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Research Article

Feedback Control Variables Have No Influence on the Permanence of a Discrete N-Species Cooperation System

Liujuan Chen,[1] Xiangdong Xie,[2] and Lijuan Chen[3]

[1] *Ministry of Science Training, Fujian Institute of Education, Fuzhou, Fujian 350001, China*
[2] *Department of Mathematics, Ningde Teachers College, Ningde, Fujian 352100, China*
[3] *College of Mathematics and Computer Science, Fuzhou University, Fuzhou, Fujian 350002, China*

Correspondence should be addressed to Liujuan Chen, clj1018@sohu.com

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A new set of sufficient conditions for the permanence of a discrete N-species cooperation system with delays and feedback controls are obtained. Our result shows that feedback control variables have no influence on the persistent property of the discrete cooperative system, thus improves and supplements the main result of F. D. Chen (2007).

1. Introduction

The aim of this paper is to investigate the permanent property of the following nonautonomous discrete n-species cooperation system with time delays and feedback controls of the form:

$$\begin{aligned} x_i(k+1) &= x_i(k)\exp\left\{r_i(k)\left[1-\frac{x_i(k-\tau_{ii})}{a_i(k)+\sum_{j=1,\, j\neq i}^{n} b_{ij}(k)x_j(k-\tau_{ij})}-c_i(k)x_i(k-\tau_{ii})\right]\right. \\ &\qquad \left. -d_i(k)u_i(k)-e_i(k)u_i(k-\eta_i)\right\}, \\ \Delta u_i(k) &= -\alpha_i(k)u_i(k)+\beta_i(k)x_i(k)+\gamma_i(k)x_i(k-\sigma_i), \end{aligned} \tag{1.1}$$

where $x_i(k)$ $(i=1,\ldots,n)$ is the density of cooperation species x_i, $u_i(k)$ $(i=1,\ldots,n)$ is the control variable (see [1, 2]).

Throughout this paper, we assume the following.

(H_1) $r_i(k), a_i(k), b_{ij}(k), c_i(k), d_i(k), e_i(k), \alpha_i(k), \beta_i(k), \gamma_i(k)$, $i, j = 1, 2, \ldots, n$ are all bounded nonnegative sequences such that

$$\begin{gathered} 0 < r_i^l \le r_i^u, \qquad 0 < a_i^l \le a_i^u, \qquad 0 < b_{ij}^l \le b_{ij}^u, \\ 0 < c_i^l \le c_i^u, \qquad 0 < d_i^l \le d_i^u, \qquad 0 \le e_i^l \le e_i^u, \\ 0 < \alpha_i^l \le \alpha_i^u < 1, \qquad 0 < \beta_i^l \le \beta_i^u, \qquad 0 < \gamma_i^l \le \gamma_i^u. \end{gathered} \tag{1.2}$$

Here, for any bounded sequence $\{h(k)\}$ and $N = \{0, 1, 2, \ldots\}$, $h^u = \sup_{k \in N}\{h(k)\}$ and $h^l = \inf_{k \in N}\{h(k)\}$.

(H_2) $\tau_{ij}, \eta_i, \sigma_i$, $i, j = 1, 2, \ldots, n$ are all nonnegative integers.

Let $\tau = \max\{\tau_{ij}, \sigma_i, \eta_i, \ i, j = 1, 2, \ldots, n\}$; we consider (1.1) together with the following initial conditions:

$$\begin{gathered} x_i(\theta) = \varphi_i(\theta) \ge 0, \quad \theta \in N[-\tau, 0] = \{-\tau, -\tau + 1, \ldots, 0\}, \ \varphi_i(0) > 0, \\ u_i(\theta) = \psi_i(\theta) \ge 0, \quad \theta \in N[-\tau, 0] = \{-\tau, -\tau + 1, \ldots, 0\}, \ \psi_i(0) > 0. \end{gathered} \tag{1.3}$$

It is not difficult to see that the solutions of (1.1)–(1.3) are well defined for all $k \ge 0$ and satisfy

$$x_i(k) > 0, \quad u_i(k) > 0, \quad \text{for } k \in Z, \ i = 1, 2, \ldots, n, \tag{1.4}$$

where Z is the set of integer numbers.

