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Research Article

New Improved Exponential Stability Criteria for Discrete-Time Neural Networks with Time-Varying Delay

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The robust stability of uncertain discrete-time recurrent neural networks with time-varying delay is investigated. By decomposing some connection weight matrices, new Lyapunov-Krasovskii functionals are constructed, and serial new improved stability criteria are derived. These criteria are formulated in the forms of linear matrix inequalities (LMIs). Compared with some previous results, the new results are less conservative. Three numerical examples are provided to demonstrate the less conservatism and effectiveness of the proposed method.

1. Introductionn

In recent years, recurrent neural networks (see [1–7]), such as Hopfield neural networks, cellular neural networks, and other networks have been widely investigated and successfully applied in all kinds of science areas such as pattern recognition, image processing, and fixed-point computation. However, because of the finite switching speed of neurons and amplifiers, time delay is unavoidable in nature and technology. It can make important effects on the stability of dynamic systems. Thus, the studies on stability are of great significance. There has been a growing research interest on the stability analysis problems for delayed neural networks, and many excellent papers and monographs have been available. On the other hand, during the design of neural network and its hardware implementation, the convergence of a neural network may often be destroyed by its unavoidable uncertainty due to the existence of modeling error, the deviation of vital data, and so on. Therefore, the studies on robust convergence of delayed neural network have been a hot research direction. Up to now, many sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the global robust asymptotic or exponential stability for different class of delayed neural networks (see [8–13]).

It is worth pointing out that most neural networks have been assumed to be in continuous time, but few in discrete time. In practice, the discrete-time neural networks are more applicable to problems that are inherently temporal in nature or related to biological realities. And they can ideally keep the dynamic characteristics, functional similarity, and even the physical or biological reality of the continuous-time networks under mild restriction. Thus, the stability analysis problems for discrete-time neural networks have received more and more interest, and some stability criteria have been proposed in literature (see [14–25]). In [14], Liu et al. researched a class of discrete-time RNNs with time-varying delay, and proposed a delay-dependent condition guaranteeing the global exponential stability. By using a similar technique to that in [21], the result obtained in [14] has been improved by Song and Wang in [15]. The results in [15] are further improved by Zhang et al. in [16] by introducing some useful terms. In [17], Yu et al. proposed a new less conservative result than that obtained in [16] via constructing a new augment Lyapunov-Krasovskii functional.

In this paper, the connection weight matrix C is decomposed, and some new Lyapunov-Krasovskii functionals are constructed. Combined with linear matrix inequality (LMI) technique, serial new improved stability criteria are derived. Numerical examples show that these new criteria are less conservative than those obtained in [14–17].

Notation 1. The notations are used in our paper except where otherwise specified. $\|\cdot\|$ denotes a vector or a matrix norm; $\mathbb{R}, \mathbb{R}^n$ are real and n-dimension real number sets, respectively; $\mathbb{N}^+$ is nonnegative integer set. I is identity matrix; $*$ represents the elements below the main diagonal of a symmetric block matrix; Real matrix $P > 0 < (0)$ denotes that P is a positive definite (negative definite) matrix; $\mathbb{N}[a, b] = \{a, a+1, \ldots, b\}$; $\lambda_{\min}(\lambda_{\max})$ denotes the minimum and maximum eigenvalue of a real matrix.

2. Preliminaries

Consider a discrete-time recurrent neural network with time-varying delays [17] described by

$$\Sigma : x(k+1) = C(k)x(k) + A(k)f(x(k)) + B(k)f(x(k-\tau(k))) + J, \quad k = 1, 2, \ldots, \tag{2.1}$$

where $x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T \in \mathbb{R}^n$ denotes the neural state vector; $f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), \ldots, f_n(x_n(k))]^T$, $f(x(k-\tau(k))) = [f_1(x_1(k-\tau(k))), f_2(x_2(k-\tau(k))), \ldots, f_n(x_n(k-\tau(k)))]^T$ are the neuron activation functions; $J = [J_1, J_2, \ldots, J_n]^T$ is the external input vector; Positive integer $\tau(k)$ represents the transmission delay that satisfies $0 < \tau_m \leq \tau(k) \leq \tau_M$, where τ_m, τ_M are known positive integers representing the lower and upper bounds of the delay. $C(k) = C + \Delta C(k)$, $A(k) = A + \Delta A(k)$, $B(k) = B + \Delta B(k)$. $C = \operatorname{diag}(c_1, c_2, \ldots, c_n)$ with $|c_i| < 1$ describes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; $C, A, B \in \mathbb{R}^{n\times n}$ represent the weighting matrices; $\Delta C(k), \Delta A(k), \Delta B(k)$ denote the time-varying structured uncertainties which are of the following form:

$$[\Delta C(k), \Delta A(k), \Delta B(k)] = KF(k)[E_c, E_a, E_b], \tag{2.2}$$

where K, E_c, E_a, E_b are known real constant matrices with appropriate dimensions, $F(k)$ is unknown time-varying matrix function satisfying $F^T(k)F(k) \leq I$, for all $k \in \mathbb{N}^+$.

The nominal Σ_0 of Σ can be defined as

$$\Sigma_0 : x(k+1) = Cx(k) + Af(x(k)) + Bf(x(k-\tau(k))) + J, \quad k = 1, 2, \ldots, \tag{2.3}$$

To obtain our main results, we need to introduce the following assumption, definition and lemmas.

Assumption 1. For any $x, y \in \mathbb{R}$, $x \neq y$,

$$\sigma_i^- \leq \frac{f_i(x) - f_i(y)}{x - y} \leq \sigma_i^+, \quad i = 1, 2, \ldots, n, \tag{2.4}$$

where σ_i^-, σ_i^+ are known constant scalars.

As pointed out in [16] under Assumption 1, system (2.3) has equilibrium points. Assume that $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ is an equilibrium point of (2.3) and let $y_i(k) = x_i(k) - x_i^*$, $g_i(y_i(k)) = f_i(y_i(k) + x_i^*) - f_i(x_i^*)$. Then, system (2.3), can be transformed into the following form:

$$y(k+1) = Cy(k) + Ag\left(y(k)\right) + Bg\left(y(k-\tau(k))\right), \quad k = 1, 2, \ldots, \tag{2.5}$$

where $y(k) = [y_1(k), y_2(k), \ldots, y_n(k)]^T$, $g(y(k)) = [g_1(y_1(k)), g_2(y_2(k)), \ldots, g_n(y_n(k))]^T$, $g(y(k - \tau(k))) = [g_1(y_1(k-\tau(k))), g_2(y_2(k-\tau(k))), \ldots, g_n(y_n(k-\tau(k)))]^T$. From Assumption 1, for any $x, y \in \mathbb{R}$, $x \neq y$, functions $g_i(\cdot)$ satisfy $\sigma_i^- \leq (g_i(x) - g_i(y))/(x-y) \leq \sigma_i^+$, $i = 1, 2, \ldots, n$, and $g_i(0) = 0$.

Remark 2.1. Assumption 1 is widely used for dealing with the stability problem for neural networks. As pointed out in [13, 14, 16, 17, 26, 27], constants σ_i^-, σ_i^+ $(i = 1, 2, \ldots, n)$ can be positive, negative, and zero. Thus, this assumption is less restrictive than traditional Lipschitz condition.

Definition 2.2. The delayed discrete-time recurrent neural network in (2.5) is said to be globally exponentially stable if there exist two positive scalars $\alpha > 0$ and $0 < \beta < 1$ such that

$$\|y(k)\| \leq \alpha \cdot \beta^k \sup_{s \in \mathbb{N}[-\tau_M, 0]} \|y(s)\|, \quad \forall k \geq 1. \tag{2.6}$$

Lemma 2.3 (Tchebychev Inequality [28]). *For any given vectors $v_i \in \mathbb{R}^n$, $i = 1, 2, \ldots, n$, the following inequality holds:*

$$\left[\sum_{i=1}^{n} v_i\right]^T \left[\sum_{i=1}^{n} v_i\right] \leq n \sum_{i=1}^{n} v_i^T v_i. \tag{2.7}$$

Lemma 2.4 (see [29]). *For given matrices $Q = Q^T$, H, E and $R = R^T > 0$ of appropriate dimensions, then*

$$Q + HFE + E^T F^T H^T < 0, \tag{2.8}$$

for all F satisfying $F^T F \leq R$, if and only if there is an $\varepsilon > 0$, such that

$$Q + \varepsilon^{-1} H H^T + \varepsilon E^T R E < 0. \tag{2.9}$$

Lemma 2.5 (see [16]). *If Assumption 1 holds, then for any positive-definite diagonal matrix $D = \mathrm{diag}(d_1, d_2, \ldots, d_n) > 0$, the following inequality holds:*

$$g(y(k))^T D g\left(y\left(k\right)\right) - y^T\left(k\right) D \left(\prod_1 + \prod_2\right) g\left(y\left(k\right)\right) + y^T\left(k\right) \prod_1 D \prod_2 y\left(k\right) \leq 0, \quad k \in \mathbb{N}^+, \tag{2.10}$$

where $\prod_1 = \mathrm{diag}(\sigma_1^-, \sigma_2^-, \ldots, \sigma_n^-)$, $\prod_2 = \mathrm{diag}(\sigma_1^+, \sigma_2^+, \ldots, \sigma_n^+)$.

