C_1 e_i(t) + A_1 [f_1(y_i(t)) - \sigma f_1(x_i(t))] \\ & + \sum_{j=1}^{N_1} G_{ij} \Gamma_1 [h_1(y_j(t)) - \sigma h_1(x_j(t))] + u_i(t), \quad i \in \mathcal{O}_1. \end{aligned} \quad (3.27)$$

Then, a simpler corollary can be produced. □

Corollary 3.7. *The following controllers:*

$$u_i(t) = -A_1 [f_1(\sigma x_i(t)) - \sigma f_1(x_i(t))] - \sum_{j=1}^{N_1} G_{ij} \Gamma_1 [h_1(y_j(t)) - \sigma h_1(x_j(t))] - k_i e_i(t), \quad i \in \mathcal{O}_1, \quad (3.28)$$

can be applied to synchronize the system (3.27) when the feedback control gain k satisfies the inequality $k \geq -C_1 + \alpha(A_1)(\beta^2 + 1)$.

Similarly, in the drive network (2.1) and the response network (2.2), if $A_1 = I_{N_1}$, $A_2 = I_{N_2}$, $B_1 = B_2 = 0$, h_1, h_2 are linear functions and $G = H = 0$, the error system (3.1) can be rewritten as follows:

$$\dot{e}_i(t) = -(C_1 + C_2) e_i(t) + f_2(e_i(t)) + \sum_{j=1}^{N_2} H_{ij}^T \sigma_2 e_j(t - \tau_2) - k_i e_i(t), \quad i \in \mathcal{O}_2. \quad (3.29)$$

Thus, we can obtain the following corollary for synchronizing the error system (3.24).

Corollary 3.8. *By applying the nonlinear controllers*

$$\begin{aligned} u_i(t) = & \sigma C_2 x_i(t) - C_1 y_i(t) - f_2(\sigma x_i(t)) + \sigma f_1(x_i(t)) - \sigma \sum_{j=1}^{N_2} H_{ij}^T \Gamma_2 x_j(t - \tau_2) \\ & + \sigma \sum_{j=1}^{N_1} G_{ij}^T \Gamma_2 x_j(t - \tau_2) - k_i e_i(t), \quad i \in \mathcal{O}_2, \end{aligned} \quad (3.30)$$

where k_i are the feedback control gains, denote $k = \min_{i \in \mathcal{O}_2} \{k_i\}$, and when $k \geq -\lambda_{\min}(C_1 + C_2) + 2\alpha(H^T \otimes \Gamma^2) + 1$, the GPS between the two neural networks (2.1) and (2.2) can be achieved.

Proof. We construct the Lyapunov function as follows:

$$V(t) = \frac{1}{2} \sum_{i=1}^{N_2} e_i^T(t) e_i(t) + \alpha (H^T \otimes \Gamma^2) \int_{t-\tau_2}^t e_i^T(s) e_i(s) ds. \quad (3.31)$$

We can conclude that $k \geq -\lambda_{\min}(C_1 + C_2) + 2\alpha(H^T \otimes \Gamma^2) + 1$ by using the same method as that in the above theorems. Meanwhile, this conclusion also contains the result of [32].

It is easy to see that the theoretical feedback gains given in the above results (Theorems 3.1–3.4) are too conservative, usually much larger than the needed value; clearly, it is desirable to make the feedback gains as small as possible. Here, the adaptive technique is adopted to achieve this goal. \square

Theorem 3.9. *Suppose that Assumption 2.3 holds and the feedback controllers are chosen as (3.20). If the feedback control gains satisfy the update law*

$$\dot{k}_i = \varrho_i e_i^T(t) e_i(t), \quad i \in \mathcal{O}_2, \quad (3.32)$$

(ϱ_i are arbitrary positive constants), then the GPS between the neural network (2.1) and (2.2) can be realized.

Proof. We construct the Lyapunov function as follows:

$$\begin{aligned} V(t) = & \frac{1}{2} \sum_{i=1}^{N_2} e_i^T(t) e_i(t) + \alpha (B_2) \gamma^2 \sum_{i=1}^{N_2} \int_{t-\tau_1}^t e_i^T(s) e_i(s) ds + \frac{1}{2} \sum_{i=1}^{N_2} \frac{1}{\varrho_i} (k_i - \varrho)^2 \\ & + \alpha (H^T \otimes \Gamma_2) \delta^2 \sum_{i=1}^{N_2} \int_{t-\tau_2}^t e_i^T(s) e_i(s) ds, \end{aligned} \quad (3.33)$$

where ϱ is a sufficient large positive constant to be determined.

