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Research Article

Delay-Dependent Stability Analysis and Synthesis for Uncertain Impulsive Switched System with Mixed Delays

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This paper studies the asymptotic stability problem for a class of uncertain impulsive switched systems with discrete and distributed delays. Based on Lyapunov functional theory, delay-dependent sufficient LMI conditions are established for the asymptotic stability of the considered systems. Moreover, an appropriate feedback controller is constructed for stabilizing the corresponding closed-loop system. The results are illustrated to be efficient through an example.

1. Introduction

A switched system is a type of hybrid system which is a combination of discrete and continuous dynamical systems. These systems arise as models for phenomena which cannot be described by exclusively continuous or exclusively discrete processes. Recently, on the basis of Lyapunov functions and other analysis tools, the stability and stabilization for switched systems have been investigated and many variable results have been obtained; see [1–4]. In general, the switched systems which have been widely studied in the literature can be classified into two groups: continuous switched systems and discrete switched systems. However, both of these classes do not cover some useful switched systems existing in the real world displaying a certain kind of dynamics with impulse effect at the switching points, that is, the states jump. Studies on the dynamic systems with impulsive effect and switching have arisen in various fields of science and engineering in recent years; see [5–11]. These systems are called impulsive switched systems, which are useful to model those physical phenomena that exhibit abrupt changes at certain time points due to impulsive inputs or switching. For these systems, there is an increasing interest among the control community in terms of stability analysis and the design of stabilizing feedback controller so as to achieve a required stability performance. For example, in [6], the authors studied a class of uncertain impulsive switched systems with delay input; in [7], the author studied a class of impulsive switched systems; by constructing appropriate Lyapunov-Krasovskii functions and using LMI approach, some asymptotic stability criteria were obtained and some appropriate feedback controllers were constructed. To the best of the authors' knowledge, most of the papers have studied the delay-independent stability criteria, and few delay-dependent results have been reported in the literature concerning the problem of robust stability for the impulsive switched systems. This motivates our research.

On the other hand, time delays and uncertainties happen frequently in various engineering, biological, and economical systems, and they many result in instability. Many stability criteria have been derived for continuous dynamical systems with time delays or uncertainties; see [12–14]. However, such fewer results have been reported for stability analysis and control of impulsive switched systems with distributed time delays.

In this paper, the problem of delay-dependent stability analysis and synthesis for impulsive switched system with discrete and distributed delays is studied. The uncertainties under consideration are norm bounded. Based on Lyapunov functional approach and linear matrix inequality technology, some new delay-dependent stability and stabilization conditions are derived. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

Notations. Throughout the paper, A^T stands for matrix transpose of the matrix A. R^n is the n-dimensional Euclidean space. $R^{n\times m}$ is the set of all $n \times m$-dimensional matrices. I denotes the identity matrix of appropriate dimensions. $P > 0$ ($P < 0$) means that P is a symmetric positive definite (negative definite) matrix. $*$ represents the elements below the main diagonal of a symmetric matrix.

2. Problem Formulation and Preliminaries

Consider the following impulsive switched system with mixed delays:

$$\begin{gathered}
\dot{x}(t) = \overline{A}_{i_k} x(t) + \overline{B}_{i_k} x(t-h_1) + C_{i_k} \int_{t-h_2}^{t} x(s)\mathrm{d}s + D_{i_k} u(t), \quad t \neq t_k, \\
\Delta x(t) = G_k x(t), \quad t = t_k, \\
x(t) = \varphi(t), \quad -h \leq t \leq 0,
\end{gathered} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^p$, $n, p \in \mathbb{N}$, are the state vector and the control input, respectively. $h_1 > 0$, $h_2 > 0$ are time delays, $h = \max\{h_1, h_2\}$. $\Delta x(t) = x(t^+) - x(t^-)$, where $x(t^+) = \lim_{v\to 0^+} x(t+v)$, $x(t^-) = \lim_{v\to 0^+} x(t-v)$. $x(t_k) = x(t_k^-)$, which means that the solution of the system (2.1) is left continuous at the impulsive switched time point t_k which satisfies $t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to+\infty} t_k = +\infty$. $i_k \in \{1, 2, \ldots, m\}$, $k, m \in \mathbb{N}$, is a discrete state variable. $\{i_k, t_k\}$ represents a switching rule of the system (2.1), that is, at t_k time point, the system switches to the i_k subsystem from the i_{k-1} subsystem:

$$\overline{A}_{i_k} = A_{i_k} + \Delta A_{i_k}(t), \qquad \overline{B}_{i_k} = B_{i_k} + \Delta B_{i_k}(t), \tag{2.2}$$