Lemma 2.6 (see [30]). *Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1^T = \Sigma_1$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if*

$$\begin{pmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{pmatrix} < 0, \quad or, \quad \begin{pmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{pmatrix} < 0. \tag{2.11}$$

Lemma 2.7 (see [13]). *Let N and E be real constant matrices with appropriate dimensions, matrix $F(k)$ satisfying $F^T(k)F(k) \leq I$, then, for any $\epsilon > 0$, $EF(k)N + N^T F^T(k) E^T \leq \epsilon^{-1} E E^T + \epsilon N^T N$, $k \in \mathbb{N}^+$.*

3. Main Results

To obtain our main results, we decompose the connection weight matrix C as follows:

$$C = C_1 + C_2. \tag{3.1}$$

Then, we can get the following stability results.

Theorem 3.1. *For any given positive scalars $0 < \tau_m < \tau_M$, then, under Assumption 1, system* (2.5) *without uncertainty is globally exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive-definite matrices P_1, Q_1, Q_2, Q_3, positive-definite diagonal matrices D_1, D_2, Q_4, Q_5, Q_6, and arbitrary matrices $P_i, H_i, i = 2, 3, \ldots, 21$ with appropriate dimensions, such that the following LMI holds:*

$$\Xi \triangleq \begin{pmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19} & \Xi_{1,10} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} & \Xi_{2,10} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} & \Xi_{3,10} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} & \Xi_{4,10} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} & \Xi_{5,10} \\ * & * & * & * & * & \Xi_{66} & \Xi_{67} & \Xi_{68} & \Xi_{69} & \Xi_{6,10} \\ * & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} & \Xi_{7,10} \\ * & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} & \Xi_{8,10} \\ * & * & * & * & * & * & * & * & \Xi_{99} & \Xi_{9,10} \\ * & * & * & * & * & * & * & * & * & \Xi_{10,10} \end{pmatrix} < 0, \tag{3.2}$$

where

$$\begin{aligned} \Xi_{11} = {}& 2C_1^T P_1 C_1 - C_1^T P_{12} C_2 - C_2^T P_{12}^T C_1 - 2P_1 + H_{12} C_2 + C_2^T H_{12}^T + Q_2 + Q_3 \\ & + (\tau_M - \tau_m + 1) Q_1 + (1 + \tau_m) Q_4 + (1 + \tau_M) Q_5 + (\tau_M - \tau_m) Q_6 - 2\prod_1 D_1 \prod_2, \end{aligned}$$

$$\Xi_{12} = C_2^T (H_{13} - P_{13})^T, \qquad \Xi_{13} = C_1^T (P_1 + P_{12}) - H_{12} + C_2^T (H_{14} - P_{14} - P_{12})^T + C_1^T P_1^T,$$

$$\Xi_{14} = C_1^T P_2 - H_2 + C_2^T (H_{15} - P_{15})^T, \qquad \Xi_{15} = -C_1^T P_2 + H_2 + C_2^T (H_{16} - P_{16})^T,$$

$$\Xi_{16} = -C_1^T P_{12} A + H_{12} A + C_2^T (H_{17} - P_{17})^T + D_1 \left(\prod_1 + \prod_2 \right),$$

$$\Xi_{17} = -C_1^T P_{12} B + H_{12} B + C_2^T (H_{18} - P_{18})^T, \qquad \Xi_{18} = -C_1^T P_2 + H_2 + C_2^T (H_{19} - P_{19})^T,$$

$$\Xi_{19} = C_1^T P_2 - H_2 + C_2^T (H_{20} - P_{20})^T, \qquad \Xi_{1,10} = C_1^T P_2 - H_2 + C_2^T (H_{21} - P_{21})^T,$$

$$\Xi_{22} = -Q_1 - 2\prod_1 D_2 \prod_2, \qquad \Xi_{23} = P_{13} - H_{13}, \qquad \Xi_{24} = P_3 - H_3,$$

$$\Xi_{25} = -P_3 + H_3, \qquad \Xi_{26} = -P_{13} A + H_{13} A,$$

$$\Xi_{27} = -P_{13} B + H_{13} B + D_2 \left(\prod_1 + \prod_2 \right), \qquad \Xi_{28} = -P_3 + H_3,$$

$$\Xi_{29} = P_3 - H_3, \qquad \Xi_{2,10} = P_3 - H_3,$$
$$\Xi_{33} = P_{12} + P_{14} - H_{14} + P_{12}^T + P_{14}^T - H_{14}^T + 2P_1, \qquad \Xi_{34} = P_2 + P_4 - H_4 + P_{15}^T - H_{15}^T$$
$$\Xi_{35} = H_4 - P_2 - P_4 + P_{16}^T - H_{16}^T, \qquad \Xi_{36} = (H_{14} - P_{12} - P_{14})\, A + P_{17}^T - H_{17}^T,$$
$$\Xi_{37} = (H_{14} - P_{12} - P_{14})\, B + P_{18}^T - H_{18}^T, \qquad \Xi_{38} = H_4 - P_2 - P_4 + P_{19}^T - H_{19}^T,$$
$$\Xi_{39} = P_4 + P_2 - H_4 + P_{20}^T - H_{20}^T, \qquad \Xi_{3,10} = P_4 + P_2 - H_4 + P_{21}^T - H_{21}^T,$$
$$\Xi_{44} = P_5 - H_5 + P_5^T - H_5^T - Q_3, \qquad \Xi_{45} = H_5 - P_5 + P_6^T - H_6^T,$$
$$\Xi_{46} = H_{15}A - P_{15}A + P_7^T - H_7^T, \qquad \Xi_{47} = H_{15}B - P_{15}B + P_8^T - H_8^T,$$
$$\Xi_{48} = H_5 - P_5 + P_9^T - H_9^T, \qquad \Xi_{49} = P_5 - H_5 + P_{10}^T - H_{10}^T,$$
$$\Xi_{49} = P_5 - H_5 + P_{11}^T - H_{11}^T,$$
$$\Xi_{55} = H_6 - P_6 + H_6^T - P_6^T - Q_2, \qquad \Xi_{56} = H_{16}A - P_{16}A + H_7^T - P_7^T,$$
$$\Xi_{57} = H_{16}B - P_{16}B + H_8^T - P_8^T, \qquad \Xi_{58} = H_6 - P_6 + H_9^T - P_9^T,$$
$$\Xi_{59} = P_6 - H_6 + H_{10}^T - P_{10}^T, \qquad \Xi_{5,10} = P_6 - H_6 + H_{11}^T - P_{11}^T,$$
$$\Xi_{66} = H_{17}A - P_{17}A + A^T H_{17}^T - A^T P_{17}^T - D_1 - D_1^T,$$
$$\Xi_{67} = H_{17}B - P_{17}B + A^T H_{18}^T - A^T P_{18}^T,$$
$$\Xi_{68} = H_7 - P_7 + A^T H_{19}^T - A^T P_{19}^T, \qquad \Xi_{69} = P_7 - H_7 + A^T H_{20}^T - A^T P_{20}^T,$$
$$\Xi_{6,10} = P_7 - H_7 + A^T H_{21}^T - A^T P_{21}^T,$$
$$\Xi_{77} = H_{18}B - P_{18}B + B^T H_{18}^T - B^T P_{18}^T - D_2 - D_2^T,$$
$$\Xi_{78} = H_8 - P_8 + B^T H_{19}^T - B^T P_{19}^T, \qquad \Xi_{79} = P_8 - H_8 + B^T H_{20}^T - B^T P_{20}^T,$$
$$\Xi_{7,10} = P_8 - H_8 + B^T H_{21}^T - B^T P_{21}^T,$$
$$\Xi_{88} = H_9 - P_9 + H_9^T - P_9^T - (1 + \tau_M)^{-1} Q_5, \qquad \Xi_{89} = P_9 - H_9 + H_{10}^T - P_{10}^T,$$
$$\Xi_{8,10} = P_9 - H_9 + H_{11}^T - P_{11}^T,$$
$$\Xi_{9,9} = P_{10} - H_{10} + P_{10}^T - H_{10}^T - (1 + \tau_m)^{-1} Q_4, \qquad \Xi_{9,10} = P_{10} - H_{10} + H_{11}^T - P_{11}^T,$$
$$\Xi_{10,10} = P_{11} - H_{11} + P_{11}^T - H_{11}^T - (\tau_M - \tau_m)^{-1} Q_6. \tag{3.3}$$