Recently, Chen [3] proposed and studied the permanence of system (1.1). Set

$$M_{i1} = \frac{\exp\{r_i^u(\tau_{ii} + 1) - 1\}}{c_i^l r_i^u}, \qquad M_{i2} = \frac{(\beta_i^u + \gamma_i^u) M_{i1}}{\alpha_i^l}. \tag{1.5}$$

Using the comparison theorem, he obtained the following result.

Theorem A (see [3]). *Assume that (H_1) and (H_2) hold, and assume further that*

(H_3)

$$r_i^l > (d_i^u + e_i^u) M_{i2}, \quad i = 1, 2, \ldots, n \tag{1.6}$$

holds, then system (1.1) *is permanent.*

However, as was pointed out by Fan and Wang [4], "if we use the method of comparison theorem, then the additional condition, in some extent, is necessary. But for the system itself, this condition may not necessary." In [4], by establishing a new difference inequality, Fan and Wang showed that feedback control has no influence on the permanence of a single species discrete model. Their success motivated us to consider the persistent property of system (1.1). Indeed, in this paper, we will develop the analysis idea of [3] and apply the difference inequality obtained by Fan and Wang [4] to prove the following result.

Theorem 1.1. *Assume that (H_1) and (H_2) hold, then system* (1.1) *is permanent.*

Remark 1.2. Theorem 1.1 shows that feedback control variables have no influence on the permanent property of system (1.1). It is natural to ask whether the feedback control variables have the influence on the stability property of the system or not. At present, we had difficulty to give an affirm answer to this problem, and we will leave this in our future study.

We will prove Theorem 1.1 in the next section. For more works on cooperative system and feedback control ecosystem, one could refer to [1–23] and the references cited therein.

2. Proof of Theorem 1.1

Now we state several lemmas which will be useful for the proof of our main result.

Lemma 2.1 (see [5, page 125]). *Consider the first-order difference equation*

$$y(k+1) = Ay(k) + B, \quad k = 1, 2, \dots, \tag{2.1}$$

where A and B are positive constants. Assume that $|A| < 1$, for any initial value $y(0)$, there exist a unique solution $y(k)$ of (2.1) *which can be expressed as follows: $y(k) = A^k(y(0) - y^*) + y^*$, where $y^* = B/(1-A)$. Thus, for any solution $\{y(k)\}$ of system* (2.1), *one has*

$$\lim_{k\to+\infty} y(k) = y^*. \tag{2.2}$$

Lemma 2.2 (see [5, page 241] (Comparison theorem)). *Let $k \in N_{k_0}^+ = \{k_0, k_0+1, \dots, k_0+l, \dots\}$, $r \geq 0$. For any fixed k, $g(k,r)$ is a nondecreasing function, and for $k \geq k_0$, the following inequalities hold:*

$$\begin{aligned} y(k+1) &\leq g(k, y(k)), \\ u(k+1) &\geq g(k, u(k)). \end{aligned} \tag{2.3}$$

If $y(k_0) \leq u(k_0)$, then $y(k) \leq u(k)$ for all $k \geq k_0$.

Lemma 2.3 (see [6, Theorem 2.1]). *Consider the following single species discrete model:*

$$N(k+1) = N(k)\exp\left(r(k)\left(1 - \frac{N(k)}{h(k)}\right)\right), \tag{2.4}$$

where $\{r(k)\}$ and $\{h(k)\}$ are strictly positive sequences of real numbers defined for $k \in N = \{0, 1, 2, \dots\}$ and $0 < h^l \leq h^u$, $0 < r^l \leq r^u$. Any solution of system (2.4) *with initial condition $N(0) > 0$ satisfies $m \leq \liminf_{k\to+\infty} N(k) \leq \limsup_{k\to+\infty} N(k) \leq M$, where $M = (h^u/r^u)\exp(r^u - 1)$, $m = h^l \exp(r^u(1 - M/h^l))$.*