$A_{i_k}, B_{i_k}, C_{i_k}, G_k \in \mathbb{R}^{n\times n}$, $D_{i_k} \in \mathbb{R}^{n\times p}$ are known constant real matrices. $\Delta A_{i_k}(\cdot)$ and $\Delta B_{i_k}(\cdot)$ are unknown real norm-bounded matrix functions which represent time-varying parameter uncertainties, which are of the following form:

$$\begin{pmatrix}\Delta A_{i_k}(t) & \Delta B_{i_k}(t)\end{pmatrix} = E_{i_k} F_{i_k}(t) \begin{pmatrix} H_{i_k} & J_{i_k}\end{pmatrix}, \tag{2.3}$$

where E_{i_k}, H_{i_k}, and J_{i_k} are known constant real matrices of appropriate dimensions and $F_{i_k}(t)$ is an unknown real time-varying matrix satisfying $F_{i_k}^T(t)F_{i_k}(t) \le I$.

Lemma 2.1. *Let D, E, and F be matrices with appropriate dimensions. Suppose that $F^TF \le I$; then, for any real scale $\lambda > 0$, one has that*

$$DFE + E^TF^TD^T \le \lambda DD^T + \lambda^{-1}E^TE. \tag{2.4}$$

Lemma 2.2. *For any constant matrix $M \in \mathbb{R}^{n\times n}$, $M > 0$, and a scalar $\gamma > 0$, vector function $w$is such that the integrations concerned are well defined; then*

$$\left(\int_0^{\gamma} w(s)\mathrm{d}s\right)^T M \left(\int_0^{\gamma} w(s)\mathrm{d}s\right) \le \gamma \int_0^{\gamma} w^T(s)Mw(s)\mathrm{d}s. \tag{2.5}$$

3. Stability Analysis

Theorem 3.1. *Suppose that there exist symmetric positive definite matrices P_{i_k}, Q_{i_k}, T_{i_k}, and R_{i_k} and some positive scalars ε_1, ε_2 such that for $i_k = 1, 2, \ldots, m$ the following LMIs hold:*

(a)

$$\begin{pmatrix} -Q_{i_k} & 0 & 0 & Q_{i_k} & 0 \\ * & -h_1T_{i_k} & 0 & h_1T_{i_k} & 0 \\ * & * & -h_2R_{i_k} & h_2R_{i_k} & 0 \\ * & * & * & Z_{i_k} & P_{i_k}E_{i_k} \\ * & * & * & * & -(\varepsilon_1+\varepsilon_2)^{-1}I \end{pmatrix} < 0, \tag{3.1}$$

where $Z_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^TP_{i_k} + \varepsilon_1^{-1}H_{i_k}^TH_{i_k} + I$,

(b)

$$\begin{pmatrix} -I & P_{i_k}B_{i_k} & 0 & P_{i_k}C_{i_k} \\ * & -Q_{i_k} + \varepsilon_1^{-1}J_{i_k}^TJ_{i_k} & 0 & 0 \\ * & * & -\dfrac{1}{h_1}T_{i_k} & 0 \\ * & * & * & -\dfrac{1}{h_2}R_{i_k} \end{pmatrix} < 0, \tag{3.2}$$

(c)

$$\begin{pmatrix} P_{i_{k-1}} & (I+G_k)^T P_{i_k} \\ * & P_{i_k} \end{pmatrix} > 0. \tag{3.3}$$

Then the trivial solution of the impulsive switched system (2.1) *with* $u(t) = 0$ *is robustly asymptotically stable.*

