Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2011, Article ID 696017, 9 pages
doi:10.1155/2011/696017

Research Article

Parameter Estimation of a Class One-Dimensional Discrete Chaotic System

Lidong Liu, Jinfeng Hu, Huiyong Li, Jun Li, Zishu He, and Chunlin Han

School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China

Correspondence should be addressed to Lidong Liu, liulidong_1982@126.com

Received 6 March 2011; Revised 1 June 2011; Accepted 20 June 2011

Academic Editor: Antonia Vecchio

It is of vital importance to exactly estimate the unknown parameters of chaotic systems in chaos control and synchronization. In this paper, we present a method for estimating one-dimensional discrete chaotic system based on mean value method (MVM). It is proposed by exploiting the ergodic and synchronization features of chaos. It can effectively estimate the parameter value, and it is more exact than MVM. Finally, numerical simulations on Chebyshev map and Tent map show that the proposed method has better performance of parameter estimation than MVM.

1. Introduction

Chaos phenomena have been widely found in physical and communication systems [1–8]. The parameters of these systems provide insight into their complex behaviors. However, direct measurement of system parameters is often difficult. So it is necessary to have robust and efficient algorithms for estimating the parameters of chaotic system.

Several approaches have been proposed for estimating the parameters of chaotic system [5–17]. In [5], an adaptive scheme for synchronization-based parameter estimation of continuous chaotic system was performed. It could estimate all the parameters of the response system using the driving signal only. In [6], a method for estimating continuous chaotic system parameters by optimizing synchronization with a genetic algorithm is proposed. It can effectively find the actual parameter value from a rugged fitness landscape, even with strong measurement noise. In [7], a method, which can detect the dynamical structure from short hiding behind complex chaotic series by comparing prediction performance of trial functions, is proposed. This method is valid for chaotic parameter estimation even when the original system is contaminated with noise. In [8], a method, which is very useful in validating the global models, is proposed. It is focused on the dynamical properties of the model, and it can estimate the parameters of the system based on the dynamical features of the model. In [9], a method, which builds polynomial models from data on a Poincaré section and prior knowledge about the first period doubling bifurcation, is proposed. Similar to [8], it is also useful for the parameter estimation of the discrete chaotic system from the suitable models. In [9, 10], a method called mean value method (MVM) for estimating the parameters of discrete chaotic system was proposed. It exploited the ergodic properties of chaotic sequence and got the estimated parameter based on the monotonic character of the mean value function of the parameter. But usually the monotonic is not strict. Thus MVM is not quite suitable for exact estimation though it is accomplished easily. Since chaos is sensitive to the parameter, a small change, for example, 10^{-6} change of the parameter, two chaotic sequences will diverge from each other rapidly; even the two chaotic sequences have the same initial condition. So a more exact estimation method for discrete chaotic sequence is needed.

In this paper, a method for estimating one-dimensional discrete chaotic system based on mean value method (MVM) is proposed. It is by exploiting the ergodicity and synchronization features of chaos. It can effectively estimate the parameter value. Our attention herein is focused on estimating the parameters of one-dimensional (1-D) discrete chaotic sequence, since the 1-D map is potentially useful and well understood. However, the proposed method is not restricted to 1-D systems, the method in this paper can still be extended.

This paper is organized as follows. In Section 2, the brief introduction of mean value method (MVM) for Parameter Estimation is introduced. In Section 3, the improved MVM is proposed, which is based on the ergodic property and synchronization property of chaos. In Section 4, numerical simulations have been done to verify the effectiveness of the proposed method in this paper. Brief conclusion of this paper is drawn in Section 5.

2. The Mean Value Method for Parameter Estimation

The proposed method is based on the mean value method (MVM); thus we introduce the MVM in this section briefly.

Before illustrating the MVM, some notation should be introduced. Let $f(\cdot)$ be a chaotic map defined on some certain closed interval and let the parameter θ of the chaotic map lie between $[\theta_a, \theta_b]$. Let $\{x_\theta(n)\}$ be a chaotic sequence generated by $f(\cdot)$; that is

$$x_\theta(n) = f(\theta, \mathbf{x}_\theta(n-1)), \tag{2.1}$$

where $\mathbf{x}_\theta(n-1) = [x_\theta(n-1), x_\theta(n-2), \dots, x_\theta(n-d)]^{\mathrm{T}}$ is the d-dimensional state vector.