Lemma 2.4 (see [7]). *Assume that $\{x(k)\}$ satisfies*

$$x(k+1) \geq x(k)\exp\{a(k) - b(k)x(k)\}, \quad k \geq N_0, \tag{2.5}$$

$\limsup_{k\to+\infty} x(k) \le x^*$, *and* $x(N_0) > 0$, *where* $a(k)$ *and* $b(k)$ *are nonnegative sequences bounded above and below by positive constants and* $N_0 \in N$. *Then*

$$\liminf_{k\to+\infty} x(k) \ge \min\left\{ \frac{a^l}{b^u} \exp\left\{ a^l - b^u x^* \right\}, \frac{a^l}{b^u} \right\}. \tag{2.6}$$

Lemma 2.5 (see [4]). *Assume that* $A > 0$ *and* $y(0) > 0$. *Further suppose that*

(i)

$$y(n+1) \le Ay(n) + B(n), \quad n = 1, 2, \ldots, \tag{2.7}$$

then for any integer $k \le n$, $y(n) \le A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1)$. *Especially, if* $A < 1$ *and* B *is bounded above with respect to* M, *then* $\limsup_{t\to+\infty} y(n) \le M/(1-A)$;

(ii)

$$y(n+1) \ge Ay(n) + B(n), \quad n = 1, 2, \ldots, \tag{2.8}$$

then for any integer $k \le n$, $y(n) \ge A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1)$. *Especially, if* $A < 1$ *and* B *is bounded below with respect to* m^*, *then* $\liminf_{t\to+\infty} y(n) \ge m^*/(1-A)$.

Lemma 2.6. *Let* $(x(k), u(k))^T = (x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_n(k))^T$ *be any positive solution of system* (1.1), *there exists a positive constant* M, *which is independent of the solution of system* (1.1), *such that*

$$\limsup_{k\to+\infty} x_i(k) \le M; \quad \limsup_{k\to+\infty} u_i(k) \le M, \quad i = 1, 2, \ldots, n. \tag{2.9}$$

Proof. Let $(x(k), u(k))^T = (x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_n(k))^T$ be any positive solution of system (1.1); similarly to the proof of Theorem 2.1 in [3], we have

$$\limsup_{k\to+\infty} x_i(k) \le M_{i1}, \qquad \limsup_{k\to+\infty} u_i(k) \le M_{i2}, \tag{2.10}$$

where M_{i1}, M_{i2}, $i = 1, 2, \ldots, n$ are defined by (1.5). In fact, from the ith equation of (1.1), it follows that

$$x_i(k+1) \le x_i(k) \exp\{r_i(k)\}. \tag{2.11}$$

Let $x_i(k) = \exp\{N_i(k)\}$, then (2.11) is equivalent to

$$N_i(k+1) - N_i(k) \le r_i(k). \tag{2.12}$$

Summing both sides of (2.12) from $k-\tau_{ii}$ to $k-1$ leads to

$$\sum_{j=k-\tau_{ii}}^{k-1}\left(N_i(j+1)-N_i(j)\right)\leq\sum_{j=k-\tau_{ii}}^{k-1}r_i(j)\leq r_i^u\tau_{ii}. \tag{2.13}$$

We obtain that $N_i(k-\tau_{ii})\geq N_i(k)-r_i^u\tau_{ii}$ and hence,

$$x_i(k-\tau_{ii})\geq x_i(k)\exp\left\{-r_i^u\tau_{ii}\right\}. \tag{2.14}$$

Substituting (2.14) to the ith equation of (1.1), it immediately follows that

$$\begin{aligned}x_i(k+1)&\leq x_i(k)\exp[r_i(k)(1-c_i(k)x_i(k-\tau_{ii}))]\\&\leq x_i(k)\exp\left[r_i(k)\left(1-c_i(k)x_i(k)\exp\left\{-r_i^u\tau_{ii}\right\}\right)\right].\end{aligned} \tag{2.15}$$

By applying Lemmas 2.2 and 2.3 to (2.15), we have

$$\limsup_{k\to+\infty}x_i(k)\leq\frac{\exp\left\{r_i^u(\tau_{ii}+1)-1\right\}}{c_i^l r_i^u}=M_{i1}. \tag{2.16}$$