Proof. When $t \in (t_k, t_{k+1}]$, consider the Lyapunov-Krasovskii function candidate

$$\begin{aligned} V(x(t)) &= x^T(t)P_{i_k}x(t) + \int_{t-h_1}^{t} x^T(s)Q_{i_k}x(s)\mathrm{d}s \\ &\quad + \int_{-h_1}^{0}\int_{t+s}^{t} x^T(u)T_{i_k}x(u)\mathrm{d}u\,\mathrm{d}s + \int_{-h_2}^{0}\int_{t+s}^{t} x^T(u)R_{i_k}x(u)\mathrm{d}u\,\mathrm{d}s. \end{aligned} \tag{3.4}$$

Taking the right upper derivative of $V(x(t))$ along the solution of the impulsive switched system (2.1), we have that

$$\begin{aligned} D^+V(x(t)) \le{}& x^T(t)(2P_{i_k}(A_{i_k} + E_{i_k}F_{i_k}H_{i_k}) + Q_{i_k} + h_1T_{i_k} + h_2R_{i_k})x(t) \\ &- x^T(t-h_1)Q_{i_k}x(t-h_1) + 2x^T(t)P_{i_k}(B_{i_k} + E_{i_k}F_{i_k}J_{i_k})x(t-h_1) \\ &- \int_{t-h_1}^{t} x^T(s)\mathrm{d}s\left(\frac{1}{h_1}T_{i_k}\right)\int_{t-h_1}^{t} x(s)\mathrm{d}s - \int_{t-h_2}^{t} x^T(s)\mathrm{d}s\left(\frac{1}{h_2}R_{i_k}\right)\int_{t-h_2}^{t} x(s)\mathrm{d}s \\ &+ 2x^T(t)P_{i_k}C_{i_k}\int_{t-h_2}^{t} x(s)\mathrm{d}s. \end{aligned} \tag{3.5}$$

Define

$$\xi(t) = \begin{pmatrix} x(t) \\ x(t-h_1) \\ \displaystyle\int_{t-h_1}^{t} x(s)\mathrm{d}s \\ \displaystyle\int_{t-h_2}^{t} x(s)\mathrm{d}s \end{pmatrix}, \qquad \Phi_{i_k} = \begin{pmatrix} \Phi_{11} & \Phi_{12} & 0 & P_{i_k}C_{i_k} \\ * & -Q_{i_k} & 0 & 0 \\ * & * & -\dfrac{1}{h_1}T_{i_k} & 0 \\ * & * & * & -\dfrac{1}{h_2}R_{i_k} \end{pmatrix}, \tag{3.6}$$

$\Phi_{11} = P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + Q_{i_k} + h_1T_{i_k} + h_2R_{i_k} + P_{i_k}E_{i_k}F_{i_k}H_{i_k} + H_{i_k}^T F_{i_k}^T E_{i_k}^T P_{i_k}$, and $\Phi_{12} = P_{i_k}B_{i_k} + P_{i_k}E_{i_k}F_{i_k}J_{i_k}$; then we have that

$$D^+V(x(t)) \le \xi^T(t)\Phi_{i_k}\xi(t). \tag{3.7}$$

Let

$$\Psi_{i_k} = \begin{pmatrix} P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + Q_{i_k} + h_1 T_{i_k} + h_2 R_{i_k} & P_{i_k}B_{i_k} & 0 & P_{i_k}C_{i_k} \\ * & -Q_{i_k} & 0 & 0 \\ * & * & -\dfrac{1}{h_1}T_{i_k} & 0 \\ * & * & * & -\dfrac{1}{h_2}R_{i_k} \end{pmatrix}. \tag{3.8}$$

