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Nonlinearity in Social Dynamics – Order Versus Chaos

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This paper discusses the significance of applying nonlinear theory to examine the complexity of social and economic evolution. First, we generally examine possible implications of nonlinear theory for analyzing the complexity of human societies. Second, we select two socio-economic models to illustrate our viewpoints.

Keywords: Nonlinearity, Complexity, Evolution, Chaos, Economic networks

1 ORDER VERSUS CHAOS

Due to the great efforts by mathematicians and scientists of various fields in the last few decades, our vision of social and economic development has been fundamentally changed: a change toward the multiple, the temporal, the unpredictable, and the complex.

It was generally believed, even about three decades ago, by most of scientists that simple systems behave in simple ways and complex behavior had to be explained by complicated causes. A mechanical contraption like a pendulum, an economic model like the Solow–Swan growth model, as long as these systems could be reduced to a few perfectly understood, perfectly deterministic laws, their long-run behavior would be stable and predictable. On the other hand, systems such as wildlife population and a national economy which were visibly unstable and unpredictable must be subjected to random external shocks. Now, all that has been changed. In the last three decades, nonlinear theory has created an alternative vision of dynamics. Simple deterministic nonlinear dynamic systems, such as the logistic map, may give rise to complex behavior and complex systems, while seemingly complicated equilibrium models may generate very simple behavior.

Mathematical interest in deterministic dynamic systems that generate catastrophes, bifurcations and chaos has dated back to Poincaré's work in the late 1800s. But the emphasis on the complexity of nonlinear dynamics was highlighted by the seminal paper by the meteorologist Edward Lorenz (1963). He discovered an extremely simplified model of Earth's climate that displays sensitive dependence on initial conditions. The long-run behavior of the system will be dramatically different even by changes in data on a scale finer than our capacity to measure. Ruelle and

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Takens (1971) argued that the traditional model of fluid flow turbulence was structurally unstable and that a dynamical system that converged to a lower dimensional deterministic chaotic attractor was a better model of certain types of fluid flow turbulence than the traditional one. They examined transitions to turbulence by first writing down a system consisting of differential equations, $dx/dt = F(x, r)$, where x is the state vector of the system and r denotes a vector of slowly moving parameters. The dimension of the state vector may be either finite or infinite. They considered how the long-run behavior of the system may converge to some attractor set $A(r)$. Indeed, the long-run behavior will structurally change as one shifts the values of the parameters r.

The theoretical ecologist Robert May found that the simple logistic equation, $x(t+1) = rx(t)(1-x(t))$, may yield chaotic behavior for suitable choices of the parameter r. If we interpret $x(t)$ as the population at time t, this implies that a small change in the parameter, such as small shifts in biological, economic or political conditions, may result in fluctuations which are beyond our capacity to forecast in the long term. The most interesting phenomenon in this chaos is that it exhibits certain aspects of order. The physicist Mitchell Feigenbaum found that the logistic mapping exhibits a period-doubling cascade that is duplicated by many other systems. When one increases the parameter, r, the long-run behavior progressively passes from a fixed point, to a two-cycle, to a four-cycle,..., to a 2^n-cycle,..., to chaos. These types of models are also found in economics. For instance, in Grandmont (1985) it was the risk aversion of the old agents in a two period overlapping generations model that played the role of r in his example of a period doubling route to economic chaos. It was the discount rate on future utility in the Boldrin and Montrucchio model (1986) that was treated as the bifurcation parameter. And it was the population sustainable capacity parameter in the Stutzer model (1980) that makes his system behave chaotically. Since the economists have had access to modern analytical methods for analyzing nonlinear dynamic phenomena such as catastrophe, bifurcation and chaos, many other nonlinear dynamic models related to social, regional, geographical and economic issues have been constructed and examined with nonlinear methods (e.g., Chiarella, 1990; Dendrinos and Sonis, 1990; Puu, 1989; Rosser, 1991; Flaschel *et al.*, 1995; Lorenz, 1993).

