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Inefficient Equilibria in Transition Economy

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The paper studies a general equilibrium in an economy where all market participants face a bid-ask spread. The spread may be caused by indirect business taxes, middlemen rent-seeking, delays in payments or liquidity constraints or price uncertainty. Wherever it comes from the spread causes inefficiency of the market equilibrium. We discuss some institutions that can decrease the inefficiency. One is second currency (barter exchange) in the inter-firm transactions. It is shown that the general equilibrium in an economy with second currency is effective though is still different from Arrow–Debreu equilibrium. Another solution can be introduction of mutual trade credit. In the economy with trade credit there are multiple equilibria that are more efficient than original bid-ask spread but still not as efficient as Arrow–Debreu one, too. The implications for firms' integration and applicability to Russian economy are discussed.

The paper generalizes some results obtained of research work that has been done in the Department of Mathematical Modeling of Economic Systems of Computing Center, Russian Academy of Science under Academician Petrov over last few years (Petrov *et al.*, 1996, *Essays on Mathematical Modelling of Economy:* Energoatomizdat.) We have successfully used some models of inefficient equilibrium in several applied projects.

1 INTRODUCTION

The transition to market economy happening now in more than twenty countries all over the world is one of the most important current economic developments. In most countries the reform has been designed along lines suggested by the International Monetary Fund according to which the government should first liberalize prices, then balance budget in order to achieve financial stabilization which is necessary (and almost sufficient) condition for investment and therefore economic recovery and growth. Some economies have quickly passed the periods of inflation and financial stabilization and have made sustained turnarounds to growth. Yet, in others (especially in former Soviet Union countries) the financial stabilization is achieved but the economic recovery has not yet begun. In Russian economy the inflation has been low and the exchange rate has been stable since 1995.

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However the growth has not been observed yet – the recent period is characterized by stable or slowly declining GDP. It is not the purpose of this paper to provide an answer why there is no growth. Rather, we shall try to suggest a theory that describes the economy at the present stage combining in a static general equilibrium model several issues such as:

- *Inefficiency of market equilibrium* Although Russian economy has been significantly liberalized and almost all price controls are abandoned, the first welfare theorem does not apply. Both objective and subjective measures of welfare are reported to decrease by tens of percents compared with pre-reform standards (Goskomstat, 1992; 1993; 1994; 1995; 1996).
- *Huge nonmonetary transactions* Since mid-95 when the Russian government has tightened the money supply, firms' managers complain about desperate shortage of working capital. In this period, various money substitutes have emerged such as firms' and banks' IOUs and promissory notes, treasury notes etc. Being used extensively in inter-firm transactions, these financial instruments have been referred to as 'quasi-money'. It is not uncommon for a firm's (and to a greater extent for a bank's) IOU to change several hands before being accepted by the issuer.[**] Also a lot of firms are engaged in barter exchanges. It is hard to estimate the volume of nonmonetary transactions. The reports vary from 30% to 80% of inter-firm turnover (Delyagin, 1997; Klepach, 1997; Makarov and Kleiner, 1997). Note that increase in nonmonetary transactions does not mean actual GDP growth – these are almost exclusively used in the inter-firm sales.
- *Neplatezhi* (nonpayments and underpayments[†] of firms to each other, arrears) Although the neplatezhi have been first perceived as an evil of high inflation they still persisted in the recent time and actually rose up to 25% GDP (Goskomstat, 1997).
- *Integration* In the present Russian economy one can clearly see the urge of firms and banks to integrate. Moreover the success of some consortia established formally or informally is quite noticeable. Certainly, this is not transition-specific but still very important.

The welfare theorems claim that under perfect competition the market economy first provides best allocation of factors of production. This feature of perfect competition market is the consequence of basic assumption that for each good there is a single market price so that marginal cost will be equal to marginal utility.

We depart from this assumption building a model in which there is a bid-ask spread every market participant faces. The nature of this bid-ask spread may be different. While building applied-general equilibrium models in transition economy in recent years (see Petrov *et al.*, 1996) we have often discovered situations in which this spread, wherever it stems from, is significant. In Section 3 we point out some of these examples.

In the presence of the spread, one should expect the general equilibrium to be inefficient.[‡] We believe that introduction of quasi-money, barter and underpayment (that we consider to be a system of mutual credit) transactions between firms decrease the inefficiency caused by the spread.

In some sense the paper follows the main idea of new institutional economics, in particular, the transaction costs economics (Coase, 1984; Williamson, 1987). We believe that if there can be institutions (namely barter and underpayments) to decrease the inefficiency of equilibria, such institutions should emerge. There is certain general equilibrium literature that analyzes the structure of financial markets (e.g. Bisin, 1997) or the emergence of a medium of

[**]Usually IOUs are paid back in kind rather than in cash i.e. accepted as a payment for the goods supplied.

[†]The word 'neplatezhi' literally translates as 'nonpayments'. However the neplatezhi refer to situation when some payment (yet below the contract price) is made. So we will use the word 'underpayments' rather than 'nonpayments'.

[‡]Certainly we do not claim the spread to be the only source of inefficiency. The latter may be caused or deepened by other factors such as imperfect competition, information asymmetries and externalities that are very significant in Russian economy.

exchange (Niehans, 1971) from the point of view of transaction costs structure. Our model is different since it is not really a model with transaction costs. The bid-ask spread that we consider is not caused by physical frictions and is in certain way fully appropriated by consumers. Thus the Walras law is still the case. There are also a few papers (e.g. Williamson and Wright, 1994) which compare money and barter taking into account search costs and information asymmetries. Usually these models aim at explaining why money as a medium of exchange emerges in the economy. Our model, though much less sophisticated, is to answer the question why money, still being a universal medium of exchange, can lose the function of unit of account and how the economy works without a common unit of account.

2 THE STATIC MODEL OF CLOSED ECONOMY AND COMPETITIVE GENERAL EQUILIBRIUM (C-EQUILIBRIUM)

Consider an economy with n goods produced, consumed and used in production of each other. There are N independent price-taking firms, which are of main interest to us, and some consumers. We do not intend to discuss here the most general case, so we assume that consumers can be described by concave monotone aggregate utility function $U(\mathbf{c})$ of total consumption $\mathbf{c} = \{c_1, \ldots, c_n\}$.

We consider here the closed economy, i.e. economy without import and export, economy where all prices are determined on home markets. This is certainly not correct nowadays for Russia but qualitative properties of economic mechanisms may be investigated only with the closed model.

We assume that there are two markets: the first ('consumer') market where firms can sell and both firms and consumers can buy goods by prices $\mathbf{p} = \{p_1, \ldots, p_n\}$ and the second ('inter-firm') market for inter-firm transactions only. Below we will put different assumptions of the mechanisms of the inter-firm market.