In [8], the authors consider that $f(\theta, \mathbf{x}_\theta(n-1))$ has a unique invariant ergodic measure, and the mean value function $M(\theta)$ of the chaotic map is monotonic for many chaotic maps, where $M(\theta)$ is defined by.

$$M(\theta) = \frac{1}{N}\sum_{n=1}^{N} x_\theta(n). \tag{2.2}$$

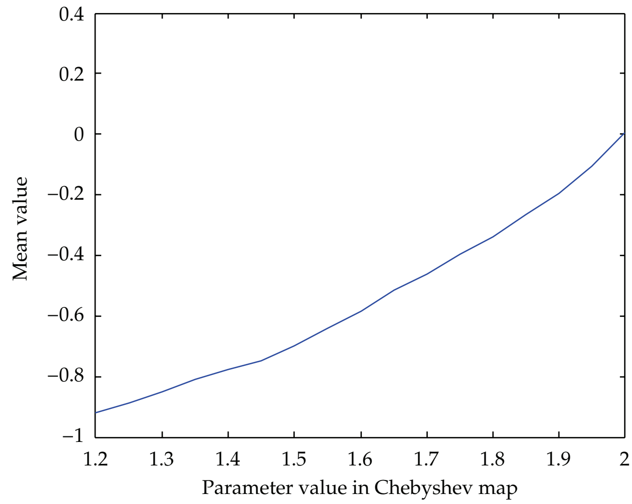

Figure 1: Mean value estimation $M(\theta)$ of chaotic sequence generated by Chebyshev map with different parameters (parameter sampling length is 0.05).

Since $M(\theta)$ is monotonic, the existence of M^{-1} is guaranteed, and the estimated parameter $\hat{\theta}$ could be got by

$$\hat{\theta} = M^{-1}(M(\theta)). \tag{2.3}$$

However, after experiments we find that $M(\theta)$ is not strictly monotonic, and this can be illustrated by Figures 1 and 2 using Chebyshev chaotic map. Figure 1 shows that the mean value function $M(\theta)$ is monotone increasing when the parameter sampling interval is large. But $M(\theta)$ is not strictly monotone increasing when the parameter sampling interval is small which can be seen in Figure 2. Thus when the parameter sampling interval is small, we cannot use (2.3), since $\hat{\theta}$ is not unique. Because (2.3) is the base of MVM, thus the accuracy of MVM is not high though it is easy to be accomplished.

3. The Improved Mean Value Method

In order to get a more accurate estimation method for 1-D discrete chaotic sequences, in this section we offer an improved method based on MVM.

According to the character that the mean value functions of discrete chaotic maps are nearly monotonic, we can get the small interval l which includes the value of the unknown parameter. First, we compute the mean value of the observation chaotic sequence $M(\theta_0)$ by (3.1) ($M(\theta_0)$ is a constant if the observation chaotic sequence is certain). Then, we compute the mean value of different parameters $M(\theta_i)$, respectively, by (3.2). $M(\theta_i)$ is variable with different θ_i. Herein, the initial value of chaotic map is random selected in the range value when using (3.2). Finally, let $M(\theta_0)$ intersect with $M(\theta_i)$, and their intersection is defined as the required interval l. These three steps can be illustrated by Figure 3. The left dotted line is the lower bound of the interval l and the right dotted line upper bound of that.

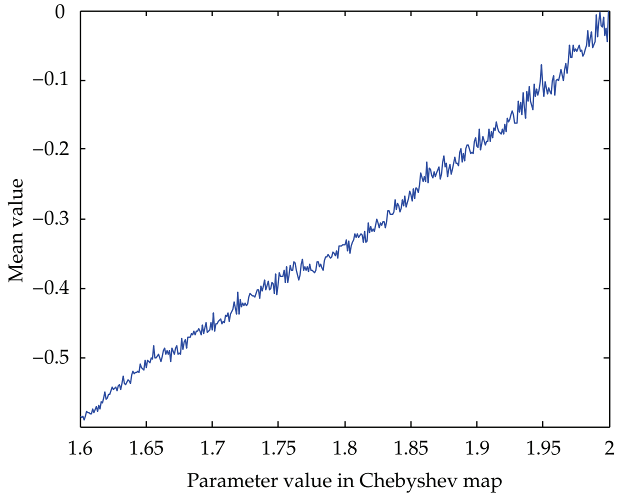

Figure 2: Mean value estimation $M(\theta)$ of chaotic sequence generated by Chebyshev map with different parameters (parameter sampling length is 0.001).