Let

$$V_1(t) = \frac{1}{2} \sum_{i=1}^{N_2} \frac{1}{Q_i} (k_i - Q)^2. \quad (3.34)$$

Then we have

$$\dot{V}_1(t) = \sum_{i=1}^{N_2} (k_i - Q) e_i^T(t) e_i(t). \quad (3.35)$$

Combining (3.18) and (3.35) with (3.33), one can obtain the following inequality:

$$\begin{aligned} \dot{V}(t) \leq & \left[-\lambda_{\min}(C_1 + C_2) + \alpha(A_2)(\beta^2 + 1) + \alpha(B_2)(1 + \gamma^2) \right. \\ & \left. + \alpha(H \otimes \Gamma_1)(\delta^2 + 1) + \alpha(H^T \otimes \Gamma_2)(1 + \delta^2) - Q \right] e^T(t) e(t). \end{aligned} \quad (3.36)$$

Therefore, according to the above proof of the three theorems, it is easy to verify the conclusion of Theorem 3.9. \square

4. Numerical Simulations

In this section, several examples are given to verify the conclusions established above. Consider the two neural networks (2.1) and (2.2) with the following parameters

$$\begin{aligned} C_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.97 & 0 \\ 0 & 1.1 \end{bmatrix}, & A_1 &= \begin{bmatrix} 2 & -0.11 \\ -5 & 3.2 \end{bmatrix}, & A_2 &= \begin{bmatrix} 2.1 & -0.1 \\ -5.1 & 3.2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} -1.6 & 0.1 \\ -0.18 & -2.4 \end{bmatrix}, & B_2 &= \begin{bmatrix} -1.5 & 0 \\ -0.15 & -2.3 \end{bmatrix}, & \Gamma_1 &= \Gamma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ G &= \begin{bmatrix} -5 & 1 & 1 & 1 & 1 & 1 \\ 1 & -5 & 1 & 1 & 1 & 1 \\ 1 & 1 & -3 & 1 & 0 & 0 \\ 1 & 1 & 1 & -4 & 1 & 0 \\ 1 & 1 & 0 & 1 & -4 & 1 \\ 1 & 1 & 0 & 0 & 1 & -3 \end{bmatrix}, & G^T &= \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}, \\ H &= \begin{bmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & -3 \end{bmatrix}, & H^T &= \begin{bmatrix} -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}, \end{aligned} \quad (4.1)$$

and $g_i^j(x) = f_i^j(x) = \tanh(x(t))$ for $j = 1, 2$, $N_1 = 6$ and $N_2 = 4$.

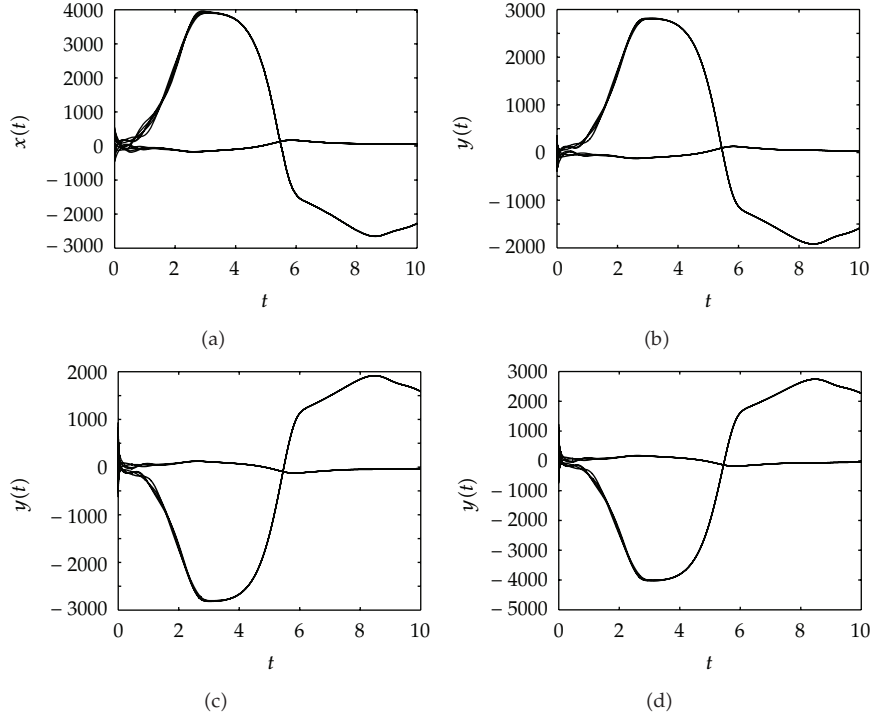


Figure 1: (a) Trajectory of the drive system. (b) trajectory of the response system with $\alpha = 0.7$. (c) trajectory of the response system with $\alpha = -0.7$. (d) Trajectory of the response system with $\alpha = -1$.