Then by Lemma 2.1 and Schur complement, we have that

$$\begin{aligned} \Phi_{i_k} &= \Psi_{i_k} + \begin{pmatrix} P_{i_k}E_{i_k}F_{i_k}H_{i_k} + H_{i_k}^T F_{i_k}^T E_{i_k}^T P_{i_k} & P_{i_k}E_{i_k}F_{i_k}J_{i_k} & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix}, \\ &\le \Psi_{i_k} + \begin{pmatrix} (\varepsilon_1 + \varepsilon_2)P_{i_k}E_{i_k}E_{i_k}^T P_{i_k} + \varepsilon_1^{-1}H_{i_k}^T H_{i_k} & 0 & 0 & 0 \\ * & \varepsilon_2^{-1}J_{i_k}^T J_{i_k} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix}, \\ &= \begin{pmatrix} -I & P_{i_k}B_{i_k} & 0 & P_{i_k}C_{i_k} \\ * & -Q_{i_k} + \varepsilon_2^{-1}J_{i_k}^T J_{i_k} & 0 & 0 \\ * & * & -\dfrac{1}{h_1}T_{i_k} & 0 \\ * & * & * & -\dfrac{1}{h_2}R_{i_k} \end{pmatrix} + \begin{pmatrix} \overline{Z}_{i_k} & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix}, \end{aligned} \tag{3.9}$$

where $\overline{Z}_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + Q_{i_k} + h_1 T_{i_k} + h_2 R_{i_k} + (\varepsilon_1 + \varepsilon_2)P_{i_k}E_{i_k}E_{i_k}^T P_{i_k} + \varepsilon_1^{-1}H_{i_k}^T H_{i_k} + I$.

The stability condition $D^+V(x(t)) < 0$ can be obtained if the following inequalities hold:

$$\begin{pmatrix} -I & P_{i_k}B_{i_k} & 0 & P_{i_k}C_{i_k} \\ * & -Q_{i_k} + \varepsilon_2^{-1}J_{i_k}^T J_{i_k} & 0 & 0 \\ * & * & -\dfrac{1}{h_1}T_{i_k} & 0 \\ * & * & * & -\dfrac{1}{h_2}R_{i_k} \end{pmatrix} < 0, \qquad \overline{Z}_{i_k} < 0. \tag{3.10}$$

By condition (b) of the theorem, the former inequality is satisfied. $\overline{Z}_{i_k} < 0$ will hold if the following condition is satisfied:

$$\operatorname{diag}\left\{-I, -I, -I, \overline{Z}_{i_k}, -I\right\} < 0. \tag{3.11}$$

Define

$$W_{i_k} = \begin{pmatrix} Q_{i_k}^{1/2} & 0 & 0 & 0 & 0 \\ 0 & h_1^{1/2}T_{i_k}^{1/2} & 0 & 0 & 0 \\ 0 & 0 & h_2^{1/2}R_{i_k}^{1/2} & 0 & 0 \\ -Q_{i_k}^{1/2} & -h_1^{1/2}T_{i_k}^{1/2} & -h_2^{1/2}R_{i_k}^{1/2} & I & -(\varepsilon_1+\varepsilon_2)^{1/2}P_{i_k}E_{i_k} \\ 0 & 0 & 0 & 0 & (\varepsilon_1+\varepsilon_2)^{-1/2}I \end{pmatrix}. \quad (3.12)$$

Then by left multiplying and right multiplying (3.11) by W_{i_k} and $W_{i_k}^T$, respectively, we have inequality (3.1).

From conditions (3.1) and (3.2), $D^+V(x(t)) < 0$. It means that the impulsive switched system is robustly asymptotically stable, except possibly at the impulsive switching points.

Next, for the impulsive switching time point t_k, we have that

$$\begin{aligned} V\left(x\left(t_k^+\right)\right) - V(x(t_k)) &= x^T\left(t_k^+\right)P_{i_k}x\left(t_k^+\right) - x^T(t_k)P_{i_{k-1}}x(t_k) \\ &= x^T(t_k)\left((I+G_k)^T P_{i_k}(I+G_k) - P_{i_{k-1}}\right)x(t_k). \end{aligned} \quad (3.13)$$

Obviously, if $(I+G_k)^T P_{i_k}(I+G_k) - P_{i_{k-1}} < 0$, we have that $V(x(t_k^+)) - V(x(t_k)) < 0$. On the other hand, by using Schur complement, $(I+G_k)^T P_{i_k}(I+G_k) - P_{i_{k-1}} < 0$ is equivalent to condition (c) of the theorem given by (3.3). Thus, by (3.1), (3.2), and (3.3), the impulsive switched system (2.1) is robustly asymptotically stable.