Proof. Construct a new augmented Lyapunov-Krasovskii functional candidate as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k) + V_6(k), \tag{3.4}$$

where

$$V_1(k) = 2\Upsilon^T(k)\begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{10n\times 10n} \Upsilon(k), \tag{3.5}$$

$\Upsilon^T(k) = [y^T(k), y^T(k-\tau(k)), \eta^T(k), y^T(k-\tau_M), y^T(k-\tau_m), g^T(y(k)), g^T(y(k-\tau_M)), \sum_{i=k-\tau_M}^{k} y^T(i), \sum_{i=k-\tau_m}^{k} y^T(i), \sum_{i=k-\tau_M+1}^{k-\tau_m} y^T(i)]^T$, $\eta(k) = y(k+1) - C_1 y(k)$; 0 is zero matrix with appropriate dimensions:

$$\begin{aligned} V_2(k) &= \sum_{i=k-\tau(k)}^{k-1} y^T(i) Q_1 y(i), \\ V_3(k) &= \sum_{i=k-\tau_m}^{k-1} y^T(i) Q_2 y(i) + \sum_{i=k-\tau_M}^{k-1} y^T(i) Q_3 y(i), \\ V_4(k) &= \sum_{j=k-\tau_m}^{k-1} \sum_{i=j}^{k-1} y^T(i) Q_4 y(i) + \sum_{j=k-\tau_M}^{k-1} \sum_{i=j}^{k-1} y^T(i) Q_5 y(i), \\ V_5(k) &= \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} y^T(i) Q_1 y(i), \\ V_6(k) &= \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} y^T(i) Q_6 y(i). \end{aligned} \tag{3.6}$$

Set $\widetilde{\Upsilon}^T(k+1) = [y^T(k+1), y^T(k-\tau(k)), \eta^T(k), y^T(k-\tau_M), y^T(k-\tau_m), g^T(y(k)), g^T(y(k-\tau_M)), \sum_{i=k-\tau_M}^{k} y^T(i), \sum_{i=k-\tau_m}^{k} y^T(i), \sum_{i=k-\tau_M+1}^{k-\tau_m} y^T(i)]^T = [y^T(k)C_1^T + \eta^T(k), \eta^T(k), y^T(k-\tau_M), y^T(k-\tau_m), g^T(y(k)), g^T(y(k-\tau_M)), \sum_{i=k-\tau_M}^{k} y^T(i), \sum_{i=k-\tau_m}^{k} y^T(i), \sum_{i=k-\tau_M+1}^{k-\tau_m} y^T(i)]^T$. Define $\Delta V(k) = V(k+1) - V(k)$. Then along the solution of system (2.5) we have

$$\begin{aligned} \Delta V_1(k) &= 2\Upsilon^T(k+1)\begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}\Upsilon(k+1) - 2\Upsilon^T(k)\begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}\Upsilon(k) \\ &= 2\widetilde{\Upsilon}^T(k+1)\begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}\widetilde{\Upsilon}(k+1) - 2\Upsilon^T(k)\begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}\Upsilon(k) \\ &\triangleq 2I_1 - 2I_2, \end{aligned} \tag{3.7}$$

$$I_1 = Y^T(k) \begin{pmatrix} C_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{pmatrix} \begin{pmatrix} P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \widetilde{Y}(k+1)$$

$$= Y^T(k) \begin{pmatrix} C_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{pmatrix} \begin{pmatrix} P_1 & P_2 & P_{12} \\ 0 & P_3 & P_{13} \\ 0 & P_4 & P_{14} \\ 0 & P_5 & P_{15} \\ 0 & P_6 & P_{16} \\ 0 & P_7 & P_{17} \\ 0 & P_8 & P_{18} \\ 0 & P_9 & P_{19} \\ 0 & P_{10} & P_{20} \\ 0 & P_{11} & P_{21} \end{pmatrix} \begin{pmatrix} y(k+1) \\ 0 \\ 0 \end{pmatrix}. \tag{3.8}$$

On the other hand, since $\eta(k) - C_2 y(k) - Ag(y(k)) - Bg(y(k-\tau(k))) = 0$, $\sum_{i=k-\tau_m}^{k} y(i) - \sum_{i=k-\tau_M}^{k} y(i) + \sum_{i=k+1-\tau_M}^{k-\tau_m} y(i) - y^T(k-\tau_m) + y^T(k-\tau_M) = 0$, we have

$$\begin{pmatrix} y(k+1) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} C_1 y(k) + \eta(k) \\ \sum\limits_{i=k-\tau_m}^{k} y(i) - \sum\limits_{i=k-\tau_M}^{k} y(i) + \sum\limits_{i=k+1-\tau_M}^{k-\tau_m} y(i) - y^T(k-\tau_m) + y^T(k-\tau_M) \\ \eta(k) - C_2 y(k) - Ag\left(y(k)\right) - Bg\left(y(k-\tau(k))\right) \end{pmatrix} \tag{3.9}$$

$$= \begin{pmatrix} C_1 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & -I & 0 & 0 & -I & I & I \\ -C_2 & 0 & I & 0 & 0 & -A & -B & 0 & 0 & 0 \end{pmatrix} Y(k), \tag{3.10}$$

$$I_2 = Y^T(k) \begin{pmatrix} P_1 & H_2 & H_{12} \\ 0 & H_3 & H_{13} \\ 0 & H_4 & H_{14} \\ 0 & H_5 & H_{15} \\ 0 & H_6 & H_{16} \\ 0 & H_7 & H_{17} \\ 0 & H_8 & H_{18} \\ 0 & H_9 & H_{19} \\ 0 & H_{10} & H_{20} \\ 0 & H_{11} & H_{21} \end{pmatrix} \begin{pmatrix} y(k) \\ 0 \\ 0 \end{pmatrix}$$

$$= Y^T(k) \begin{pmatrix} P_1 & H_2 & H_{12} \\ 0 & H_3 & H_{13} \\ 0 & H_4 & H_{14} \\ 0 & H_5 & H_{15} \\ 0 & H_6 & H_{16} \\ 0 & H_7 & H_{17} \\ 0 & H_8 & H_{18} \\ 0 & H_9 & H_{19} \\ 0 & H_{10} & H_{20} \\ 0 & H_{11} & H_{21} \end{pmatrix} \begin{pmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & -I & 0 & 0 & -I & I & I \\ -C_2 & 0 & I & 0 & 0 & -A & -B & 0 & 0 & 0 \end{pmatrix} Y(k). \tag{3.11}$$

$$\Delta V_2(k) \leq y^T(k) Q_1 y(k) - y^T(k-\tau(k)) Q_1 y(k-\tau(k)) + \sum_{i=k+1-\tau_M}^{k-\tau_m} y^T(i) Q_1 y(i), \tag{3.12}$$

$$\Delta V_3(k) = y^T(k)(Q_2+Q_3) y(k) - y^T(k-\tau_m) Q_2 y(k-\tau_m) - y^T(k-\tau_M) Q_3 y(k-\tau_M). \tag{3.13}$$