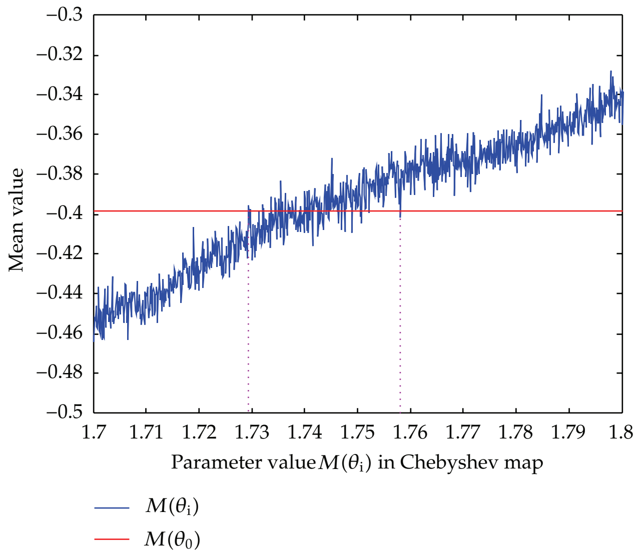

Figure 3: $M(\theta_0)$ intersects with $M(\theta_i)$ to define the needed interval which is between the two dot lines for Chebyshev map with $\theta_0 = 1.7527$.

The aforementioned way is the difference to MVM. In MVM, they use (2.3) to get estimation value. Here, we do not need an approximate estimation value by using (2.3) but use (3.1) and (3.2) together to get the required small interval l:

$$M(\theta_0) = \left(\frac{1}{N}\right)\sum_{n=1}^{N} x_{\theta_0}(n), \tag{3.1}$$

$$M(\theta_i) = \left(\frac{1}{N}\right)\sum_{n=1}^{N} x_{\theta_i}(n), \tag{3.2}$$

where θ_i is a sample value in the parameter range.

When l is got, we can get the more accurate estimation value according to the character that the synchronization error of the discrete chaotic systems is sensitive to the parameters. Next, we will describe how to use chaotic synchronization way to estimate parameter.

Consider two maps. One is the master system with the unknown parameter θ_0. The other is the slave system with the parameter $\widehat{\theta} \in l$. The master system is defined by (3.3), and the slave system is defined by (3.4):

$$x_{\theta_0}(n) = f(\theta_0, \mathbf{x}_{\theta_0}(n-1)), \tag{3.3}$$

$$y_{\widehat{\theta}}(n) = f_{\widehat{\theta}}\left(\widehat{\theta}, \mathbf{y}_{\widehat{\theta}}(n-1)\right). \tag{3.4}$$

We assume that the obtained chaotic sequence is generated by the master system. In order to make the two system synchronization we should add a controller $u(n)$ to slave system:

$$y_{\widehat{\theta}}(n) = f_{\widehat{\theta}}\left(\widehat{\theta}, \mathbf{y}_{\widehat{\theta}}(n-1)\right) + u(n), \tag{3.5}$$

where $u(n)$ is given by

$$u(n) = k\left|y_{\widehat{\theta}}(n-1) - x_{\theta_0}(n-1)\right|, \tag{3.6}$$

k is the coupling coefficient. In order to get a more accurate estimation, we pick up the estimation value $\widehat{\theta}_0$ in l according to

$$\widehat{\theta}_0 = \arg \inf_{\widehat{\theta}_0 \in l} \left\| \mathbf{x}_{\theta_0}(n) - \mathbf{y}_{\widehat{\theta}_0}(n) \right\|, \tag{3.7}$$

where $\inf(f(x))$ denotes the infimum of $f(x)$ and $\|\mathbf{x}(n)\| = \sqrt{\langle \mathbf{x}(n), \mathbf{x}^*(n) \rangle}$. The reason of using (3.7) is according to the feature that synchronization error of the discrete chaotic systems is sensitive to the parameters. That means, only when the estimated parameter value approaches the real parameter value, can the synchronization error e which is defined by (3.8) approach zero in the improved MVM in this paper. In essence, the estimated parameter is chosen in the interval l. The rule is the following: choose the estimated parameter value which can make least the synchronization error by (3.7).

$$e = \mathbf{x}_{\theta_0}(n) - \mathbf{y}_{\widehat{\theta}_0}(n). \tag{3.8}$$

The proposed method is summarized as follows.