Denote by $\mathbf{x}^\nu = \{x_1^\nu, \ldots, x_n^\nu\}$ and $\mathbf{y}^\nu = \{y_1^\nu, \ldots, y_n^\nu\}$, sales of goods $1, \ldots, n$ of the firm

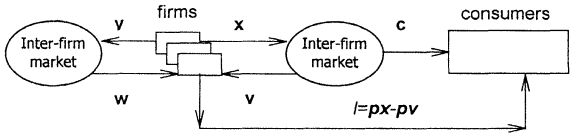


FIGURE 1

$\nu = 1, \ldots, N$, in the first and the second markets and by $\mathbf{v}^\nu = \{v_1^\nu, \ldots, v_n^\nu\}$ and $\mathbf{w}^\nu = \{w_1^\nu, \ldots, w_n^\nu\}$, purchases of the firm on these markets (see Fig. 1). In a closed economy sales and purchases of agents in each market should be in a balance:

$$\sum_\nu \mathbf{x}^\nu = \sum_\nu \mathbf{v}^\nu + \mathbf{c}, \tag{1}$$

$$\sum_\nu \mathbf{y}^\nu = \sum_\nu \mathbf{w}^\nu. \tag{2}$$

We consider static economy, i.e. economy without growth of capital. It meets the present situation in Russia where as is known, the industrial investments ensuring growth are so insignificant that hardly cover capital depreciation.

We assume working capital in the form of raw materials, fuel etc. to be the only limiting factor of production. It also meets the modern Russian economy where constant capital labor and natural resources are in excess supply.

These two assumptions allow to describe production possibilities of a firm ν by constant *technological set* T^ν. Each vector $\mathbf{z} = \{z_1, \ldots, z_n\} \in T^\nu$ corresponds to some possible combination of production inputs and outputs. Positive component of vector z represents net output of corresponding good, negative component represents net input. Following the neoclassic theory of production we assume T^ν to be strictly convex, closed and compact. We also require that a firm cannot produce anything without expenditures but can produce nothing without expenditures ($\mathbf{z} \geq 0$ and $\mathbf{z} \in T^\nu \Leftrightarrow \mathbf{z} = \mathbf{0}$). The last is not so trivial as it may seem. For doing nothing some of Russian industrial enterprises have to spend on heating, lighting and so on, up to one-third of expenditures they pay working at full capacity.

Technological set restricts net sales of a firm ν:

$$\mathbf{x}^\nu + \mathbf{y}^\nu - \mathbf{v}^\nu - \mathbf{w}^\nu \le \mathbf{z}^\nu;\ \mathbf{x}^\nu, \mathbf{y}^\nu, \mathbf{v}^\nu, \mathbf{w}^\nu \ge \mathbf{0};\quad \mathbf{z}^\nu \in T^\nu. \tag{3}$$

Comparison of (3) with (1) and (2) shows that all possible vectors of consumption lie in the *aggregate technological set*

$$\mathbf{c} \in T = \sum_\nu T^\nu = \left\{ \mathbf{z} \,|\, \mathbf{z} = \sum_\nu \mathbf{z}^\nu,\ \mathbf{z}^\nu \in T^\nu \right\}. \tag{4}$$

We assume that $\mathbf{0}$ is internal point of T i.e. the economy as a whole can produce positive quantity of all goods for final consumption.

General equilibrium is purified description of the result of self-adjustment of a market economy. It is defined by four basic hypothesis which, for our model, have the following form:

- Consumers maximize utility of consumption under budget constraint.

$$U(\mathbf{c}) \Rightarrow \max \text{ over } \mathbf{c} \ge 0 \text{ subject to } \mathbf{pc} \le I, \tag{5}$$

  where I is the total income (budget) of consumers and $\mathbf{p} = \{p_1, \ldots, p_n\}$ is a vector of consumer prices of goods. The solution of the problem (5) determines aggregate *demand* which will be a function of prices,

$$\mathbf{c} = \mathbf{c}(\mathbf{p}, I). \tag{6}$$

- Each firm ν maximizes its net *real* income (profit) P^ν under technological constraint (3):

$$P^\nu \Rightarrow \max \text{ over } \mathbf{x}^\nu, \mathbf{y}^\nu, \mathbf{v}^\nu, \mathbf{w}^\nu, \mathbf{z}^\nu \text{ subject to (3)}. \tag{7}$$

  Explicit expression of P^ν depends on mechanism of payments under consideration. The cases considered below will formally differ from each other by the form of expressions of P^ν and some additional restrictions. Solution of the problem (7) determines *supply* functions of producers.

- 'Invisible hand' of market puts prices at the level balancing demand and supply so that (1), (2) will hold.
- Income I of consumers should be equal to total net *nominal* income of producers (Walras law)

$$\sum_\nu (\mathbf{px}^\nu - \mathbf{pv}^\nu) = I = \mathbf{pc}. \tag{8}$$

We start with perfect competition case (Arrow–Debreu economy, see for example Nikaido, 1968) which we will take as a benchmark. Under perfect competition each good costs the same price for any agent in any transaction. In particular the prices in the inter-firm market are the same as at consumer market and net income of the firm ν is

$$P^\nu_{\mathrm{C}} = \mathbf{px}^\nu - \mathbf{pv}^\nu + \mathbf{py}^\nu - \mathbf{pw}^\nu. \tag{9}$$

General competitive equilibrium in our model is the state of economy $\{\mathbf{p} = \mathbf{p}_{\mathrm{C}}, I = I_{\mathrm{C}}, \mathbf{c} = \mathbf{c}_{\mathrm{C}}, \mathbf{x}^\nu = \mathbf{x}^\nu_{\mathrm{C}}, \mathbf{y}^\nu = \mathbf{y}^\nu_{\mathrm{C}}, \mathbf{v}^\nu = \mathbf{v}^\nu_{\mathrm{C}}, \mathbf{w}^\nu = \mathbf{w}^\nu_{\mathrm{C}}, \mathbf{z}^\nu = \mathbf{z}^\nu_{\mathrm{C}}\}$ which satisfies (1), (2), (5) and (8) providing that $\mathbf{x}^\nu, \mathbf{y}^\nu, \mathbf{v}^\nu, \mathbf{w}^\nu, \mathbf{z}^\nu$ maximize (9) subject to (3).