For any small enough $\epsilon>0$, it follows from (2.16) that there exists enough large K_1 such that

$$x_i(k)\leq M_{i1}+\epsilon,\quad \text{for } k\geq K_1. \tag{2.17}$$

This, together with $(n+i)$th equation of (1.1), leads to

$$\Delta u_i(k)\leq-\alpha_i(k)u_i(k)+\left(\beta_i(k)+\gamma_i(k)\right)(M_{i1}+\epsilon),\quad \text{for } k\geq K_1+\tau. \tag{2.18}$$

And so,

$$u_i(k+1)\leq\left(1-\alpha_i^l\right)u_i(k)+\left(\beta_i^u+\gamma_i^u\right)(M_{i1}+\epsilon),\quad \text{for } k\geq K_1+\tau. \tag{2.19}$$

Notice that $0<1-\alpha_i^l<1$; it follows from (2.19) and Lemmas 2.1 and 2.2 that $\limsup_{k\to+\infty}u_i(k)\leq(\beta_i^u+\gamma_i^u)(M_{i1}+\epsilon)/\alpha_i^l$. Let $\epsilon\to 0$ in above inequality, then

$$\limsup_{k\to+\infty}u_i(k)\leq\frac{\left(\beta_i^u+\gamma_i^u\right)M_{i1}}{\alpha_i^l}=M_{i2}. \tag{2.20}$$

Set $M=\max_i\{M_{i1},M_{i2}\}$. The conclusion of Lemma 2.6 holds. The proof is complete. □

Lemma 2.7. *Let* $(x(k), u(k))^T = (x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_n(k))^T$ *be any positive solution of system* (1.1)*, there exists a positive constant* m*, which is independent of the solution of system* (1.1)*, such that*

$$\liminf_{k\to+\infty} x_i(k) \ge m; \qquad \liminf_{k\to+\infty} u_i(k) \ge m. \tag{2.21}$$

Proof. Let $(x(k), u(k))^T = (x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_n(k))^T$ be a solution of system (1.1) satisfying the initial condition (1.3). From Lemma 2.6, there exists a K_1 such that for all $k \ge K_1$, $x_i(t) \le 2M_{i1}$, $u_i(k) \le 2M_{i2}$. Thus, for $k > K_1 + \tau$, from the ith equation of system (1.1), it follows that

$$\begin{aligned} x_i(k+1) &\ge x_i(k)\exp\left\{ r_i(k)\left(1 - \frac{x_i(k-\tau_{ii})}{a_i^l} - c_i^u x_i(k-\tau_{ii})\right) - 2\left(d_i^u + e_i^u\right)M_{i2}\right\} \\ &\ge x_i(k)\exp\left\{ r_i^l\left(1 - \frac{2M_{i1}}{a_i^l} - 2c_i^u M_{i1}\right) - 2\left(d_i^u + e_i^u\right)M_{i2}\right\} \\ &\ge x_i(k)\exp\left\{ -\frac{2r_i^l M_{i1}}{a_i^l} - 2r_i^l c_i^u M_{i1} - 2\left(d_i^u + e_i^u\right)M_{i2}\right\} \\ &\overset{\text{def}}{=} x_i(k)\exp\{\zeta_i\}. \end{aligned} \tag{2.22}$$

Obviously, ζ_i is a negative constant. Let $x_i(k) = \exp\{N_i(k)\}$, the above inequality is equivalent to

$$N_i(k+1) - N_i(k) \ge \zeta_i. \tag{2.23}$$

Summing both sides of (2.23) from $k-m$ to $k-1$ leads to $\sum_{j=k-m}^{k-1}(N_i(j+1) - N_i(j)) \ge \zeta_i m$, and so, $N_i(k-m) \le N_i(k) - \zeta_i m$, therefore,

$$x_i(k-m) \le x_i(k)\exp\{-\zeta_i m\}. \tag{2.24}$$

Specially, we have

$$\begin{aligned} x_i(k-\sigma_i) &\le x_i(k)\exp\{-\zeta_i\sigma_i\} \le x_i(k)\exp\{-\zeta_i\tau\}, \\ x_i(k-\tau_{ii}) &\le x_i(k)\exp\{-\zeta_i\tau_{ii}\} \le x_i(k)\exp\{-\zeta_i\tau\}, \\ x_i(k-\eta_i) &\le x_i(k)\exp\{-\zeta_i\eta_i\} \le x_i(k)\exp\{-\zeta_i\tau\}. \end{aligned} \tag{2.25}$$