Simulation 1

The four graphs in Figure 1 represent the motion trace of drive and response systems. Comparing Figures 1(b), 1(c), and 1(d) with Figure 1(a), we can find a trivial conclusion. Because the parameters in response system and the ones in drive system are very similar, the shape of the motion orbit of response systems looks like the one of drive system. But the biases between the four traces and the “0” orbit is rather different.

Simulation 2

Setting $\theta = 1$, $\mu = 1$, $\pi = 1$, $\beta = 1$, $\gamma = 1$, $\delta = 1$, it is easy to verify that Assumption 2.3 holds. Choose $x_{ij}(t) = 100(12 - 3i - j)$ and $x_{ij}(t) = 100(-8 + 3i + j)$ for $t \leq 0$ as the initial values and define

$$E_x = \sum_{i>j} \|x^i(t) - x^j(t)\|, \quad E_y = \sum_{i>j} \|y^i(t) - y^j(t)\|, \quad E_{xy} = \sum_{i=1}^6 \sum_{j=1}^4 \|x^i(t) + y^j(t)\|, \quad (4.2)$$

for measuring the process of projective synchronization.

In Figure 2, the top three plots present the synchronization process of drive system with $\alpha = 0.7, -0.7, -1$, respectively; the middle three graphs demonstrate the change trend of E_y as $t \rightarrow \infty$ with the former three projective factors; the bottom three ones represent the

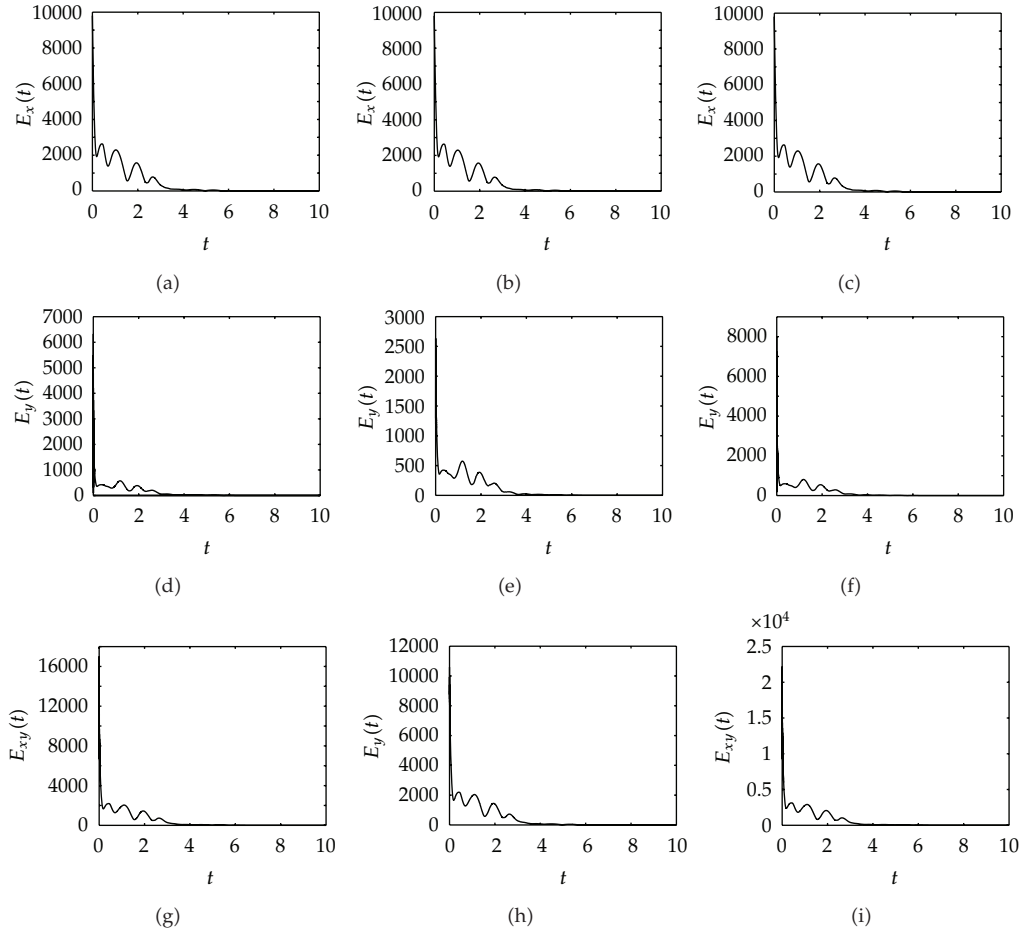


Figure 2: The change process of E_x , E_y , and E_{xy} as time $t \rightarrow \infty$ for different projective factors.