From Lemma 2.3 we can obtain

$$\begin{aligned}
\Delta V_4(k) &= \sum_{j=k+1-\tau_m}^{k}\sum_{i=j}^{k} y^T(i)\,Q_4 y(i) - \sum_{j=k-\tau_m}^{k-1}\sum_{i=j}^{k-1} y^T(i)\,Q_4 y(i) \\
&\quad + \sum_{j=k+1-\tau_M}^{k}\sum_{i=j}^{k} y^T(i)\,Q_5 y(i) - \sum_{j=k-\tau_M}^{k-1}\sum_{i=j}^{k-1} y^T(i)\,Q_5 y(i) \\
&= \sum_{j=k-\tau_m}^{k-1}\sum_{i=j+1}^{k} y^T(i)\,Q_4 y(i) - \sum_{j=k-\tau_m}^{k-1}\sum_{i=j}^{k-1} y^T(i)\,Q_4 y(i) \\
&\quad + \sum_{j=k-\tau_M}^{k-1}\sum_{i=j+1}^{k} y^T(i)\,Q_5 y(i) - \sum_{j=k-\tau_M}^{k-1}\sum_{i=j}^{k-1} y^T(i)\,Q_5 y(i) \\
&= \sum_{j=k-\tau_m}^{k-1}\Big(y^T(k)\,Q_4 y(k) - y^T(j)\,Q_4 y(j)\Big) \\
&\quad + \sum_{j=k-\tau_M}^{k-1}\Big(y^T(k)\,Q_5 y(k) - y^T(j)\,Q_5 y(j)\Big) \\
&\le (1+\tau_m)\,y^T(k)\,Q_4 y(k) - \sum_{j=k-\tau_m}^{k} y^T(j)\,Q_4 y(j) \\
&\quad + (1+\tau_M)\,y^T(k)\,Q_5 y(k) - \sum_{j=k-\tau_M}^{k} y^T(j)\,Q_5 y(j) \\
&\le (1+\tau_m)\,y^T(k)\,Q_4 y(k) - \frac{1}{1+\tau_m}\left[\sum_{j=k-\tau_m}^{k} y(j)\right]^T Q_4 \left[\sum_{j=k-\tau_m}^{k} y(j)\right] \\
&\quad + (1+\tau_M)\,y^T(k)\,Q_5 y(k) - \frac{1}{1+\tau_M}\left[\sum_{j=k-\tau_M}^{k} y(j)\right]^T Q_5 \left[\sum_{j=k-\tau_M}^{k} y(j)\right],
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\Delta V_5(k) &= \sum_{j=k+2-\tau_M}^{k+1-\tau_m}\sum_{i=j}^{k} y^T(i)\,Q_1 y(i) - \sum_{j=k+1-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1} y^T(i)\,Q_1 y(i) \\
&= \sum_{j=k+1-\tau_M}^{k-\tau_m}\sum_{i=j+1}^{k} y^T(i)\,Q_1 y(i) - \sum_{j=k+1-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1} y^T(i)\,Q_1 y(i) \\
&= \sum_{j=k+1-\tau_M}^{k-\tau_m}\Big(y^T(k)\,Q_4 y(k) - y^T(j)\,Q_1 y(j)\Big) \\
&= (\tau_M-\tau_m)\,y^T(k)\,Q_1 y(k) - \sum_{j=k+1-\tau_M}^{k} y^T(j)\,Q_1 y(j).
\end{aligned} \tag{3.15}$$

Similarly,

$$\begin{aligned}\Delta V_6(k) &= (\tau_M-\tau_m)\,y^T(k)\,Q_6 y(k) - \sum_{j=k+1-\tau_M}^{k-\tau_m} y^T(j)\,Q_6 y(j)\\ &\le (\tau_M-\tau_m)\,y^T(k)\,Q_6 y(k) - \frac{1}{\tau_M-\tau_m}\left[\sum_{j=k+1-\tau_M}^{k-\tau_m} y(j)\right]^T Q_6\left[\sum_{j=k+1-\tau_M}^{k-\tau_m} y(j)\right].\end{aligned}\tag{3.16}$$

From Lemma 2.5, for any positive diagonal matrix D_1, D_2, it follows that

$$\begin{aligned}&2y^T(k-\tau(k))\,D_2\left(\prod_1+\prod_2\right)g\left(y(k-\tau(k))\right)-2g^T\left(y(k-\tau(k))\right)D_2 g\left(y(k-\tau(k))\right)\\ &\quad -2y^T(k-\tau(k))\prod_1 D_2\prod_2 y(k-\tau(k))\ge 0\\ &2y^T(k)\,D_1\left(\prod_1+\prod_2\right)g\left(y(k)\right)-2g^T\left(y(k)\right)D_1 g\left(y(k)\right)-2y^T(k)\prod_1 D_1\prod_2 y(k)\ge 0.\end{aligned}\tag{3.17}$$

Combining (3.7)–(3.17), we get

$$\Delta V(k)\le Y^T(k)\,\Xi Y(k).\tag{3.18}$$

If the LMI (3.2) holds, it follows that there exists a sufficient small scalar $\varepsilon>0$ such that

$$\Delta V(k)\le -\varepsilon\left\|y(k)\right\|^2.\tag{3.19}$$

On the other hand, it can easily to get that

$$\begin{aligned}V(k) &\le 2\lambda_m(P_1)\,\|y(k)\|^2+\lambda_{\max}(Q_1)\sum_{i=k-\tau(k)}^{k-1}\left\|y(i)\right\|^2+\lambda_{\max}(Q_2)\sum_{i=k-\tau_m}^{k-1}\left\|y(i)\right\|^2\\ &\quad+\lambda_{\max}(Q_3)\sum_{i=k-\tau_M}^{k-1}\left\|y(i)\right\|^2+\lambda_{\max}(Q_4)\sum_{j=k-\tau_m}^{k}\sum_{i=j}^{k-1}\left\|y(i)\right\|^2+\lambda_{\max}(Q_5)\sum_{j=k-\tau_M}^{k}\sum_{i=j}^{k-1}\|y(i)\|^2\\ &\quad+\lambda_{\max}(Q_1)\sum_{j=k-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1}\left\|y(i)\right\|^2+\lambda_{\max}(Q_6)\sum_{j=k+1-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1}\left\|y(i)\right\|^2\\ &\le 2\lambda_{\max}(P)\left\|y(k)\right\|^2+\lambda\sum_{i=k-\tau_M}^{k-1}\left\|y(i)\right\|^2,\end{aligned}\tag{3.20}$$

where $\lambda = \lambda_{\max}(Q_1) + \lambda_{\max}(Q_2) + \lambda_{\max}(Q_3) + (1+\tau_m)\lambda_{\max}(Q_4) + (1+\tau_M)\lambda_{\max}(Q_5) + (1+\tau_M - \tau_m)(\lambda_{\max}(Q_1) + \lambda_{\max}(Q_6))$. Choose a scalar $\theta > 1$ such that $-\varepsilon\theta + 2(\theta-1)\lambda_{\max}(P_1) + (\theta-1)\lambda \cdot \tau_M \theta^{\tau_M} = 0$. Then by (3.19) and (3.20), we get

$$\begin{aligned}\theta^{k+1} V(k+1) - \theta^k V(k) &= \theta^{k+1}\Delta V(k) + \theta^k(\theta-1)V(k) \\ &\le \varepsilon_1 \theta^k \|y(k)\|^2 + \varepsilon_2 \theta^k \sum_{i=k-\tau_M}^{k-1} \|y(i)\|^2,\end{aligned} \tag{3.21}$$

where $\varepsilon_1 = -\varepsilon\theta + 2\lambda_{\max}(P)(\theta-1)$, $\varepsilon_2 = \lambda(\theta-1)$. Therefore, for arbitrary positive integer $N \ge \tau_M + 1$, summing up both sides of (3.21) from 0 to $N-1$, we can obtain

$$\begin{aligned}\theta^N V(N) - V(0) &\le \varepsilon_1 \sum_{k=0}^{N-1} \theta^k \|y(k)\|^2 + \varepsilon_2 \sum_{k=0}^{N-1} \sum_{i=k-\tau_M}^{k-1} \theta^k \|y(i)\|^2 \\ &\le \varepsilon_2 \tau_M (\tau_M+1)\theta^{\tau_M} \sup_{i\in\mathbb{N}[-\tau_M,0]} \|y(i)\|^2 + (\varepsilon_1 + \varepsilon_2 \tau_M \theta^{\tau_M}) \sum_{k=0}^{N-1} \theta^k \|y(\mathrm{k})\|^2.\end{aligned} \tag{3.22}$$