Substituting the first inequality into the $(n+i)$th equation of system (1.1) leads to

$$\begin{aligned} u_i(k+1) &\leq (1-\alpha_i(k))u_i(k) + \beta_i(k)x_i(k) + \gamma_i(k)x_i(k)\exp\{-\zeta_i\tau\} \\ &\leq \left(1-\alpha_i^l\right)u_i(k) + \beta_i^u x_i(k) + \gamma_i^u x_i(k)\exp\{-\zeta_i\tau\} = A_i u_i(k) + B_i x_i(k), \end{aligned} \tag{2.26}$$

where $A_i = 1-\alpha_i^l$, $B_i = \beta_i^u + \gamma_i^u \exp\{-\zeta_i\tau\}$. Then Lemma 2.5 and (2.24) imply that, for any integer $s \leq k$,

$$\begin{aligned} u_i(k) &\leq A_i^s u_i(k-s) + \sum_{j=0}^{s-1} B_i x_i(k-j-1) \\ &\leq A_i^s u_i(k-s) + \sum_{j=0}^{s-1} B_i \exp\{-\zeta_i(j+1)\} x_i(k). \end{aligned} \tag{2.27}$$

Note that $0 < 1-\alpha_i^l < 1$ and for enough large k, s, which satisfy $k-s \geq K_1$, then $u_i(k-s) \leq 2M$ and $\lim_{s\to+\infty} A_i^s = 0$. Thus, for $k, s \to +\infty$ and $k-s \geq K_1$, $0 \leq A_i^s u_i(k-s) \leq 2A_i^s M \to 0$. Then, there exists a positive integer $K_2 > K_1$ such that for any positive solution of system (1.1), $2(d_i^u + e_i^u)A_i^s M \leq (1/2)r_i^l$, for all $s \geq K_2$ and $i = 1, 2, \ldots, n$. In fact, we could choose $K_2 = \max_i\{|\ln C_i / \ln A_i|\}$, where $C_i = (1/2)r_i^l / 2M(d_i^u + e_i^u)$, $i = 1, 2, \ldots, n$. Fix K_2, for $k > K_2 + K_1$, we get

$$\begin{aligned} u_i(k) &\leq A_i^{K_2} u_i(k-K_2) + \sum_{j=0}^{K_2-1} B_i x_i(k-j-1) \\ &\leq 2A_i^{K_2} M + \sum_{j=0}^{K_2-1} B_i \exp\{-\zeta_i(j+1)\} x_i(k) \\ &\overset{\text{def}}{=} 2A_i^{K_2} M + D_i x_i(k). \end{aligned} \tag{2.28}$$

And so, for $k > K_2 + K_1 + \tau$, we have

$$u_i(k-\eta_i) \leq 2A_i^{K_2} M + D_i x_i(k-\eta_i). \tag{2.29}$$

Substituting (2.28) and (2.29) into the ith equation of system (1.1), this together with (2.25) leads to (note that $2(d_i^u + e_i^u)A_i^{K_2}M \le (1/2)r_i^l$)