This completes the proof. □

4. Design of Feedback Controller

In this section, we focus on designing a memoryless state feedback controller in the form of $u(t) = K_{i_k}x(t)$, which stabilizes the uncertain impulsive switched systems with discrete and distributed delays considered.

Theorem 4.1. *Suppose that there exist symmetric positive definite matrices P_{i_k}, Q_{i_k}, T_{i_k}, and R_{i_k} and some positive scalars $\varepsilon_1 > 0$, ε_2, such that for $i_k = 1, 2, \ldots, m$ LMIs* (3.2), (3.3), *and the following LMIs hold:*

$$\begin{pmatrix} -Q_{i_k} & 0 & 0 & Q_{i_k} & 0 \\ * & -h_1T_{i_k} & 0 & h_1T_{i_k} & 0 \\ * & * & -h_2R_{i_k} & h_2R_{i_k} & 0 \\ * & * & * & Z_{i_k} & P_{i_k}X_{i_k} \\ * & * & * & * & -I \end{pmatrix} < 0, \quad (4.1)$$

where $X_{i_k}X_{i_k}^T = (\varepsilon_1+\varepsilon_2)E_{i_k}E_{i_k}^T - D_{i_k}D_{i_k}^T$, $Z_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^T P_{i_k} + \varepsilon_1^{-1}H_{i_k}^T H_{i_k} + I$.

Then the trivial solution of the impulsive switched system (2.1) *is asymptotically stable. Moreover,*

$$u(t) = K_{i_k}x(t), \qquad K_{i_k} = -\frac{1}{2}D_{i_k}^{T}P_{i_k}, \tag{4.2}$$

is a feedback controller which stabilizes the corresponding closed-loop impulsive switched system.

Proof. Substitute $u(t) = K_{i_k}x(t)$ and $K_{i_k} = -(1/2)D_{i_k}^{T}P_{i_k}$ into the system (2.1). Then the corresponding closed-loop impulsive switched system is of the form

$$\begin{gathered}\dot{x}(t) = \widetilde{A}_{i_k}x(t) + \overline{B}_{i_k}x(t-h_1) + C_{i_k}\int_{t-h_2}^{t} x(s)\mathrm{d}s, \quad t \neq t_k,\\ \Delta x(t) = G_k x(t), \quad t = t_k,\\ x(t) = \varphi(t), \quad -h \le t \le 0,\end{gathered} \tag{4.3}$$

where $\widetilde{A}_{i_k} = A_{i_k} - (1/2)D_{i_k}D_{i_k}^{T}P_{i_k} + \Delta A_{i_k}(t)$.

Replacing A_{i_k} with $A_{i_k} - (1/2)D_{i_k}D_{i_k}^{T}P_{i_k}$ in the matrix Φ_{i_k} defined in Theorem 3.1, we have that

$$\overline{\Phi}_{i_k} = \begin{pmatrix} \overline{\Phi}_{11} & \Phi_{12} & 0 & P_{i_k}C_{i_k} \\ * & -Q_{i_k} & 0 & 0 \\ * & * & -\dfrac{1}{h_1}T_{i_k} & 0 \\ * & * & 0 & -\dfrac{1}{h_2}R_{i_k} \end{pmatrix}, \tag{4.4}$$

where $\overline{\Phi}_{11} = P_{i_k}(A_{i_k} - (1/2)D_{i_k}D_{i_k}^{T}P_{i_k}) + (A_{i_k} - (1/2)D_{i_k}D_{i_k}^{T}P_{i_k})^{T}P_{i_k} + Q_{i_k} + h_1T_{i_k} + h_2R_{i_k} + P_{i_k}E_{i_k}F_{i_k}H_{i_k} + H_{i_k}^{T}F_{i_k}^{T}E_{i_k}^{T}P_{i_k}$, and