Noting that

$$V(N) \ge \lambda_{\min}(P_1) \|y(N)\|^2, \qquad V(0) \le (\lambda\tau_M + 2\lambda_{\max}(P_1)) \sup_{i\in\mathbb{N}[-\tau_M,0]} \|y(i)\|^2. \tag{3.23}$$

It follows that $\|y(N)\| \le \alpha \cdot \beta^N \sup_{i\in\mathbb{N}[-\tau_M,0]} \|y(i)\|$, where $\beta = (\sqrt{\theta})^{-1}$, $\alpha = \sqrt{(\lambda\tau_M + 2\lambda_{\max}(P) + \varepsilon_2\tau_M(\tau_M+1)\theta^{\tau_M})/\lambda_{\min}(P)}$. By Definition 2.2, system (2.5) is globally exponentially stable, which completes the proof of Theorem 3.1. □

Remark 3.2. By constructing the new augmented Lyapunov functional, free-weighting matrices $P_i, H_i, i = 2, 3, \ldots, 21$ are introduced so as to reduce the conservatism of the delay-dependent result. Moreover, the decomposition of matrix $C = C_1 + C_2$ makes the conservatism of the stability criterion reduce further, since the elements of matrices C_1, C_2 are not restricted to $(-1, 1)$ any more.

Remark 3.3. Since Theorem 3.1 holds for arbitrary matrices C_1, C_2 satisfying $C_1 + C_2 = C$, then, when $C_1 = 0$ or $C_2 = 0$, respectively, we can easily obtains the following simplified useful corollaries.

Corollary 3.4. *For any given positive scalars* $0 < \tau_m < \tau_M$, *then, under Assumption 1, system* (2.5) *is globally exponentially stable for any time-varying delay* $\tau(k)$ *satisfying* $\tau_m \le \tau(k) \le \tau_M$, *if there exist positive-definite matrices* P_1, Q_1, Q_2, Q_3, *positive-definite diagonal matrices* D_1, D_2, Q_4, Q_5, Q_6, *and arbitrary matrices* $P_i, H_i, i = 2, 3, \ldots, 21$ *with appropriate dimensions, such that the following LMI holds:*

$$\tilde{\Xi} \triangleq \begin{pmatrix} \tilde{\Xi}_{11} & \tilde{\Xi}_{12} & \tilde{\Xi}_{13} & \tilde{\Xi}_{14} & \tilde{\Xi}_{15} & \tilde{\Xi}_{16} & \tilde{\Xi}_{17} & \tilde{\Xi}_{18} & \tilde{\Xi}_{19} & \tilde{\Xi}_{1,10} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} & \Xi_{2,10} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} & \Xi_{3,10} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} & \Xi_{4,10} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} & \Xi_{5,10} \\ * & * & * & * & * & \Xi_{66} & \Xi_{67} & \Xi_{68} & \Xi_{69} & \Xi_{6,10} \\ * & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} & \Xi_{7,10} \\ * & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} & \Xi_{8,10} \\ * & * & * & * & * & * & * & * & \Xi_{99} & \Xi_{9,10} \\ * & * & * & * & * & * & * & * & * & \Xi_{10,10} \end{pmatrix} < 0, \tag{3.24}$$

where

$$\begin{aligned}
\tilde{\Xi}_{11} &= -2P_1 + H_{12}C + C^T H_{12}^T + Q_2 + Q_3 + (\tau_M - \tau_m + 1)\,Q_1 \\
&\quad + (1+\tau_m)\,Q_4 + (1+\tau_M)\,Q_5 + (\tau_M - \tau_m)\,Q_6 - 2\prod_1 D_1 \prod_2, \\
\tilde{\Xi}_{12} &= C^T (H_{13} - P_{13})^T, \qquad \tilde{\Xi}_{13} = C^T (H_{14} - P_{14} - P_{12})^T - H_{12}, \\
\tilde{\Xi}_{14} &= C^T (H_{15} - P_{15})^T - H_2, \qquad \tilde{\Xi}_{15} = H_2 + C^T (H_{16} - P_{16})^T, \\
\tilde{\Xi}_{16} &= H_{12}A + C^T (H_{17} - P_{17})^T + D_1 \left(\prod_1 + \prod_2\right), \qquad \tilde{\Xi}_{17} = H_{12}B + C^T (H_{18} - P_{18})^T, \\
\tilde{\Xi}_{18} &= H_2 + C^T (H_{19} - P_{19})^T, \qquad \tilde{\Xi}_{19} = C^T (H_{20} - P_{20})^T - H_2, \qquad \tilde{\Xi}_{1,10} = C^T (H_{21} - P_{21})^T - H_2.
\end{aligned} \tag{3.25}$$

Corollary 3.5. *For any given positive scalars* $0 < \tau_m < \tau_M$*, then, under Assumption 1, system* (2.5) *is globally exponentially stable for any time-varying delay* $\tau(k)$ *satisfying* $\tau_m \le \tau(k) \le \tau_M$*, if there exist positive-definite matrices* P_1, Q_1, Q_2, Q_3*, positive-definite diagonal matrices* D_1, D_2, Q_4, Q_5, Q_6*, and arbitrary matrices* $P_i, H_i, i = 2, 3, \ldots, 21$ *with appropriate dimensions, such that the following LMI holds:*

$$\widehat{\Xi} \triangleq \begin{pmatrix} \widehat{\Xi}_{11} & 0 & \widehat{\Xi}_{13} & \widehat{\Xi}_{14} & \widehat{\Xi}_{15} & \widehat{\Xi}_{16} & \widehat{\Xi}_{17} & \widehat{\Xi}_{18} & \widehat{\Xi}_{19} & \widehat{\Xi}_{1,10} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} & \Xi_{2,10} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} & \Xi_{3,10} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} & \Xi_{4,10} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} & \Xi_{5,10} \\ * & * & * & * & * & \Xi_{66} & \Xi_{67} & \Xi_{68} & \Xi_{69} & \Xi_{6,10} \\ * & * & * & * & * & * & \Xi_{77} & \Xi_{78} & \Xi_{79} & \Xi_{7,10} \\ * & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} & \Xi_{8,10} \\ * & * & * & * & * & * & * & * & \Xi_{99} & \Xi_{9,10} \\ * & * & * & * & * & * & * & * & * & \Xi_{10,10} \end{pmatrix} < 0, \tag{3.26}$$

where

$$\begin{gathered} \widehat{\Xi}_{11} = 2C^T P_1 C - 2P_1 + Q_2 + Q_3 + (\tau_M - \tau_m + 1)\, Q_1 + (1 + \tau_m)\, Q_4 \\ + (1 + \tau_M)\, Q_5 + (\tau_M - \tau_m)\, Q_6 - 2\prod_1 D_1 \prod_2, \\ \widehat{\Xi}_{13} = C^T (P_1 + P_{12}) - H_{12} + C^T P_1^T, \qquad \widehat{\Xi}_{14} = C^T P_2 - H_2, \widehat{\Xi}_{15} = -C^T P_2 + H_2, \\ \widehat{\Xi}_{16} = -C^T P_{12} A + H_{12} A + D_1 \left(\prod_1 + \prod_2\right), \qquad \widehat{\Xi}_{17} = -C^T P_{12} B + H_{12} B, \\ \widehat{\Xi}_{18} = -C^T P_2 + H_2, \qquad \widehat{\Xi}_{19} = C^T P_2 - H_2, \qquad \widehat{\Xi}_{1,10} = C^T P_2 - H_2. \end{gathered} \tag{3.27}$$

Remark 3.6. It is worth pointing out that Theorem 3.1 and Corollary 3.4 can be easily extended to robust exponential stability conditions. As for the stability of system (2.1), according to Lemma 2.4, we can obtain the following robust stability results.