Maximization of (9) under (3) over $\mathbf{x}^\nu, \mathbf{y}^\nu, \mathbf{v}^\nu, \mathbf{w}^\nu, \mathbf{z}^\nu$ gives multiple solution because (a) under perfect competition two markets are equivalent and firm can chose any of them for planned transaction; (b) in the absence of transaction cost, firm may sale goods to itself and the mathematical model describes this formal possibility. Since the slightest transaction cost will diminish turnover to minimal necessary value and the slightest difference of market condition will transfer all transactions to more preferable market, we (for perfect competition case only) exclude inter-firm transactions and sales of firm to itself by additional requirements,

$$\mathbf{w}^\nu_{\mathrm{C}} = \mathbf{y}^\nu_{\mathrm{C}} = \mathbf{0};\quad \mathbf{w}^\nu_{\mathrm{C}} \cdot \mathbf{v}^\nu_{\mathrm{C}} = 0. \tag{10}$$

We will refer this state of economy as *C-equilibrium*.

The definition above characterizes equilibrium in a local, decentralized way as a balance of rational decision made be independent agents. It is easy to characterize C-equilibrium globally from the view point of the whole system.

Maximization of (9) by $\mathbf{x}^\nu$ turns inequality in (3) into equality $\mathbf{z}^\nu = \mathbf{x}^\nu + \mathbf{y}^\nu - \mathbf{v}^\nu - \mathbf{w}^\nu$ and P_C^ν becomes equal to $\mathbf{p}_C\mathbf{z}^\nu$. Further maximization of profit by $\mathbf{z}^\nu$ over T^ν determines $\mathbf{z}_C^\nu$ as the only boundary point of T^ν with normal vector $\mathbf{p}_C$. Sum of these points over ν gives $\mathbf{c}_C$ (see (4)) which will be the boundary point of T^ν with the same normal vector $\mathbf{p}_C$. On the other hand the solution of (5) requires that $\mathbf{p}_C$ be proportional to the gradient of utility function U at $\mathbf{c}_C$. The only point where normality to T is proportional to the gradient of U is the point $\mathbf{c}_C$ where U reaches its maximum value over the whole T. This gives global characterization of C-equilibrium.

Equilibrium prices are simply the vector normal to T at $\mathbf{c}_C$. According to (10) sales $\mathbf{x}_C^\nu$ and purchases $\mathbf{v}_C^\nu$ are determined uniquely as positive and negative parts of previously found $\mathbf{z}_C^\nu$. Multiplication of (1) by $\mathbf{p}_C$ shows that (8) holds.

To compare C-equilibrium with other types of equilibria considered below it is convenient to characterize each state of economy by utility of consumption $U(\mathbf{c})$ and some scalar measure E of gross expenditures of firms (sum of negative components of $\mathbf{z}_C^\nu \sum_\nu (\mathbf{z}_C^\nu)_-$). This correspondence maps a set of possible states (state of economy (1), (2) and (3)) onto two-dimensional set $\tilde{T}$, represented in Fig. 2. Suppose that measure of expenditures is chosen so that the boundary of $\tilde{T}$ is mapped on the boundary of T. Since C-equilibrium maximizes utility it will be represented by the top point of $\tilde{T}$.

The arguments above show that

- C-equilibrium exists and corresponding material flows $\mathbf{c}_C, \mathbf{x}_C^\nu, \mathbf{y}_C^\nu$ are determined *uniquely*.
- Equilibrium prices $\mathbf{p}_C$ and income I_C are determined uniquely up to an arbitrary positive *price scaling* factor.
- Consumption reaches north-east boundary of technological set at C-equilibrium. This means that the consumption $\mathbf{c}_C$ cannot be increased by all its components at once. This property of competitive equilibrium is referred to as *efficiency*. Being in effective equilibrium, economy will completely use current production factors and must feel the lack of either labor, or fixed capital, or natural resources. Such economy will demonstrate some tendency to investments.

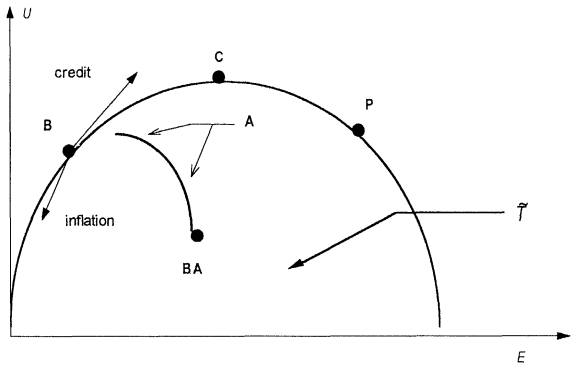


FIGURE 2

We do not see any will to invest in modern Russian economy. On the contrary, we hear complaints on desperate shortage of current production factors. It induces on an idea, that in Russian economy some inefficient equilibrium is realized.[§]

3 INEFFECTIVE EQUILIBRIUM WITH BID-ASK SPREAD (BA-EQUILIBRIUM)

The C-equilibrium effective due to the fact that all agents estimate any good by the same price. Now we depart from this hypothesis and consider economy where market participants face bid-ask spread i.e. buyer of some good pays more than its seller receives. Let us first provide some examples of economic relationships in which the difference between buying p^b and selling p^s prices emerges.

1. *Excises:* The simplest example is sales tax or excise. Let market price of some good be p and the tax rate set at n percent. If it is the seller who pays the tax the price for buyer is still $p^b = p$ while the price for seller is only $p^s = (1-n)p$. If the tax is paid by buyer then $p^b = (1+n)p$ and $p^s = p$.

[§]We mean under equilibrium in the general plan the coordination of volumes of production and consumption in economy by means of special information variables (usually prices).

2. *Middlemen profit:* In a competitive economy the difference between purchasing and selling prices of trade intermediaries is a reward for trade services that should increase the consumer value of the good. It is thus possible to consider trade services as a special good produced by middlemen. Then there should be no bid-ask spread. However the price differentials in Russian trade are so high and the corruption and racket are so common that it is rather reasonable to assume that most of the price differentials is rather rent than normal profit. This rent is received by those in ownership or control of the sales channels: corrupted bureaucrats, organized crime and managers abusing their firms' interests. Such view makes the price differentials in trade similar to taxes.

Note that expropriation of producer income by both government and the intermediaries does not imply that the money disappears from the economy. Both tax payments and rent in closed economy contribute to households' incomes and increase the aggregate demand so that Walras law (8) still holds.

3. *Delays in payments:* A less explicit reason which brings about the bid-ask spread can be delays in payments in an economy with high inflation (Guriev and Pospelov, 1994). If the seller receives sales revenues with delay τ and prices grow at the rate γ then the seller evaluates unit of product at $p^{s} = e^{-\gamma\tau}p$. Note that payment-in-advance principle does not help – in this case $p^{s} = p$ but $p^{b} = e^{\gamma\tau}p$. This factor was significant in Russian in 1991–92 when delays in payments were about some weeks while inflation was as high as 20% a month. It gives factor $e^{\gamma\tau} \approx$ 10–15%.