$$\overline{\Phi}_{i_k} \le \begin{pmatrix} -I & P_{i_k}B_{i_k} & 0 & P_{i_k}C_{i_k} \\ * & -Q_{i_k} + \varepsilon_2^{-1}J_{i_k}^{T}J_{i_k} & 0 & 0 \\ * & * & -\dfrac{1}{h_1}T_{i_k} & 0 \\ * & * & 0 & -\dfrac{1}{h_2}R_{i_k} \end{pmatrix} + \begin{pmatrix} \widetilde{Z}_{i_k} & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix}, \tag{4.5}$$

where $\widetilde{Z}_{i_k} = P_{i_k}A_{i_k} + A_{i_k}^{T}P_{i_k} + Q_{i_k} + h_1T_{i_k} + h_2R_{i_k} + \varepsilon_1^{-1}H_{i_k}^{T}H_{i_k} + I + P_{i_k}((\varepsilon_1+\varepsilon_2)E_{i_k}E_{i_k}^{T} - D_{i_k}D_{i_k}^{T})P_{i_k}$.

Similar to the proof of Theorem 3.1, in what follows, we will prove that the following LMIs hold:

$$\operatorname{diag}\left\{-I, -I, -I, \widetilde{Z}_{i_k}, -I\right\} < 0. \tag{4.6}$$

Define

$$\overline{W}_{i_k} = \begin{pmatrix} Q_{i_k}^{1/2} & 0 & 0 & 0 & 0 \\ 0 & h_1^{1/2}T_{i_k}^{1/2} & 0 & 0 & 0 \\ 0 & 0 & h_2^{1/2}R_{i_k}^{1/2} & 0 & 0 \\ -Q_{i_k}^{1/2} & -h_1^{1/2}T_{i_k}^{1/2} & -h_2^{1/2}R_{i_k}^{1/2} & I & P_{i_k}X_{i_k} \\ 0 & 0 & 0 & 0 & -I \end{pmatrix}, \tag{4.7}$$

where X_{i_k} is defined in Theorem 4.1. Then by left multiplying and right multiplying by $\overline{W}_{i_k}$ and $\overline{W}_{i_k}^T$, respectively, we have the LMIs (4.1). The rest of the proof is similar to that of Theorem 3.1 and will be omitted.

This completes the proof. □

5. Numerical Example

As an illustration, we consider a system in the form of (2.1) under the given switching rule $\Delta t_k \equiv 1$, $k \in \mathbb{N}$. Without loss of generality, assume that there are two subsystems, that is, $i_k \in \{1,2\}$, between which the dynamical system alternates. Choose the discrete time delay $h_1 = 0.1$. We consider robust performance of the system using Theorem 3.1. The parameters of the system are specified as follows:

$$\begin{gathered} A_1 = \begin{pmatrix} -2 & 0 \\ 0 & 2.4 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0.2 & 0 \\ 0.7 & 1 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 0.5 & 0 \\ 0.4 & 0.1 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 0.05 & 1 \\ 0 & 0.05 \end{pmatrix}, \\ A_2 = \begin{pmatrix} -2.5 & 0 \\ 0 & -1.2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -0.7 & 0 \\ -1 & -1 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0.3 & 0.1 \\ 0.6 & 0.4 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0.05 & 1 \\ 0 & 0.05 \end{pmatrix}, \\ E_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} -0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad H_1 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.3 \end{pmatrix}, \\ J_1 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}, \quad G_k = \begin{pmatrix} -0.1 & 0 \\ 0 & -0.1 \end{pmatrix}. \end{gathered} \tag{5.1}$$

Let $\varepsilon_1 = \varepsilon_2 = 1$. Then by solving (3.1)~(3.3) under MATLAB Toolbox, we obtain the upper bound of $h_2 = 0.1546$.

Letting $h_1 = 0.1$ and $h_2 = 0.1546$, we obtain the following linear memoryless controller by using Theorem 4.1:

$$K_1 = \begin{pmatrix} -0.1158 & 0.0127 \\ 0.0127 & -0.0322 \end{pmatrix}, \quad K_2 = \begin{pmatrix} -0.1216 & 0.0182 \\ 0.0182 & -0.0315 \end{pmatrix}. \tag{5.2}$$

6. Conclusion

In this paper, the asymptotic stability problem for a class of uncertain impulsive switched systems with discrete and distributed delays is discussed. Firstly, delay-dependent stability criteria have been obtained by choosing proper Lyapunov function. Furthermore, some appropriate feedback controllers have been constructed to ensure the asymptotic stability of the closed-loop systems. A numerical example is solved by MATLAB Toolbox to illustrate that the results obtained are effective.