$$\begin{aligned}
x_i(k+1) \ge{}& x_i(k)\exp\Bigg[r_i(k)\left(1-\left(\frac{1}{a_i^l}+c_i^u\right)\exp\{-\zeta_i\tau\}x_i(k)\right)\\
&\quad -d_i(k)\left(2A_i^{K_2}M + D_i x_i(k)\right) - e_i(k)\left(2A_i^{K_2}M + D_i x_i(k-\eta_i)\right)\Bigg]\\
\ge{}& x_i(k)\exp\Bigg[r_i(k)\left(1-\left(\frac{1}{a_i^l}+c_i^u\right)\exp\{-\zeta_i\tau\}x_i(k)\right)\\
&\quad -d_i(k)\left(2A_i^{K_2}M + D_i x_i(k)\right) - e_i(k)\left(2A_i^{K_2}M + D_i \exp\{-\zeta_i\tau\}x_i(k)\right)\Bigg]\\
={}& x_i(k)\exp\Bigg[\left(r_i(k) - 2(d_i(k)+e_i(k))2A_i^{K_2}M\right)\\
&\quad -\left(r_i(k)\left(\frac{1}{a_i^l}+c_i^u\right)\exp\{-\zeta_i\tau\} + d_i(k)D_i + e_i(k)D_i\exp\{-\zeta_i\tau\}\right)x_i(k)\Bigg]\\
\ge{}& x_i(k)\exp\Bigg[\left(r_i^l - 2(d_i^u+e_i^u)2A_i^{K_2}M\right)\\
&\quad -\left(r_i^u\left(\frac{1}{a_i^l}+c_i^u\right)\exp\{-\zeta_i\tau\} + d_i^u D_i + e_i^u D_i\exp\{-\zeta_i\tau\}\right)x_i(\mathrm{k})\Bigg]\\
\ge{}& x_i(k)\exp\left[\frac{1}{2}r_i^l - E_i x_i(k)\right],
\end{aligned} \tag{2.30}$$

where $E_i = r_i^u(1/a_i^l + c_i^u)\exp\{-\zeta_i\tau\} + d_i^u D_i + e_i^u D_i \exp\{-\zeta_i\tau\}$.

By applying Lemma 2.4 to (2.30), it immediately follows that

$$\liminf_{k\to+\infty} x_i(k) \ge m_{i1}. \tag{2.31}$$

where $m_{i1} = \min\{(1/2)r_i^l/E_i, ((1/2)r_i^l/E_i)\exp\{(1/2)r_i^l - E_i M\}\}$.

From (2.31), we know that there exists enough large $K_3 > K_2 + K_1 + \tau$ such that

$$x_i(k) \ge \frac{1}{2}m_{i1}, \quad \text{for } k \ge K_3 + \tau. \tag{2.32}$$

This together with the $(n+i)$th equation of (1.1) leads to

$$\Delta u_i(k) \ge -\alpha_i(k)u_i(k) + \frac{1}{2}\left(\beta_i(k)+\gamma_i(k)\right)m_{i1}, \quad \text{for } k \ge K_3 + \tau. \tag{2.33}$$

And so,

$$u_i(k+1) \geq (1-\alpha_i^u)u_i(k) + \frac{1}{2}\left(\beta_i^l + \gamma_i^l\right)m_{i1}, \quad \text{for } k \geq K_3 + \tau. \tag{2.34}$$

Noticing that $0 < 1 - \alpha_i^u < 1$ and applying Lemmas 2.1 and 2.2 to (2.34), we have

$$\liminf_{k\to+\infty} u_i(k) \geq \frac{(1/2)\left(\beta_i^l + \gamma_i^l\right)m_{i1}}{\alpha_i^u} \stackrel{\text{def}}{=} m_{i2}. \tag{2.35}$$

Setting $m = \min_i\{m_{i1}, m_{i2}\}$, the conclusion of Lemma 2.7 follows. This ends the proof of Lemma 2.7. □

Proof of Theorem 1.1. Lemmas 2.6 and 2.7 show that under the assumptions (H_1) and (H_2), for any positive solution $(x(k), u(k)) = (x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_n(k))^T$ of system (1.1), one has

$$\begin{aligned} m &\leq \liminf_{k\to+\infty} x_i(k) \leq \limsup_{k\to+\infty} x_i(k) \leq M, \\ m &\leq \liminf_{k\to+\infty} u_i(k) \leq \limsup_{k\to+\infty} u_i(k) \leq M, \end{aligned} \tag{2.36}$$

where m and M are independent of the solution of system (1.1), thus, system (1.1) is permanent. This ends the proof of Theorem 1.1. □

3. Conclusions

Stimulated by the works of Fan and Wang [4], in this paper, we revisit the model proposed by Chen [3]. We showed that condition (H_3) in [3] is not necessary to ensure the permanence of the system, which means that feedback control variables have no influence on the persistent property of system (1.1).

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