4. *Liquidity constraints:* Even less obvious case is account for liquidity (or cash-in-advance) constraints in a dynamic model with cash flow discounting. Suppose that a firm sell output X at price p and buys inputs V at price q. Then the cash holdings M change over time according to

$$\frac{dM}{dt} = p(t)X(t) - q(t)V(t) - F(t),$$

where F are withdrawn earnings. The firm needs to hold some cash balances for transaction purposes (liquidity constraint):

$$M(t) \geq \theta q(t)X(t), \tag{11}$$

where θ is a certain time constant. Let the firm maximizes discounted withdrawn earnings

$$\int_{t}^{\infty} e^{-\delta(\xi - t)}F(\xi) \cdot d\xi.$$

Here δ is pure time preference rate which may be interpreted as a reciprocal to average planning horizon (see e.g. Blanchard and Fischer, 1989). It is easy to show (see Pospelov, 1995) that if the firm expects prices to be constant over time $p(\xi) = p(t)$, $q(\xi) = q(t)$, the level of production (inputs V and outputs X) should be set to maximize

$$\frac{1}{1 + \delta\theta}p(t)X - q(t)V \tag{12}$$

rather than profit $p(t)X - q(t)V$. This actually means that the producer evaluates the product at price $p^{s} = p(1 + \delta\theta)^{-1}$ and leads to lower level of both inputs and outputs. This result may seem counterintuitive since under constant prices the cash holdings M do not change and the instantaneous cash inflow or withdrawn earnings equals profit rather than (12). The point is that by decreasing both input and output the firm is able to withdraw some cash and this one-time withdrawal can overweight discounted permanent increase in cash inflows in future. Withdrawing money a firm then will explain underproduction by shortage of money.

There is a lot of direct and indirect evidence that many entrepreneurs in Russian as well as other transition economies look at short-term perspective so that discount rate δ may be quite large. Meanwhile, the liquidity constraints are also reported to be very restrictive as they should be in the absence of well developed working capital market. Both factors increase importance of this effect.

5. *Price uncertainty:* Another conventional example of bid-ask spread is difference between effective buying and selling prices when the price

is uncertain. Even if both buyer and seller have the same information about the probability distribution of price the perceived buying and selling prices will differ: $p^s < Ep < p^b$, where Ep is expected price (e.g. see the firm's model in Demers and Demers, 1990).

The examples of bid-ask spread given above can be described as follows. We again assume that there are some market prices p the same for both markets. However due to the reasons above the objective function of firm does not equal (9) i.e. the difference between gross sales and gross purchases. Due to distortive taxation or rent-seeking by middlemen or liquidity constraint or delays in payments etc. the firms evaluate the sales at prices generally lower than p and purchases at prices higher than p. We assume that producer maximizes

$$P^\nu_{BA} = \mathbf{p}A\mathbf{x}^\nu - \mathbf{p}B\mathbf{v}^\nu + \mathbf{p}A\mathbf{y}^\nu - \mathbf{p}B\mathbf{w}^\nu, \qquad (13)$$

where A and B are given nonnegative constant diagonal matrices. All diagonal elements a_i of matrix A are less or equal than 1 and all diagonal elements b_i of matrix B are greater or equal than 1 and $a_i < b_i$:

$$A \le E \le B, \quad A < B.$$

All results below were proved in more general and realistic case when matrices A and B are different for different firms (A and B depend on ν). For description of middleman profit it might be better to consider A and B depending on price/volume of transaction and we have investigated some of such cases also. But the simplest assumption (13) is quite enough to illustrate and discuss the principle results.

General equilibrium with bid-ask spread in our model is the state of economy

$$\begin{cases} \mathbf{p} = \mathbf{p}_{BA}, \ I = I_{BA}, \ \mathbf{c} = \mathbf{c}_{BA}, \\ \mathbf{x}^\nu = \mathbf{x}^\nu_{BA}, \ \mathbf{y}^\nu = \mathbf{y}^\nu_{BA}, \ \mathbf{v}^\nu = \mathbf{v}^\nu_{BA}, \ \mathbf{w}^\nu = \mathbf{w}^\nu_{BA}, \ \mathbf{z}^\nu = \mathbf{z}^\nu_{BA} \end{cases}$$

which satisfies (1), (2), (5) and (8) providing that $\mathbf{x}^\nu, \mathbf{y}^\nu, \mathbf{v}^\nu, \mathbf{w}^\nu, \mathbf{z}^\nu$ maximize (13) subject to (3). To make the solution of the last problem unique we again exclude the second market by requirement[¶]

$$\mathbf{w}^\nu_C = \mathbf{y}^\nu_C = 0.$$

We will refer this state of economy as *BA-equilibrium.*

It may be proved that

- BA-equilibrium exists[#] and corresponding material flows $\mathbf{c}_{BA}, \mathbf{x}^\nu_{BA}, \mathbf{y}^\nu_{BA}$ are determined *uniquely*. If A is too small or B is too large BA-equilibrium may be degenerated: $\mathbf{c}_{BA} = \mathbf{0}$.
- Equilibrium prices $\mathbf{p}_C$ and income I_C are determined uniquely up to an arbitrary positive *price scaling* factor.
- Points $\mathbf{z}^\nu_{BA}$ maximizing profit (13) lie on the boundaries of T^ν but have different normal vectors for different ν. Hence their sum $\mathbf{c}_{BA}$ cannot be on the boundary of aggregate set T. BA-equilibrium is *ineffective*. Each enterprise works as hard as possible but suffers from shortage of working capital and economy as a whole and does not use completely labor, fixed capital and natural resources. Such economy will not demonstrate any tendency to investments.
- Both gross sales and gross expenses at BA-equilibrium will be less if compared with C-equilibrium: $\sum_\nu \mathbf{x}^\nu_{BA} < \sum_\nu \mathbf{x}^\nu_C, \sum_\nu \mathbf{v}^\nu_{BA} < \sum_\nu \mathbf{v}^\nu_C$. BA-equilibrium is placed within the set $\tilde{T}$ to the left-bottom of C-equilibrium (point 'BA' in Fig. 2).

Generally speaking, the inefficiency not always means smaller than in C-equilibrium issues and expenses. Studying mechanisms of functioning of a centrally planned economy (Petrov *et al.*, 1996), we came to a conclusion that it may be described by a specific P-equilibrium. P-equilibrium is characterized by shortage of goods of normal quality, overproduction and excessive inter-firm transactions. This 'P-equilibrium' is represented by point 'P' on Fig. 2. Thus it is possible to say, that during reforms

[¶]Here we need not exclude sales to itself (see (10)). Due to transaction costs A, B maximization of profit cats excessive turnover automatically.