(1) Compute the mean value function $M(\theta)$ of the chaotic sequence $\{x(n)\}$ using time average, that is, $M(\theta_i) = (1/N)\sum_{n=1}^{N} x(n)$, $i = 1, 2, \ldots, T$, where T is the total parameter sampling number.

(2) Compute the mean value of the estimated chaotic sequence $\{x_{\theta_0}(n)\}$ by using time average in (3.1).

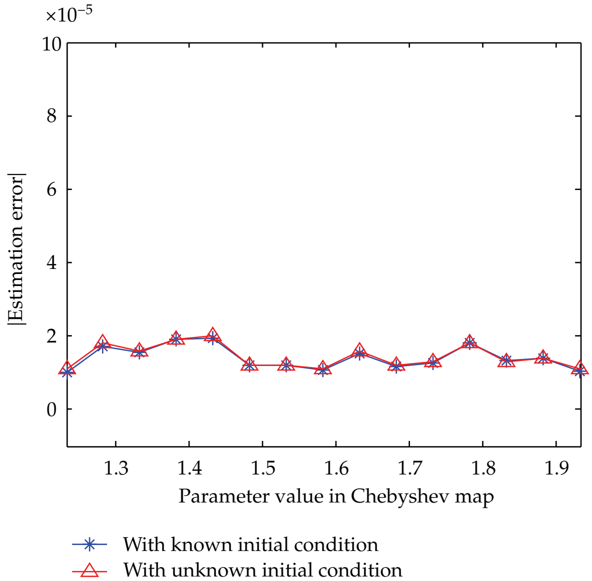

Figure 4: Comparison of improved MVM with and without the knowledge of initial condition, the chaotic map is the Chebyshev map.

(3) Get the required small interval by using the (3.2).

(4) Use the synchronization way to get the $\widehat{\theta}_0$ in the small interval by using (3.7).

4. Numerical Simulation

In order to verify the effectiveness of the proposed parameter estimation method in this paper, simulations on Chebyshev map and Tent map have been done.

Chebyshev map is given by $x(n) = \cos(\theta\cos^{-1}(x(n-1)))$, and Tent map is given by $x(n) = \theta - 1 - \theta|x(n-1)|$, where $x(n) \in (-1, 1]$. We let $N = 2000$, parameter sampling interval 10^{-6}, and coupling coefficient $k = -1$ for both maps. Figures 4 and 5 show the estimation performance of proposed method with and without the knowledge of initial condition, respectively. See from Figures 4 and 5, the estimation performance of the improved MVM is independent of the initial condition. One reason is that the MVM is independent of the initial condition, which has been proved in [8]. The other reason is that the chaotic synchronization is independent of the initial condition. Thus the proposed method is also independent of the initial condition.

The estimation error of the improved MVM in this paper and the estimation error of MVM are shown in Figures 6 and 7, respectively. We can see that the estimation error of improved MVM is less than that of MVM.

5. Conclusion

In summary, in this paper we have developed an improved mean value method for parameter estimation of a class of chaotic sequences derived from the one-dimensional nonlinear map. Based on the ergodic theory and monotone property of MVM, we can get the small interval