synchronization trace of the two neural networks for the three α s. The nine figures say that, for different α , the required synchronization times are all in the interval $[4, 6]$. However, the change amplitudes of the nine curves show obvious differences. By the same way, for the three projective factors, we can provide the three infimums of feedback control gains that are $k \geq 40.65$ with $\alpha = 0.7$, $k \geq 40.65$ with $\alpha = -0.7$ and $k \geq 39.33$ with $\alpha = -1$, respectively.

Remark 4.1. The reason why we apply three different nonlinear controllers for obtaining Theorems 1, 2, and 3 is that we want to find smaller feedback control gain. By computing, we have $k \geq 40.65$ for Theorem 3.1, $k \geq 21.43$ for Theorem 3.2, and $k \geq 20.43$ for Theorem 3.4, respectively. It shows that our conjecture and simulation meet these results.

Simulation 3

In this simulation, we want to verify the synchronization process of the two neural networks when an adaptive control is applied to the response system. Figure 3 shows the time evolution of E_x , E_y , E_{xy} and $k(t)$, as well as the fourth plot Figure 3(d) tell us that the feedback control gain trends towards 11.15 approximately as $t \rightarrow \infty$ which is far lower than 39.33.

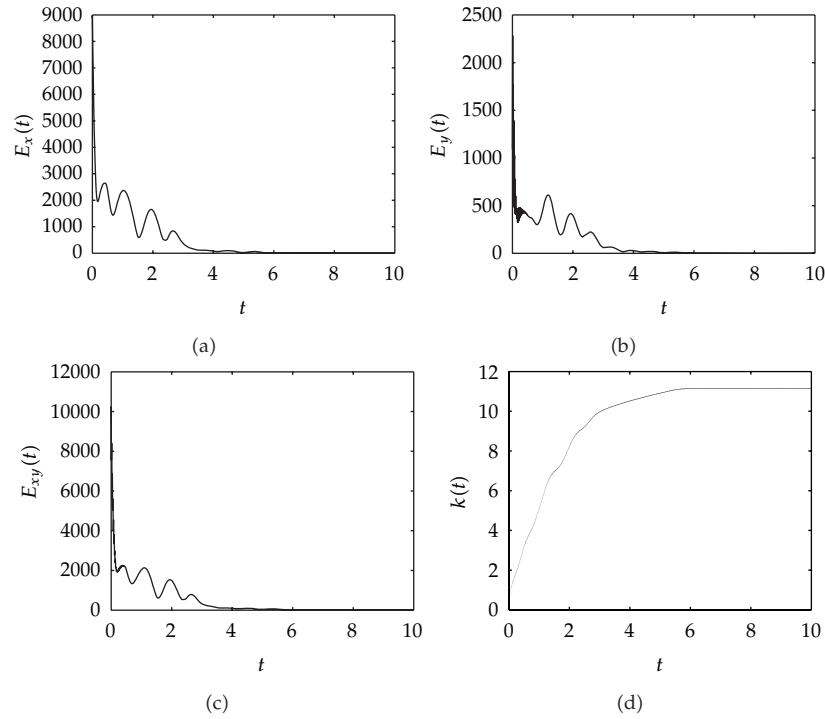


Figure 3: (a), (b), and (c) show the change process of E_x , E_y , and E_{xy} with the adaptive control (3.32); (d) shows the change curve of the adaptive control gain.

Actually, this proves that the adaptive control method can diminish the feedback control gain.

5. Conclusion

The GPS between two neural networks with mixed time delays and different parameters was investigated in this paper. By means of the Lyapunov stability theories, the GPS was realized under control of three nonlinear controllers. By comparison, we found that the nonlinear controller in Theorem 3.4 was simpler and easier to ensure that the GPS was achieved. Therefore, it was also applied in practical design. According to the optimal nonlinear controller, an adaptive update technique was designed to guarantee that the feedback control gain was sufficiently small. Eventually, several numerical simulations verified the validity of those results.

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