A synthesis of the basic ideas in this new vision of nonlinear dynamics was clearly developed in Hermann Haken's synergetics (1977; 1983) and Ilya Prigogine's self-organization theory (Nicolis and Prigogine, 1977). The purpose of this study to explain how these new ideas can be used to explore the complexity of social and economic dynamics. The remainder of this paper is organized as follows. Section 2 discusses issues related to change speeds and time scales of analysis in socio-economic dynamic theory. Section 3 illustrates the ideas of the preceding sections by explaining the development of inter-regional economic relations in the world from the years around 1000 A.D. until 2000 in terms of Andersson's four logistical revolutions. Section 4 utilizes Zhang's economic development model for the Chinese economy to provide some insight into the complicated interactions among capital and knowledge accumulation and economic and cultural opening to the outside world. Section 5 concludes the study.

2 CHANGE SPEEDS AND TIME SCALES OF ANALYSIS

We argue that social changes consist of dynamic processes of self-organization with spontaneous formation of increasingly subtle and complicated structures. It resembles a turbulent movement of liquid in which varied and relatively stable forms of current and whirlpools, constantly influence one another. The accidental nature and the presence of structural changes like catastrophe and

bifurcation which are characteristic of nonlinear systems and whose further trajectory is determined by chance make social dynamics irreversible, i.e., deprive the evolution process of a time symmetry. But there is one important character of social changes. Social evolution has a hierarchical nature in the sense that new structures do not replace the old ones, instead, they evolve against their cultural background. This is the reason why history is important for social sciences.

It is obvious that the society has to be politically and economically analyzed and to be scientifically treated as a single whole. On the other hand, our capacity to generally handle the complexity of dynamic systems is rather limited. Even with the help of the most advanced mathematics and computers, we cannot completely know the behavior of some three-dimensional nonlinear dynamic systems, not to mention higher dimensional ones. As social and economic systems are obviously nonlinear and highly dimensional, it is of great importance to find out some analytical methods to effectively handle such dynamic analyses. As shown in Andersson (1992) and Zhang (1991), explicit classification of change speeds of social and economic variables may not only help us to recognize differences in different schools of social and economic sciences and in cultural characteristics of different nations, but also help us to handle complicated nonlinear analysis.

A serious debate in social and economic dynamics is related to the adjustment speeds of variables. For some economists a competitive economic system adjusts smoothly and quickly to disturbances in its way toward to the long-run equilibrium. For others a competitive economic system can hardly simultaneously guarantee both (social, political and cultural) stability and economic efficiency because the assumption of a perfectly competitive market structure, perfect information and perfect expectations cannot coexist in the real world.

Distinction between speed changes in varied cultural circumstances is the key to understand differences among cultures (e.g., Andersson *et al.*, 1992). Various cultures have different traditions directing man's actions, different conventions expressed by symbols, and different artificial objects, institutions and processes. Each culture provides each individual with behavior patterns and endows him with a specific perspective, attitude and intuition. Hence, social and economic variables have different change speeds under different social and cultural circumstances. Prices are determined very quickly in a competitive economy; construction of transportation and communication infrastructures usually takes much longer time in developing countries than in developed ones. The speed of transactions, the time needed to take decisions, the speed in which new ideas are created in laboratories, the speed that data, information and new knowledge affect economic systems are obviously different among peoples, regions, and nations.

Not only economic and institutional variables, even ideologies, morals, habits, which are usually thought of as "being fixed" in daily life, are subject to changes. The loss of the power of the Church and the current collapse of the communist world indicate that ideologies are long-run endogenous variables of social evolution.

To illustrate how we can analytically handle dynamics with different speeds, let us consider a dynamic system generally described by

$$\mathrm{d}x_{ij}/\mathrm{d}t = s^{m_i}F_i(x), \tag{2.1}$$

in which s ($\ll 1$) is a parameter used to classify speeds of variables, and x_{ij} are variables describing the state of the dynamic system, and m_i are order parameters.

For simplicity of illustration, we only discuss a two-dimensional case. Then, we present a general result about the dynamics of (2.1).

We now consider a dynamic system consisting of two variables

$$\mathrm{d}x/\mathrm{d}t = f(x,y), \quad \mathrm{d}y/\mathrm{d}t = sg(x,y), \tag{2.2}$$

in which f and g are appropriate continuous functions and s is a very small parameter. We

may thus call x and y fast and slow variables, respectively.