[#]It may be proved by standard way using well Known Gale's lemma (see Nikaido, 1968).

Russia has passed by the desired point 'C' and from inefficient planning 'P' to the inefficient market 'BA'.

4 EQUILIBRIUM WITH BARTER EXCHANGE OR SECOND CURRENCY (B-EQUILIBRIUM)

The losses of enterprises in BA-equilibrium are connected with money payments. So one could expect emergence of compensating mechanisms, reducing money transactions. In Russia two such mechanisms are steadily observed. They are barter and *nonpayments* (actually underpayments). Usually the problems of barter and nonpayments are treated as one. We will separately address the issues and will show that the equilibria in these two cases are very different and hence deserve different treatments. We start with more simple case of a barter exchange. Certainly we study the idealized barter just as we considered the idealized case of perfect competition above.

Our first assumption of barter exchange is that it takes place only in the second inter-firm market which was actually put aside as above. On the other hand if a firm wishes to sell or to buy for money it should go to the first market where it still faces transaction costs A and B. So the vectors $\mathbf{y}^\nu$ and $\mathbf{w}^\nu$ represent sales and purchases of the firm ν by barter transactions. Restriction (3) does not exclude pure gamble when the enterprise buys some goods by barter not for manufacturing but to sell it for money or quite conversely.

Our second assumption is that there exists single set of barter exchange ratios (*barter prices*) $\mathbf{q} = \{q_1, \ldots, q_n\}$ in the inter-firm market and any barter transaction is fair if measured in these prices:

$$\mathbf{q}\mathbf{y}^\nu = \mathbf{q}\mathbf{w}^\nu. \tag{14}$$

This condition may be treated as budget constraint (balance sheet of account) of the firm for the second currency (compare with budget constraint of consumers in (5)). The second currency is of no interest for fiscal institutions and racketeers, so there is no transaction cost. The second currency is actually emitted by a firm when it is necessary so that no liquidity constraint occurs (see (11)). Barter exchanges contribute nothing to firm's income, hence firm's behavior may be described as maximization of

$$P_{\mathrm{B}}^\nu = \mathbf{p}A\mathbf{x}^\nu - \mathbf{p}B\mathbf{v}^\nu \tag{15}$$

subject to (3) and (14).

Note that Walras law for usual money (8) still holds. Moreover summation of (14) over ν states Walras law for second currency too.

General equilibrium with barter exchange in our model is the state of economy

$$\begin{cases} \mathbf{p} = \mathbf{p}_{\mathrm{B}},\ \mathbf{q} = \mathbf{q}_{\mathrm{B}},\ I = I_{\mathrm{B}},\ \mathbf{c} = \mathbf{c}_{\mathrm{B}}, \\ \mathbf{x}^\nu = \mathbf{x}_{\mathrm{B}}^\nu,\ \mathbf{y}^\nu = \mathbf{y}_{\mathrm{B}}^\nu,\ \mathbf{v}^\nu = \mathbf{v}_{\mathrm{B}}^\nu,\ \mathbf{w}^\nu = \mathbf{w}_{\mathrm{B}}^\nu,\ \mathbf{z}^\nu = \mathbf{z}_{\mathrm{B}}^\nu \end{cases}$$

which satisfies (1), (2), (5) and (8) providing that $\mathbf{x}^\nu, \mathbf{y}^\nu, \mathbf{v}^\nu, \mathbf{w}^\nu, \mathbf{z}^\nu$ maximize (15) subject to (3) and (14). Absence of transaction cost in barter market again allows selling to itself. We exclude them by additional requirement**

$$\mathbf{w}_{\mathrm{C}}^\nu \cdot \mathbf{y}_{\mathrm{C}}^\nu = 0.$$

We will refer this state of economy as *B-equilibrium*.

Maximization of P_{B}^ν determines supply and demand of firms in both markets as a function of two systems of prices: $\mathbf{p}$ and $\mathbf{q}$. Relations (1) and (2) give two systems of equations for these prices. Barter prices may be excluded by the following trick. Consider all firms as one enterprise seeking to maximize the total profit

$$P_{\mathrm{B}} = \sum_\nu (\mathbf{p}A\mathbf{x}^\nu - \mathbf{p}B\mathbf{v}^\nu) \tag{16}$$

under given prices $\mathbf{p}$ subject to individual constraints (3) and common balance constraint on

**Here we need not exclude sales to itself (see (10)). Due to transaction cost A, B maximization of profit cats excessive turnover automatically.

inter-firm transaction (2). The solution of this problem defines demand and supply of firms in the first market as a function of prices $\mathbf{p}$ only and also defines Lagrange multipliers to the common balance constraint (2). It is easy to prove that the latter ones may be taken as barter prices $\mathbf{q}$ and (14) will hold.

Having demand and supply of firms as functions of $\mathbf{p}$ we then state existence of equilibrium prices $\mathbf{p}_B$ by usual way. More detailed study in the direction pointed out shows that:

- B-equilibrium exists and corresponding material flows $\mathbf{c}_B, \mathbf{x}_B^\nu, \mathbf{y}_B^\nu$ are determined *uniquely*.
- Equilibrium prices $\mathbf{p}_B$, income I_B and barter prices $\mathbf{q}_B$ are determined uniquely up to *two* arbitrary positive *price scaling* factors: one for $\mathbf{p}_B$, I_B and another independent factor for $\mathbf{q}_B$.
- Firms do not buy for money in B-equilibrium. All their demand is satisfied by barter exchanges: $\mathbf{v}_B^\nu = 0$.
- Points $\mathbf{z}_B^\nu$ maximizing profit (15) lie on the boundaries of T^ν and have the same t normal vectors proportional to $\mathbf{q}_B$. Hence the sum of $\mathbf{z}_B^\nu$ put on the boundary of aggregate set T and B-equilibrium is *effective*, but differs from C-equilibrium.
- Both gross sales and gross expenses at B-equilibrium will be less if compared with C-equilibrium: $\sum_\nu \mathbf{x}_B^\nu < \sum_\nu \mathbf{x}_C^\nu$, $\sum_\nu \mathbf{v}_B^\nu < \sum_\nu \mathbf{v}_C^\nu$. B-equilibrium is placed on the boundary of $\tilde{T}$ to the left of C-equilibrium (point 'B' on Fig. 2).

The efficiency of B-equilibrium seems to point out that the well organized legalized barter could be capable of solving a problem of bid-ask spread. However closer consideration of the second and third properties shows fatal defects of the barter mechanism.