Theorem 3.7. *For any given positive scalars $0 < \tau_m < \tau_M$, then, under Assumption 1, system* (2.1) *is robustly, globally, exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive-definite matrices P_1, Q_1, Q_2, Q_3, positive-definite diagonal matrices D_1, D_2, Q_4, Q_5, Q_6, arbitrary matrices $P_i, H_i, i = 2, 3, \ldots, 21$ with appropriate dimensions, and a positive scalar ϵ such that the following LMI holds:*

$$\Xi' \triangleq \begin{pmatrix} \Xi & \xi_1 & \epsilon\xi_2^T \\ & -\epsilon I & 0 \\ & * & -\epsilon I \end{pmatrix} < 0, \tag{3.28}$$

where $\xi_1^T = [K^T(H_{12} - C_1^T P_{12})^T, K^T(H_{13} - P_{13})^T, K^T(H_{14} - P_{12} - P_{14})^T, K^T(H_{15} - P_{15})^T, K^T(H_{16} - P_{16})^T, K^T(H_{17} - P_{17})^T, K^T(H_{18} - P_{18})^T, K^T(H_{19} - P_{19})^T, K^T(H_{20} - P_{20})^T, K^T(H_{21} - P_{21})^T]$, $\xi_2 = [E_c, 0, 0, 0, 0, E_a, E_b, 0, 0, 0]$.

Proof. Replacing A, B, C_2 in inequality (3.2) with $A + KF(t)E_a$, $B + KF(t)E_b$ and $C_2 + KF(t)E_c$ respectively, inequality (3.2) for system (2.1) is equivalent to $\Xi + \xi_1 F(t)\xi_2 + \xi_2^T F^T(t)\xi_1^T < 0$. From Lemmas 2.6 and 2.7, we can easily obtain this result, this complete the proof. Similarly, we have. □

Theorem 3.8. *For any given positive scalars $0 < \tau_m < \tau_M$, then, under Assumption 1, system (2.1) is robustly, globally, exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive-definite matrices P_1, Q_1, Q_2, Q_3, positive-definite diagonal matrices D_1, D_2, Q_4, Q_5, Q_6, arbitrary matrices $P_i, H_i, i = 2, 3, \ldots, 21$ with appropriate dimensions, and a positive scalar ϵ such that the following LMI holds:*

$$\tilde{\Xi}' \triangleq \begin{pmatrix} \tilde{\Xi} & \xi_1 & \epsilon\xi_2^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{pmatrix} < 0. \tag{3.29}$$

Theorem 3.9. *For any given positive scalars $0 < \tau_m < \tau_M$, then, under Assumption 1, system (2.1) is robustly, globally, exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive-definite matrices P_1, Q_1, Q_2, Q_3, positive-definite diagonal matrices D_1, D_2, Q_4, Q_5, Q_6, and arbitrary matrices $P_i, H_i, \overline{P}_j, \overline{H}_j$, $i = 2, 3, \ldots, 21, j = 1, 2, \ldots, 6$ with appropriate dimensions, such that the following LMI holds:*

$$\Xi'' \triangleq \begin{pmatrix} \Xi''_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19} & \Xi_{1,10} & \Xi''_{1,11} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} & \Xi_{2,10} & \Xi''_{2,11} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} & \Xi_{3,10} & \Xi''_{3,11} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} & \Xi_{4,10} & \Xi''_{4,11} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} & \Xi_{5,10} & \Xi''_{5,11} \\ * & * & * & * & * & \Xi''_{66} & \Xi_{67} & \Xi_{68} & \Xi_{69} & \Xi_{6,10} & \Xi''_{6,11} \\ * & * & * & * & * & * & \Xi''_{77} & \Xi_{78} & \Xi_{79} & \Xi_{7,10} & \Xi''_{7,11} \\ * & * & * & * & * & * & * & \Xi_{88} & \Xi_{89} & \Xi_{8,10} & \Xi''_{8,11} \\ * & * & * & * & * & * & * & * & \Xi_{99} & \Xi_{9,10} & \Xi''_{9,11} \\ * & * & * & * & * & * & * & * & * & \Xi_{10,10} & \Xi''_{10,11} \\ * & * & * & * & * & * & * & * & * & * & \Xi''_{11,11} \end{pmatrix}_{13n\times 13n} < 0, \tag{3.30}$$

where

$$\begin{aligned}
&\Xi''_{11} = \Xi_{11} + E_c^T E_c, \qquad \Xi_{66} = \Xi''_{66} + E_a^T E_a, \qquad \Xi''_{77} = \Xi_{77} + E_b^T E_b,\\
&\Xi''_{1,11} = \left(H_{12} - C_1^T P_{12}\right)\overline{K} + C_2^T\left(\overline{H}_{23} - \overline{P}_{23}\right)^T, \qquad \Xi''_{2,11} = \left(H_{13} - P_{13}\right)\overline{K},\\
&\Xi''_{3,11} = \left(H_{14} - P_{14} - P_{12}\right)\overline{K} + \overline{P}_{23}^T - \overline{H}_{23}^T, \qquad \Xi''_{4,11} = \left(H_{15} - P_{15}\right)\overline{K} + \overline{P}_{22}^T - \overline{H}_{22}^T,\\
&\Xi''_{5,11} = \left(H_{16} - P_{16}\right)\overline{K} - \overline{P}_{22}^T - \overline{H}_{22}^T, \qquad \Xi''_{6,11} = \left(H_{17} - P_{17}\right)\overline{K} - A^T\overline{P}_{23}^T + A^T\overline{H}_{23}^T,\\
&\Xi''_{7,11} = \left(H_{18} - P_{18}\right)\overline{K} - B^T\overline{P}_{23}^T + B^T\overline{H}_{23}^T, \qquad \Xi''_{8,11} = \left(H_{19} - P_{19}\right)\overline{K} - \overline{P}_{22}^T + \overline{H}_{22}^T,\\
&\Xi''_{9,11} = \left(H_{20} - P_{20}\right)\overline{K} + \overline{P}_{22}^T - \overline{H}_{22}^T, \qquad \Xi''_{10,11} = \left(H_{21} - P_{21}\right)\overline{K} + \overline{P}_{22}^T - \overline{H}_{22}^T,\\
&\Xi''_{11,11} = \left(\overline{H}_{23} - \overline{P}_{23}\right)\overline{K} + \overline{K}^T\left(\overline{H}_{23} - \overline{P}_{23}\right)^T - \overline{I}, \quad \overline{K} = \begin{bmatrix} K & K & K \end{bmatrix},\\
&\overline{P}_{22} = \begin{pmatrix} \overline{P}_1 \\ \overline{P}_2 \\ \overline{P}_3 \end{pmatrix}, \qquad \overline{P}_{23} = \begin{pmatrix} \overline{P}_4 \\ \overline{P}_5 \\ \overline{P}_6 \end{pmatrix}, \qquad \overline{H}_{22} = \begin{pmatrix} \overline{H}_1 \\ \overline{H}_2 \\ \overline{H}_3 \end{pmatrix},\\
&\overline{H}_{23} = \begin{pmatrix} \overline{H}_4 \\ \overline{H}_5 \\ \overline{H}_6 \end{pmatrix}, \qquad \overline{I} = \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}.
\end{aligned} \tag{3.31}$$

Proof. Replacing A, B, C in system (2.5) with $A + KF(t)E_a$, $B + KF(t)E_b$ and $C + KF(t)E_c$, respectively. Then, system (2.5) can be transformed into the following equivalent form:

$$\begin{aligned}
y(k+1) &= Cy(k) + Ag(y(k)) + Bg(y(k-\tau(k))) + KF(t)E_c y(k) + KF(t)E_a g(y(k))\\
&\quad + KF(t)E_b g(y(k-\tau(k)))\\
&= C_1 y(k) + C_2 y(k) + Ag(y(k)) + Bg(y(k-\tau(k))) + \overline{K}\Upsilon(k),
\end{aligned} \tag{3.32}$$

where

$$\begin{aligned}
\Upsilon(k) &= \begin{pmatrix} F(t)E_c y(k) \\ F(t)E_a g(y(k)) \\ F(t)E_b g(y(k-\tau(k))) \end{pmatrix}\\
&= \operatorname{diag}(F(t), F(t), F(t))\operatorname{diag}(E_c, E_a, E_b)\begin{pmatrix} y(k) \\ g(y(k)) \\ g(y(k-\tau(k))) \end{pmatrix}.
\end{aligned} \tag{3.33}$$

Constructing a new augmented Lyapunov-Krasovskii functional candidate as follows:

$$V(k) = \overline{V}_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k) + V_6(k), \tag{3.34}$$

where

$$\overline{V}_1(k) = 2\overline{Y}^T(k) \begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{13n\times 13n} \overline{Y}(k), \tag{3.35}$$

$\overline{Y}^T(k) = [y^T(k), y^T(k-\tau(k)), \eta^T(k), y^T(k-\tau_M), y^T(k-\tau_m), g^T(y(k)), g^T(y(k-\tau_M)), \sum_{i=k-\tau_M}^{k} y^T(i), \sum_{i=k-\tau_m}^{k} y^T(i), \sum_{i=k-\tau_M+1}^{k-\tau_m} y^T(i), \Upsilon^T(k)]^T$, $\eta(k) = y(k+1) - C_1 y(k)$; $V_2(k), V_3(k), \ldots, V_6(k)$ are the same as in Theorem 3.1.