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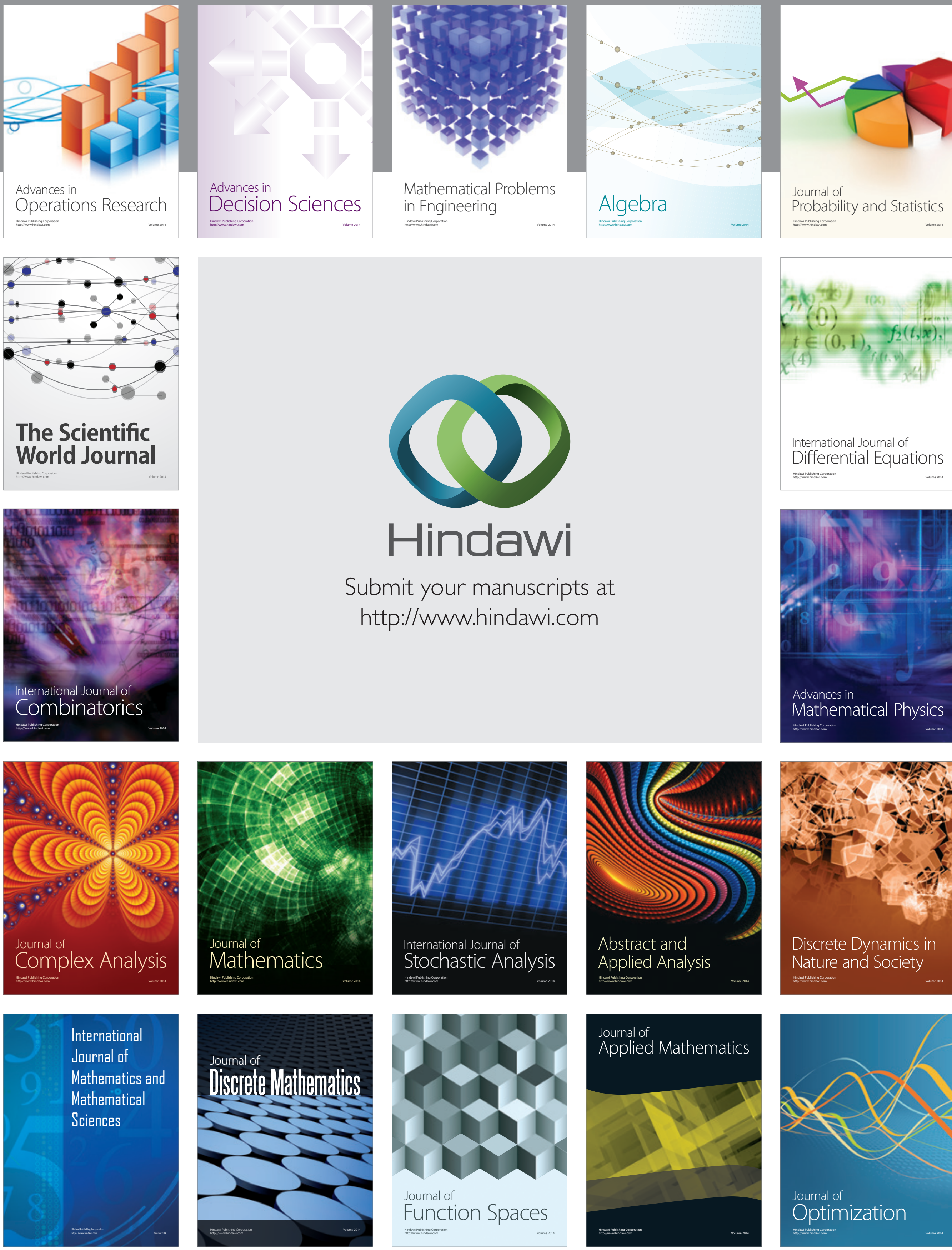

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