Consider a firm which does not produce goods of consumer demand. According to the third property such a firm cannot sell its output for money in B-equilibrium. If we ask a question, how this firm could get money, the answer is that in B-equilibrium the firm which cannot produce goods of consumer will have positive income. It will exchange its output on goods of consumer demand by barter and then resell the latter for money.

Thus, barter actually reduces to commodity money (gold, petrol, vodka etc.) with all inherent defects of such means of payment: fluctuations of consumer demand, issue not correlated with general level of development of economy etc. Let us remind that the incompatibility of commodity money with qualitative and quantitative economic growth has resulted in the beginning XX century in complete replacement of gold by modern credit money.

The specified defect of a barter exchange is aggravated by independence of scales of the market and barter prices. To legalize barter it would be necessary to double book keeping, which even for single money represents a serious economic problem. All this shows that one should not hope that barter could solve a problem of inefficiency of modern Russian market.

5 BARTER AND BANK CREDIT

It is necessary to tell some words about relations of barter and bank credit. This question is not yet investigated in general, but some conclusions can be made of research of particular models presented in Guriev and Pospelov (1994), and Pospelov (1995).

Credit money like barter is issued by necessity and so takes away liquidity constraint (see Error! Reference source not found Section 3). On the other hand credit unlike barter is measured in common currency and so no problem of twice book keeping arises. That is why cheap enough credit may play the role of second currency in diminishing inefficiency of market. The wide spread reliable credit both industrial and consumer, long since existing in the western countries, is probably the reason why barter never spread widely in western economies. This in turn is the reason why barter has been mainly dropped out of sight of the economic theory. However studying Russian economy, one should not ignore the possibilities of barter. Credit investments in Russia make only 10% GDP while in Germany they make 110% GDP and even in Czechia they reach 60% GDP.

Credit money have also some disadvantages as compared with idealized second currency. Credit money are of interest for racketeers and, what is more important, credit, being measured in common currency, may come to consumer market and provoke inflation. So we have complex contradictory situation representing schematically in Fig. 2 by arrows at point 'B'. On the one hand cheap credit removing liquidity constraints reduces inefficiency according to Error! Reference source not found, Section 3; on the other hand pushing forward inflation cheap credit increases inefficiency according to Section 3.

The end result of these opposite tendencies may be characterized as follows. If distortion of economy is slight (point 'B' is near to 'C' on Fig. 2) and hence growth of production does not increase utility of consumption the negative effect dominates and one should expect further development according to monetarist reasoning: additional money push inflation which suppresses production. If distortion is serious (point 'B' is far from 'C' on Fig. 2) and growth of production would be desirable for consumers the positive effect dominates and development will correspond Keinsian reasoning: additional money stimulate production in spite of inflation.

Moreover models predict possibility of the so-called 'deflation shock'. When crisis is deep (point 'B' is far from 'C' on Fig. 2) the tight credit policy suppress production so that output falls faster than money mass and inflation even grows. Some Russian economists, in particular G. Yavlinsky spoke of repeated deflation shocks in Russia in spring of 1992, 1993 and 1994. Experiments with our model confirm this opinion (Petrov *et al.*, 1996).

Questions above are undoubtedly an interesting and urgent subject for further research.

6 EQUILIBRIUM WITH ARREARS OR MUTUAL CREDIT OF ENTERPRISES (A-EQUILIBRIUM)

Notorious nonpayments (underpayments, arrears, inter-firm defaults) widely spread in Russian economy represent alternative to barter mechanism of reducing money payments. While perfect competition market is investigated completely and barter mechanisms are rather transparent, arrears remain questionable. There are only few works that provide formal models of the underpayments. While building such a model the economist faces at least one challenge. It is not hard to explain why the buyer is not able (or willing) to pay the full price. It is less evident how to explain why the seller agrees to sell at a lower price and if he does why pay him at all?

We are aware of several answers to this question. The most apparent one is described in Grigoriev, 1997. It essentially relies upon assumption of imperfect competition and suggests that the seller needs to discriminate buyers by prices for which the underpayments are a convenient instrument. The paper solves for equilibrium in case of asymmetric information so the underpayments are used as screening device in a principal/agent model. The drawback of the model is the exogenous parameter that measures the danger of being too indebted (probability of being announced insolvent). This means the model does not deal with the other key point of the payment crisis: If the seller agrees to sell at lower prices why to pay at all?

The other approach (Perotti, 1994; Pospelov, 1995) assumes some corporate (or, should one say, collusive) spirit that firms as a whole possess. In these models the Firms use the underpayments as an instrument for getting low-interest credit from outside lenders such as government. Being indebted the firm expects that the government is going to help and provide some cheap money. This case did take place in 1992–94. The key point here is that it is the buyer who gets the cheap money and it is the seller who is underpaid. However every firm is both a buyer and a seller in some transactions so the externality issue is resolved by cooperative behavior.

Then, there is a couple of papers that use general equilibrium models. The advantage of general equilibrium approach is obvious since in an economy with inter-firm arrears the partial equilibrium clearly misses certain some important points. The

paper by Kim and Kwon, 1995 suggests a general equilibrium model of an economy with rigid technological structure. There are n firms with linear production technology and working capital as limiting factor. The firm i buys from firm $i-1$ and supplies to firm $i+1$ and is subject to a firm-specific productivity shocks. Although our model is much less sophisticated than one by Kim and Kwon (1995) with uncertainty and dynamic setting, our analysis is more general in the sense that we do not make any special assumptions about the production sets beyond conventional ones. In Granville *et al.* (1996) all firms act under given rate of underpayments and given exogenous parameter of disutility of being indebted like the one described above. Then the underpayment rate is to be determined from equilibrium conditions.

Our model is similar to the later works though we explicitly state to which extent firms accept underpayments and allow different levels of underpayments for different goods (which really happens). Our approach is also compatible with the idea of corporate behavior but does not take into account monopolistic aspects.

Our answer to the question above is that a firm sells at lower price in order to be able to buy at lower price. A firm explains to its suppliers that it cannot pay because consumers did not pay to it.

We assume arrears are allowed only in inter-firm market. The stronger assumption is that there exist constant rates of underpayment for each good $\mathbf{s}=\{s_1,\ldots,s_2\}$ same for all firms. Nominal price in inter-firm market are $\mathbf{p}$, but actually a firm pays for unit of good i only p_i-s_i dollars. The remaining s_i dollars per unit of good i are recorded as an asset (debtors receivable) to seller and as liability (accounts for payment) to buyer's balance sheets. Monetary payments $\mathbf{p}-\mathbf{s}$ face the same transaction costs A and B as in consumer market.