Set $\overline{\overline{Y}}^T(k+1) = [y^T(k+1), y^T(k-\tau(k)), \eta^T(k), y^T(k-\tau_M), y^T(k-\tau_m), g^T(y(k)), g^T(y(k-\tau_M)), \sum_{i=k-\tau_M}^{k} y^T(i), \sum_{i=k-\tau_m}^{k} y^T(i), \sum_{i=k-\tau_M+1}^{k-\tau_m} y^T(i), \Upsilon^T(k)]^T = [y^T(k)C_1^T + \eta^T(k), \eta^T(k), y^T(k-\tau_M), y^T(k-\tau_m), g^T(y(k)), g^T(y(k-\tau_M)), \sum_{i=k-\tau_M}^{k} y^T(i), \sum_{i=k-\tau_m}^{k} y^T(i), \sum_{i=k-\tau_M+1}^{k-\tau_m} y^T(i), \Upsilon^T(k)]^T$. Define $\Delta V(k) = V(k+1) - V(k)$. Then along the solution of system (3.32) we have

$$\begin{aligned} \Delta\overline{V}_1(k) &= 2\overline{Y}^T(k+1) \begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \overline{Y}(k+1) - 2\overline{Y}^T(k) \begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \overline{Y}(k) \\ &= 2\overline{\overline{Y}}^T(k+1) \begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \overline{\overline{Y}}(k+1) - 2\overline{Y}^T(k) \begin{pmatrix} P_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \overline{Y}(k) \\ &\triangleq 2\overline{I}_1 - 2\overline{I}_2. \end{aligned} \tag{3.36}$$

$$\overline{I}_1 = \overline{Y}^T(k) \begin{pmatrix} C_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{I} \end{pmatrix} \begin{pmatrix} P_1 & P_2 & P_{12} \\ 0 & P_3 & P_{13} \\ 0 & P_4 & P_{14} \\ 0 & P_5 & P_{15} \\ 0 & P_6 & P_{16} \\ 0 & P_7 & P_{17} \\ 0 & P_8 & P_{18} \\ 0 & P_9 & P_{19} \\ 0 & P_{10} & P_{20} \\ 0 & P_{11} & P_{21} \\ 0 & \overline{P}_{22} & \overline{P}_{23} \end{pmatrix} \begin{pmatrix} y(k+1) \\ 0 \\ 0 \end{pmatrix}. \tag{3.37}$$

On the other hand, since $\eta(k)-C_2y(k)-Ag(y(k))-Bg(y(k-\tau(k)))-\overline{K}\Upsilon(k)=0$, $\sum_{i=k-\tau_m}^{k} y(i)-\sum_{i=k-\tau_M}^{k} y(i)+\sum_{i=k+1-\tau_M}^{k-\tau_m} y(i)-y^T(k-\tau_m)+y^T(k-\tau_M)=0$, we have

$$\begin{aligned} \begin{pmatrix} y(k+1) \\ 0 \\ 0 \end{pmatrix} &= \begin{pmatrix} C_1y(k)+\eta(k) \\ \displaystyle\sum_{i=k-\tau_m}^{k} y(i) - \sum_{i=k-\tau_M}^{k} y(i) + \sum_{i=k+1-\tau_M}^{k-\tau_m} y(i) - y^T(k-\tau_m) + y^T(k-\tau_M) \\ \eta(k) - C_2y(k) - Ag\left(y(k)\right) - Bg\left(y(k-\tau(k))\right) - \overline{K}\Upsilon(k) \end{pmatrix} \\ &= \begin{pmatrix} C_1 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & -I & 0 & 0 & -I & I & I & 0 \\ -C_2 & 0 & I & 0 & 0 & -A & -B & 0 & 0 & 0 & -\overline{K} \end{pmatrix} \overline{Y}(k), \end{aligned} \tag{3.38}$$

$$\overline{I}_2 = \overline{Y}^T(k) \begin{pmatrix} P_1 & H_2 & H_{12} \\ 0 & H_3 & H_{13} \\ 0 & H_4 & H_{14} \\ 0 & H_5 & H_{15} \\ 0 & H_6 & H_{16} \\ 0 & H_7 & H_{17} \\ 0 & H_8 & H_{18} \\ 0 & H_9 & H_{19} \\ 0 & H_{10} & H_{20} \\ 0 & H_{11} & H_{21} \\ 0 & \overline{H}_{22} & \overline{H}_{23} \end{pmatrix} \begin{pmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & -I & 0 & 0 & -I & I & I & 0 \\ -C_2 & 0 & I & 0 & 0 & -A & -B & 0 & 0 & 0 & -\overline{K} \end{pmatrix} \overline{Y}(k). \tag{3.39}$$

Table 1: Allowable upper bounds τ_M for given τ_m.

Cases	$\tau_m = 2$	$\tau_m = 4$	$\tau_m = 6$	$\tau_m = 8$	$\tau_m = 10$	$\tau_m = 20$
By [15]	11	11	12	13	14	21
By [16]	11	12	13	14	16	23
By [17]	13	13	17	19	21	31
By Theorem 3.1	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$

Noting that

$$\begin{aligned}\Upsilon^T(k)\Upsilon(k) \le{}& \left[y^T(k), g^T(y(k)), g^T(y(k-\tau(k)))\right]^T \\ &\times \begin{pmatrix} E_c^T E_c & 0 & 0 \\ 0 & E_a^T E_a & 0 \\ 0 & 0 & E_b^T E_b \end{pmatrix} \begin{pmatrix} y(k) \\ g(y(k)) \\ g(y(k-\tau(k))) \end{pmatrix}.\end{aligned} \tag{3.40}$$

Combining (3.12)–(3.17), (3.36)–(3.40), similar to the proof of Theorem 3.1, one can easily obtain this result, which completes the proof. □

Remark 3.10. Compared with the augmented Lyapunov functional constructed in Theorem 3.1, this new augmented Lyapunov functional include the term $\Upsilon(k)$, which makes the conservatism of the stability criterion be reduced further (details for more, see Example 4.2).

4. Numerical Examples

In this section, three numerical examples will be presented to show the validity of the main results derived above.

Example 4.1. For the convenience of comparison, let us consider a delayed discrete-time recurrent neural network in (2.5) with parameters given by

$$C = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.9 \end{pmatrix}, \qquad A = \begin{pmatrix} 0.001 & 0 \\ 0 & 0.005 \end{pmatrix}, \qquad B = \begin{pmatrix} -0.1 & 0.01 \\ -0.2 & -0.1 \end{pmatrix}. \tag{4.1}$$

The activation functions are given by $g_1(x) = g_2(x) = \tanh(x)$. It is easy to see that the activation functions satisfy Assumption 1 with $\sigma_1^- = \sigma_2^- = 0$, $\sigma_1^+ = \sigma_2^+ = 1$. For $\tau_m = 2, 4, 6, 8, 10, 20$, references [15–17] gave out the allowable upper bound τ_M of the time-varying delay, respectively. Decompose matrix C as $C = C_1 + C_2$, where

$$C_1 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.5 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}. \tag{4.2}$$

Table 1 shows that our results are less conservative than these previous results.

Table 2: Allowable upper bounds τ_M for given τ_m.