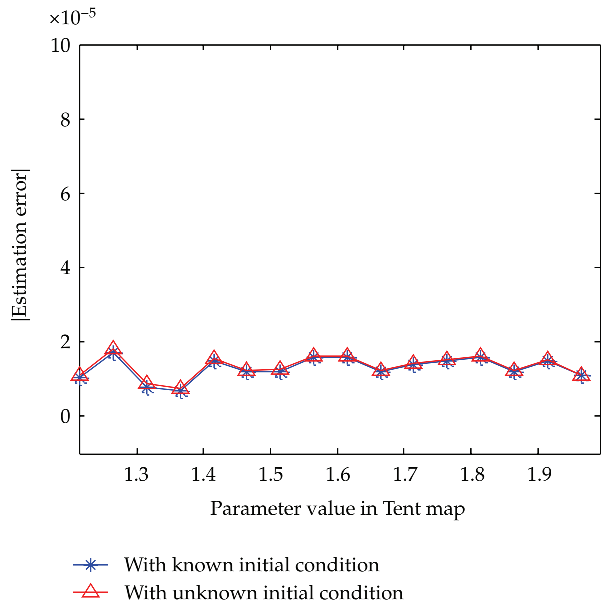

Figure 5: Comparison of improved MVM with and without the knowledge of initial condition; the chaotic map is the Tent map.

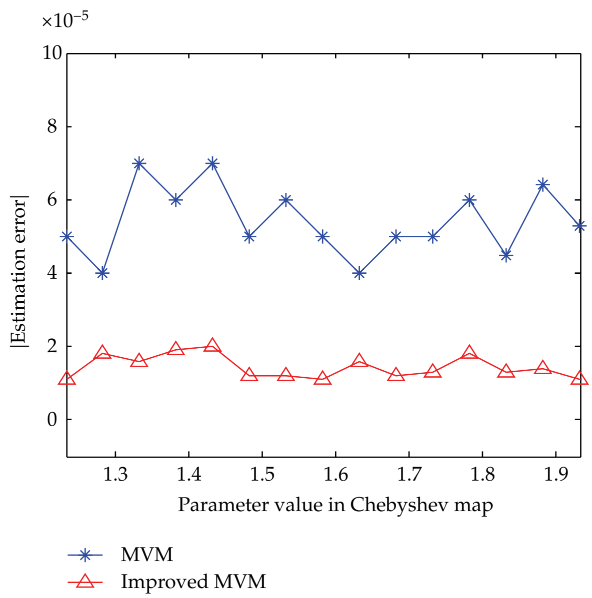

Figure 6: The estimation error of MVM in [8] and the improved MVM proposed in this paper for Chebyshev map.

that contains the real parameter value. In the interval we use chaotic synchronization way to get the more accurate estimation. The simulation of Chebyshev map and Tent map confirms that the estimation performance of the proposed method is better than that of MVM.

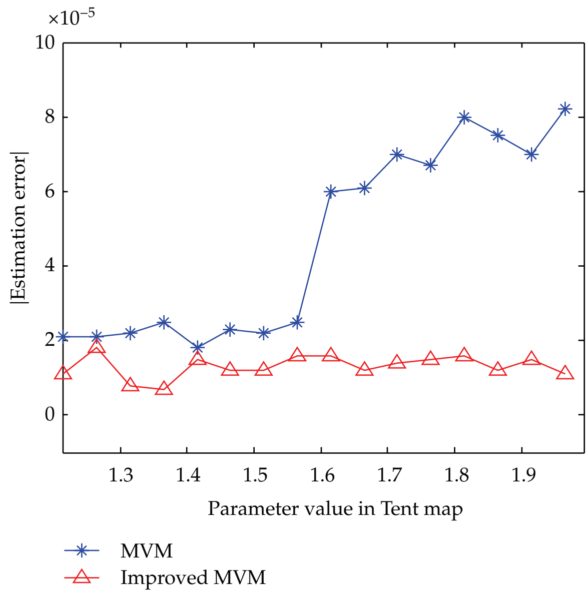

Figure 7: The estimation error of MVM in [8] and the improved MVM proposed in this paper for Tent map.

Notice that if the unknown parameters are more than one, for example, if the parameters are two dimensional, the required interval, which has been obtained based on MVM, is changed to a two-dimensional zone. When using (3.7), the parameters' values are chosen in the required two-dimensional zone. However, this makes a large computational cost. How to use optimization algorithm to reduce the large computational is our future work. What is more is that in the proposed method we do not consider the infection of noise. The reason is that the noise may be reduced by the method in [13]. In further work, we will do the research on the parameter estimation in the background of noise by using the noise reduction method such as [13].