So we consider arrears as free of interest mutual credit of firms and require that each firm holds the *solvability constraint*

$$\mathbf{s}\mathbf{y}^{\nu}\geq\mathbf{s}\mathbf{w}^{\nu}. \tag{17}$$

which means that its liabilities on inter-firm transaction do not exceed its assets or, in other words, that its own underpayments are justified by underpayments to it. Statistical data shows that (17) fulfilled for majority of Russian enterprises at least up to the middle of 1997.

Now the behavior of firms may be described as maximization of profit

$$P^{\nu}_{\mathrm{A}}=\mathbf{p}A\mathbf{x}^{\nu}-\mathbf{p}B\mathbf{v}^{\nu}+(\mathbf{p}-\mathbf{s})A\mathbf{y}^{\nu}-(\mathbf{p}-\mathbf{s})B\mathbf{w}^{\nu} \tag{18}$$

subject to (3) and (17). The presence of 'second prices' $\mathbf{s}$ in (18) does not allow to interpret (17) as budget constraint similar to (14) and differs radically mutual credit from second currency.

General equilibrium with arrears in our model is the state of economy

$$\begin{cases}\mathbf{p}=\mathbf{p}_{\mathrm{A}},\ \mathbf{s}=\mathbf{s}_{\mathrm{A}},\ I=I_{\mathrm{A}},\ \mathbf{c}=\mathbf{c}_{\mathrm{A}},\\ \mathbf{x}^{\nu}=\mathbf{x}^{\nu}_{\mathrm{A}},\ \mathbf{y}^{\nu}=\mathbf{y}^{\nu}_{\mathrm{A}},\ \mathbf{v}^{\nu}=\mathbf{v}^{\nu}_{\mathrm{A}},\ \mathbf{w}^{\nu}=\mathbf{w}^{\nu}_{\mathrm{A}},\ \mathbf{z}^{\nu}=\mathbf{z}^{\nu}_{\mathrm{A}}\end{cases}$$

which satisfies (1), (2), (5) and (8) providing that $\mathbf{x}^{\nu},\mathbf{y}^{\nu},\mathbf{v}^{\nu},\mathbf{w}^{\nu},\mathbf{z}^{\nu}$ maximize (18) subject to (3) and (17). Since both markets face participants against transaction costs no additional conditions are required. We will refer this state of economy as *A-equilibrium*.

Investigation of A-equilibrium requires specific mathematical technique. A-equilibrium exists but, contrary to C-, BA-, and B-equilibrium considered above, occurs to be *essentially not unique*.

Of course one may multiply $\mathbf{p}_{\mathrm{A}},\mathbf{s}_{\mathrm{A}},I_{\mathrm{A}}$ by a common positive factor not breaking A-equilibrium because it is equivalent to changing of currency unit. But one cannot scale $\mathbf{s}_{\mathrm{A}}$ independently on $\mathbf{p}_{\mathrm{A}}$ like in B-equilibrium because $\mathbf{s}_{\mathrm{A}}$ should remain less than $\mathbf{p}_{\mathrm{A}}$ or maximization of (18) will generate infinite demand and equilibrium will be broken. On the other hand conditions of A-equilibrium like conditions of B-equilibrium do not suffice to determine second prices by given first prices $\mathbf{p}$ uniquely.

The result is that one can arbitrarily fix by one component each of $\mathbf{s}$ and $\mathbf{p}$, say s_1 and p_1, $0 \le s_1 \le p_1$ and A-equilibrium will be determined definitely. Different values of s_1 under fixed p_1 give A-equilibria with different proportion of prices and different real values $\mathbf{c}_A, \mathbf{x}_A^\nu, \mathbf{y}_A^\nu, \mathbf{v}_A^\nu, \mathbf{w}_A^\nu, \mathbf{z}_A^\nu$. Changing s_1 we will obtain the whole line of A-equilibria (see Fig. 2).

This line begins with BA-equilibrium which is a particular case of A-equilibrium at $\mathbf{s}_A = \mathbf{0}$ and come to its end at some point where one of the components of $\mathbf{s}_A$ becomes equal to corresponding component of $\mathbf{p}_A$.

All these A-equilibria are *ineffective* (lie within $\tilde{T}$). However calculations made with abstract data show that the line of A-equilibria in the beginning (at small $\mathbf{s}_A$) goes toward C-equilibrium then close to boundary of $\tilde{T}$, turns abruptly toward B-equilibrium and only after that ends within $\tilde{T}$ (see Fig. 2). Utility of consumption in ineffective A-equilibrium in the region of turning point occurred to be more than in effective B-equilibrium.

Of course it is impossible to check this property empirically because in real economy rates of underpayment changes together with other indicators of economic conjuncture. However it is possible to carry out the test with the model.

In Computing Center of Russian Academy of Sciences, the working group under the head of the academician A. Petrov with active participation of the authors of the article has developed a mathematical model of economy of Sverdlovsk region. The model have been developed by the order of Regional Senior Management of Central Bank of Russia. This model nowadays is well verified. It reproduces dynamics of more than 50 basic indicators of regional economy with correlation more than 75% and accuracy less than 10%. The principal scheme of the model will be published in Guriev *et al.*, 1998. This model, in particular, takes into account underpayments and reaction of enterprises on them according to the scheme considered above. The regional economy is opened and prices on regional market are not determined by regional demand and supply. Prices as well as rates of underpayments are exogenous parameters of the model of regional economy. However it is possible to examine change of economic indicators depending of rate of underpayments. Figure 3 represents one of such results.

Each point in Fig. 3 corresponds the average level of consumption which would be in Sverdlovsk region according to the model calculations if the share of underpayments (s/p) in the price of fuel[†] p had the value pointed out at the horizontal axis. Cross marks the actual level of consumption vs. actual share of underpayments ($\approx$27.5%).

It is interesting to see how the growth of the share of underpayments at first stimulates (consumption grows) then suppresses (consumption falls) and at last ruins economy. (Disorder of points at the right-hand side of diagram corresponds to instability of economy predicted by the model at large rate of underpayments.) Qualitatively the picture corresponds to the trend of the line of A-equilibria described above.

Note that (a) actual state of economy (cross in Fig. 3) lies near the optimum and (b) instability emerges at the end of A-equilibria line where A-equilibrium becomes similar to B-equilibrium. These observations seem to point out that the economy has additional mechanisms which (a) keep underpayments at reasonable level and (b) make unstable the effective barter equilibrium.

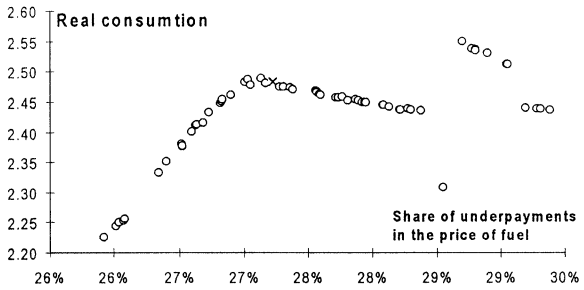


FIGURE 3

[†]Sverdlovsk region imports almost all fuel.