Cases	$\tau_m = 2$	$\tau_m = 4$	$\tau_m = 6$	$\tau_m = 8$	$\tau_m = 10$
By [16]	6	8	10	12	14
By [17]	10	12	14	16	18
By Theorem 3.7	17	19	21	23	25
By Theorem 3.9	22	24	26	28	30

Example 4.2. Consider an uncertain delayed discrete-time recurrent neural network in (2.1) with parameters given by

$$C = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.1 \end{pmatrix}, \quad A = \begin{pmatrix} 0.12 & 0.24 \\ -0.15 & 0.2 \end{pmatrix}, \quad B = \begin{pmatrix} -0.25 & 0.1 \\ 0.02 & 0.09 \end{pmatrix}, \quad K = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.3 \end{pmatrix},$$
$$E_c = \begin{pmatrix} 0.15 & 0.1 \\ 0 & -0.7 \end{pmatrix}, \quad E_a = \begin{pmatrix} 0.1 & 0.3 \\ -0.2 & 0.05 \end{pmatrix}, \quad E_b = \begin{pmatrix} 0.13 & 0.06 \\ -0.05 & 0.15 \end{pmatrix}, \quad J = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{4.3}$$

The activation functions are given by $f_1(x) = \tanh(0.55x) + \sin(0.45x)$, $f_2(x) = \tanh(0.65x) + \sin(0.45x)$. It is easy to see that the activation functions satisfy Assumption 1 with $\sigma_1^- = 0.1, \sigma_2^- = 0.2, \sigma_1^+ = 1, \sigma_2^+ = 1.1$. For $\tau_m = 2, 4, 6, 8, 10$, references [16, 17] gave out the allowable upper bound τ_M of the time-varying delay, respectively. Decompose matrix C as $C = C_1 + C_2$, where

$$C_1 = \begin{pmatrix} 0.2 & -1 \\ 0.012 & 0.05 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0.05 & 1 \\ -0.012 & 0.05 \end{pmatrix}. \tag{4.4}$$

Set $\epsilon = 50$, by using the MATLAB toolbox, the allowable upper bounds τ_M for given τ_m are showed in Table 2. Obviously, our results are less conservative than these previous results.

Example 4.3. Consider an uncertain delayed discrete-time recurrent neural network in (2.1) with parameters given by

$$C = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.9 \end{pmatrix}, \quad A = \begin{pmatrix} 0.07 & 0.1 \\ 0 & 0.05 \end{pmatrix}, \quad B = \begin{pmatrix} -0.1 & 0.01 \\ -0.2 & -0.1 \end{pmatrix}, \quad K = \begin{pmatrix} 0.02 & 0 \\ 0 & 0.03 \end{pmatrix},$$
$$E_c = \begin{pmatrix} 0.15 & 0.1 \\ 0 & -0.7 \end{pmatrix}, \quad E_a = \begin{pmatrix} 0.1 & 0.3 \\ -0.2 & 0.05 \end{pmatrix}, \quad E_b = \begin{pmatrix} 0.13 & 0.06 \\ -0.05 & 0.15 \end{pmatrix}, \quad J = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{4.5}$$

And the activation functions are the same as given in Example 4.2. Decompose matrix C as $C = C_1 + C_2$, where

$$C_1 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.5 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}. \tag{4.6}$$

Table 3: Allowable upper bounds τ_M for given τ_m.

Cases	$\tau_m = 2$	$\tau_m = 4$	$\tau_m = 6$	$\tau_m = 8$	$\tau_m = 10$	$\tau_m = 20$
By Theorem 3.7	88	90	92	94	96	105

Set $\epsilon = 50$, by using the MATLAB toolbox, the allowable upper bounds τ_M for given τ_m are showed in Table 3.

The free-weighting matrices are obtained as follows when $\tau_m = 2, \tau_M = 60$:

$$Q_1 = \begin{pmatrix} 0.0074 & -0.0002 \\ -0.0002 & 0.0007 \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} 0.0627 & -0.0191 \\ -0.0191 & 0.0094 \end{pmatrix},$$

$$Q_3 = \begin{pmatrix} 0.0627 & -0.0191 \\ -0.0191 & 0.0094 \end{pmatrix}, \qquad D_1 = \begin{pmatrix} 0.8703 & 0 \\ 0 & 0.2672 \end{pmatrix},$$

$$Q_4 = \begin{pmatrix} 0.0079 & 0 \\ 0 & 0.0012 \end{pmatrix}, \qquad Q_5 = 1.0e-003 \begin{pmatrix} 0.2326 & 0 \\ 0 & 0.0303 \end{pmatrix},$$

$$Q_6 = 1.0e-003 \begin{pmatrix} 0.2493 & 0 \\ 0 & 0.0320 \end{pmatrix}, \qquad P_1 = \begin{pmatrix} 6.3936 & -0.9854 \\ -0.9854 & 0.2599 \end{pmatrix},$$

$$P_5 = \begin{pmatrix} -304.8096 & -16.0366 \\ -131.0952 & -930.9992 \end{pmatrix}, \qquad D_2 = \begin{pmatrix} 0.0612 & 0 \\ 0 & 0.0083 \end{pmatrix},$$

$$P_6 = 1.0e+003 \begin{pmatrix} 1.1931 & 0.1097 \\ 0.3365 & 0.0269 \end{pmatrix}, \qquad P_9 = 1.0e+003 \begin{pmatrix} -1.4764 & -0.1509 \\ -0.0791 & 1.0626 \end{pmatrix},$$

$$P_{10} = \begin{pmatrix} -350.0434 & -852.4925 \\ -26.8470 & 904.2654 \end{pmatrix},$$

$$P_{11} = 1.0e+003 \begin{pmatrix} -0.5682 & -0.0257 \\ -0.0413 & 1.1255 \end{pmatrix}, \qquad P_{12} = 1.0e+004 \begin{pmatrix} -0.6548 & -1.6295 \\ 0.9399 & 4.7692 \end{pmatrix},$$

$$P_{13} = 1.0e+003 \begin{pmatrix} 2.2001 & -3.7370 \\ -0.3034 & -0.3479 \end{pmatrix},$$

$$P_{14} = 1.0e+004 \begin{pmatrix} 0.3186 & 0.7851 \\ -0.4972 & -2.3726 \end{pmatrix}, \qquad P_{17} = \begin{pmatrix} 273.4789 & -200.5789 \\ -165.3251 & -31.3854 \end{pmatrix},$$

$$P_{18} = 1.0e+003 \begin{pmatrix} 0.0888 & -3.8454 \\ 1.6080 & 4.3515 \end{pmatrix},$$

$$H_5 = \begin{pmatrix} -304.4622 & -16.0335 \\ -131.0922 & -930.6433 \end{pmatrix}, \qquad H_6 = 1.0e+003 \begin{pmatrix} 1.1927 & 0.1097 \\ 0.3365 & 0.0265 \end{pmatrix},$$

$$
\begin{aligned}
H_9 &= 1.0e+003\begin{pmatrix} -1.4768 & -0.1509 \\ -0.0791 & 1.0622 \end{pmatrix}, \\
H_{10} &= \begin{pmatrix} -349.6840 & -852.4933 \\ -26.8478 & 904.6230 \end{pmatrix}, \qquad H_{11} = 1.0e+003\begin{pmatrix} -0.5678 & -0.0257 \\ -0.0413 & 1.1259 \end{pmatrix}, \\
H_{12} &= 1.0e+004\begin{pmatrix} -0.2616 & -0.6522 \\ 0.4698 & 2.3832 \end{pmatrix}, \\
H_{13} &= 1.0e+003\begin{pmatrix} 2.2001 & -3.7370 \\ -0.3034 & -0.3479 \end{pmatrix}, \qquad H_{14} = 1.0e+004\begin{pmatrix} -0.3344 & -0.8443 \\ 0.4422 & 2.4005 \end{pmatrix}, \\
H_{17} &= 1.0e+004\begin{pmatrix} 274.5315 & -200.9016 \\ -167.3579 & -33.2092 \end{pmatrix}, \qquad H_{18} = 1.0e+003\begin{pmatrix} 0.0893 & -3.8440 \\ 1.5970 & 4.3553 \end{pmatrix}, \\
P_2 &= P_3 = P_4 = P_7 = P_8 = P_{15} = P_{16} = P_{19} = P_{20} = P_{21} \\
&= H_2 = H_3 = H_4 = H_7 = H_8 = H_{15} = H_{16} = H_{19} = H_{20} = H_{21} = 0.
\end{aligned}
\tag{4.7}
$$

5. Conclusion

By decomposing some connection weight matrices , combined with linear matrix inequality (LMI) technique, some new augmented Lyapunov-Krasovskii functionals are constructed, and serial new improved sufficient conditions ensuring exponential stability or robust exponential stability are obtained. Numerical examples show that the new criteria derived in this paper are less conservative than some previous results obtained in the references cited therein.

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