Acknowledgments

The authors would like to thank the suggestions of the anonymous reviewers, and they would like to thank the support of Defense Pre-research Fund (9140A07011609DZ0216), the Fundamental Research Funds for the Central Universities (ZYGX2009J011, ZYGX2009J015, ZYGX2010J015, 103.1.2-E022050205), and National Natural Science Foundation (61101172).

References

[1] B. Y. Ou and D. S. Liu, "Chaotic attractor generation via a simple linear time-varying system," *Discrete Dynamics in Nature and Society*, vol. 2010, Article ID 840346, 9 pages, 2010.
[2] G. H. Erjaee and M. Alnasr, "Phase synchronization in coupled Sprott chaotic systems presented by fractional differential equations," *Discrete Dynamics in Nature and Society*, vol. 2009, Article ID 753746, 10 pages, 2009.
[3] W. Z. Ji, "Chaos and control of game model based on heterogeneous expectations in electric power triopoly," *Discrete Dynamics in Nature and Society*, vol. 2009, Article ID 469564, 8 pages, 2009.
[4] J. F. Hu and J. B. Guo, "Breaking a chaotic secure communication scheme," *Chaos*, vol. 18, no. 1, Article ID 01321, 7 pages, 2008.

[5] D. Ghosh and S. Banerjee, “Adaptive scheme for synchronization-based multiparameter estimation from a single chaotic time series and its applications,” *Physical Review E*, vol. 78, no. 5, Article ID 056211, 2008.

[6] C. Tao, Y. Zhang, and J. J. Jiang, “Estimating system parameters from chaotic time series with synchronization optimized by a genetic algorithm,” *Physical Review E*, vol. 76, no. 1, Article ID 016209, 2007.

[7] C. Tao, X. Liu, and G. Du, “Detection of dynamical structure from short and noisy chaotic series,” *Physical Review E*, vol. 81, no. 4, Article ID 046209, 2010.

[8] L. Aguirre, E. Furtado, and L. Tôrres, “Evaluation of dynamical models: dissipative synchronization and other techniques,” *Physical Review E*, vol. 74, no. 6, Article ID 066203, p. 16, 2006.

[9] L. A. Aguirre and E. C. Furtado, “Building dynamical models from data and prior knowledge: the case of the first period-doubling bifurcation,” *Physical Review E*, vol. 76, no. 4, Article ID 046219, 2007.

[10] H. Leung, H. Yu, and K. Murali, “Ergodic chaos-based communication schemes,” *Physical Review E*, vol. 66, no. 3, Article ID 036203, pp. 1–8, 2002.

[11] H. Leung, S. Shanmugam, N. Xie, and S. Wang, “An ergodic approach for chaotic signal estimation at low SNR with application to ultra-wide-band communication,” *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 1091–1103, 2006.

[12] K. Wang, W. J. Pei, S. P. Wang, Y. M. Cheung, and Z. He, “Symbolic vector dynamics approach to initial condition and control parameters estimation of coupled map lattices,” *IEEE Transactions on Circuits and Systems. I*, vol. 55, no. 4, pp. 1116–1124, 2008.

[13] C. Tao, Y. Zhang, G. Du, and J. J. Jiang, “Estimating model parameters by chaos synchronization,” *Physical Review E*, vol. 69, Article ID 036204, 2004.

[14] C. Tao and G. Du, “Determinate relation between two generally synchronized spatiotemporal chaotic systems,” *Physics Letters A*, vol. 311, no. 2-3, pp. 158–164, 2003.

[15] K. Wang, W. Pei, Z. He, and Y.-m. Cheung, “Estimating initial conditions in coupled map lattices from noisy time series using symbolic vector dynamics,” *Physics Letters, Section A*, vol. 367, no. 4-5, pp. 316–321, 2007.

[16] Q. He, L. Wang, and B. Liu, “Parameter estimation for chaotic systems by particle swarm optimization,” *Chaos, Solitons and Fractals*, vol. 34, no. 2, pp. 654–661, 2007.

[17] F. Gao, J.-J. Lee, Z. Li, H. Tong, and X. Lü, “Parameter estimation for chaotic system with initial random noises by particle swarm optimization,” *Chaos, Solitons and Fractals*, vol. 42, no. 2, pp. 1286–1291, 2009.