So we have seen that though equilibrium with underpayments (A-equilibrium) does not lay on border of technological set, it can be rather effective. At the same time A-balance is deprived of the basic faults of equilibrium with barter (B-equilibrium): there is actually only one scale of prices and enterprises which is not producing goods of consumers' demand receive their income by sales of their own production and not by reselling of another's.

For these reasons we may suppose that underpayments being legalized, i.e. transformed in the lawful mutual credit of the enterprises, could serve as a suitable means to overcome systematic shortage of working capital in Russian economy.

7 INEFFICIENCY OF THE MARKET AS STIMULUS TO INTEGRATION OF THE ENTERPRISES

In the case of perfect competition (C-equilibrium, Section 2) firms have no stimuli to integrate. Well known theorem of the core of economy (Nikaido, 1968) states that no group of firms in C-equilibrium could increase their total profit by reallocation of resources within the group.

This is not true in the case of A-equilibrium. Reallocating of resources besides market firms can avoid transaction cost. It is necessary, however, always to mean, that pure administrative association of large number of the enterprises will face problems like those of a planned economy. In modern conditions of a powerful external competition, such group most likely will be compelled to organize internal systems of accounts and economic stimuli, i.e. internal submarket. Then at the end association be reduced to one of models of equilibrium but with the solidary financial responsibility of group, i.e. financial-industrial group (FPG).

To estimate the effect of integration in FIG suppose that the firm ν is allowed to relax solvability constraint (17) by buying additional assets Δ^ν (or selling excessive liabilities) at rate θ. What volume Δ^ν the firm will chose? To answer this question it is necessary to solve modified problem (18). Namely the firm should maximize

$$P_A - \theta\Delta^\nu \tag{19}$$

over $\mathbf{x}^\nu, \mathbf{y}^\nu, \mathbf{v}^\nu, \mathbf{w}^\nu, \mathbf{z}^\nu$ and Δ^ν subject to under technological constraint (3) *and* modified solvability constraint

$$\Delta^\nu + \mathbf{s}\mathbf{y}^\nu \geq \mathbf{s}\mathbf{w}^\nu. \tag{20}$$

It may be shown that the problems (17) and (20) define monotone demand/supply function $\Delta^\nu(\theta, \mathbf{p}, \mathbf{s})$. We speak of supply because at high rates θ it is more profitable to chose $\Delta^\nu < 0$, i.e. to get additional income in (20) at the expense of tightening solvability constraint (20). Demand/supply θ^ν becomes zero when the rate θ is equal to Lagrange multiplier $\Delta^\nu(\mathbf{p}, \mathbf{s})$ of constraint (17) in the initial optimization problem of maximization (18) subject to (3) and (17). If $\theta < \theta^\nu(\mathbf{p}, \mathbf{s})$ the firm ν wants to buy assets, if $\theta > \theta^\nu(\mathbf{p}, \mathbf{s})$ it wants to sell ones.

In generic case Lagrange multipliers $\theta^\nu(\mathbf{p}, \mathbf{s})$ are different for different firms and at given θ some firms will ask assets while others will offer. For any group of firms G there exists equilibrium rate $\theta^G(\mathbf{p}, \mathbf{s})$ bringing in balance supply and demand of assets within the group

$$\sum_{\nu \in G} \Delta^\nu(\theta^G(\mathbf{p}, \mathbf{s}), \mathbf{p}, \mathbf{s}) = 0.$$

Exchange of assets by equilibrium rate increases profit of each member of group for any $\mathbf{p}$, $\mathbf{s}$. The equilibrium rate may be found from the problem of maximizing of the total profit of group

$$P_A^G = \sum_{\nu \in G} P_A^\nu \tag{21}$$

subject to (3) and *solidary solvability* constraint

$$\sum_{\nu \in G} \mathbf{s}\mathbf{y}^\nu \geq \sum_{\nu \in G} \mathbf{s}\mathbf{w}^\nu. \tag{22}$$

Equilibrium rate θ^G is Lagrange multiplier to (22) and the sum of profits of members of group (19) after exchange of assets at equilibrium rate will be equal to maximum possible value of (21).

So we see that in A-equilibrium[‡] firms have certain stimulus to integrate their underpayment responsibility, say in the framework of FPG. One may ask question why all enterprises have not yet formed a single FPG? The answer may be the difficulty of estimation of solvability of enterprises. A firm can be well aware of real solvability only of its neighbors in technological chain (its consumers and suppliers) while for integration it is necessary to estimate solvability of other enterprises. Generally speaking the estimation of solvability is not a business of enterprise. It is the business of banks. Banks being well aware of real solvability of their clients may easily organize local market of underpayments discounting clients' arrears and promissory notes.

We believe one can observe the process of integration in the form of wide spread in modern Russia spontaneous grouping of industrial and trade enterprises into a sort of small FIG under domination of a bank. Bank in the head of such a group serves as guarantor of solidary solvability of the whole group.

8 THE CONCLUSION

- Market mechanisms can form inefficient equilibria of different types which are characterized by underloading of major production factors (labor, capital and natural resources) and chronic shortage of working capital. The description of these equilibria offered above was tested in applied projects. The main reasons of an inefficiency in modern Russian economy can be high expenses of realization and short horizon of planning of the economic agents.
- In inefficient equilibrium one should expect emergence of quasi-money which partially compensate inefficiency. Different types of quasi-money should not be mixed together as it is done in many publications. At least two: barter (second currency) and underpayments (mutual credit) are essentially different. They put economy in different states.
- Payment crisis in Russian economy cannot be resolved by mechanical injection of money in economy since all ineffective equilibria considered above are invariant to price scaling. The injection of money only will cause inflation but barter and arrears will be quickly restored at a new level.
- Proper institualization and legalization of quasi-money may help to overcome payment crisis. The mechanism of mutual crediting is preferable as compared with mechanism of barter exchange as latter has a lot of serious internal faults.
- Institualization of the mutual credit is in essence a function of banks. However to execute this function properly banks they should rise reliability and essentially decrease discount rate of promissory notes at the expense of expansion of sphere of circulation of these securities.

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[‡]By formal analogy one might try to consider exchange of second currency in B-equilibrium. Such exchange would not be profitable because Lagrange multipliers to budget constraints (14) in the problems of maximization of (15) are unlike $\theta''(\mathbf{p}, \mathbf{s})$ and are equal to each other. This is the consequence of the fact that barter exchange has already maximized sum of firms' profit, see (16).

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