Since s is extremely small, it is acceptable to assume that $dy/dt = 0$, i.e., to treat y as a constant in a short-run dynamic analysis. This implies that we may analyze (2.2) merely by examining the reduced system

$$dx/dt = f^*(x), \tag{2.3}$$

in which $f^* = f(x, y)$ with y fixed. But this reduced form may not describe the dynamics of the system under consideration in the long term. In order to "transform our conscience of time", we introduce the following transformation: $T = st$. We may rewrite (2.1) as

$$s\,dx/dT = f(x, y), \quad dy/dT = g(x, y). \tag{2.4}$$

We thus may approximately rewrite (2.4) as $f(x, y) = 0$ and $dy/dT = g(x, y)$, i.e.,

$$dy/dT = g(h(y), y) = g^*(y), \tag{2.5}$$

where $x = h(y)$ determined by $f(x, y) = 0$. Accordingly, the fast variable disappears as an explicit variable in the long-run analysis.

The above illustration indicates why it is often reasonable for economists to neglect studies by historians, while a historian without a profound knowledge of economics can never really understand the history of an economy. Many economic variables are fast, e.g., prices. This implies that it is acceptable for an economist to "forget" slow variables such as institutions as the latter can be treated as parameters in the short-run dynamic analysis. But (2.5) tells that the slow variable is constantly affected by the fast variable x even in long-run dynamics.

It must be emphasized that the reduction of dimensions of a nonlinear dynamic system is not often as simple as in the illustrated model (e.g., Haken, 1983; Carr, 1981; Kevorkian and Cole, 1981; Chow and Hale, 1982; O'Malley, 1988; Pliss and Sell, 1991). The validity of the above discussion is dependent upon the stability of the system. From the literature of modern dynamic analysis, we know that a small change in a parameter may cause structural changes in an unstable nonlinear system. This implies that even when s is very small, we cannot safely omit its effects upon the reduced system in a dynamic analysis.

The remainder of the study applies the above ideas to two long-run social and economic dynamic studies – (a) the development of interregional economic relations in the world from the years around 1000 A.D. until 2000 A.D. in terms of four logistical revolutions and (b) the complexity of China's opening to the outside world.

3 INFRASTRUCTURE AND ECONOMIC DEVELOPMENT

Andersson (1986; 1992) attempted to explain regional development by focusing the attention on the role of knowledge, transportation and communication networks in the determination of economic development and its industrial, spatial and other structural consequences. There it is argued that consequences of slow improvements of infrastructures will provide relative stabilization, a slow development of existing structures. During this process there is a continuous search for new structures to improve efficiencies of social and economic interactions. The search appear embryos of future communication and transportation structures. After a series of bifurcations and chaos transforming the development process, there will come a relatively stable stage of development.

Sufficiently changing the general and structural characteristics of networks may trigger change of industrial organization and spatial structure of economy. It is impossible to understand the role of transportation, financial and other communication services and other spatial and organizational flows, unless we proceed from the slowly changing networks to the fast processes of pro-

duction and market exchange. Even though there is a feedback from the fast to the slow processes, most of the time the causal links go from the slowly changing networks to the quickly adapting flow variables of the markets.

In order to develop a sound theory of regional development, Andersson (1992) suggested the following order of explanation:

1. a certain level of technological and other knowledge exists, in embodied or disembodied form;
2. a set of (built or natural) networks for transportation and communication has been established;
3. a set of property rights and other organizational rules have also been established;
4. an optimal logistical system, including nodal production and other transformation activities, is determined, implying a certain industrial and spatial economic organization;
5. on this arena, patterns of deliveries and prices will be determined so as to establish a stable equilibrium;
6. some share of the income generated by this equilibrium structure will be used for infrastructure investments in knowledge or networks;
7. if these slow but steady transformations of the arena will be progressing in some definite direction, then there must sooner or later be a complete structural change, i.e., a fast phase transition from one spatial and industrial equilibrium organizational structure into another, topologically non-equivalent stable equilibrium structure.

We now apply the above ideas and nonlinear dynamic theory to explain the complexity of regional dynamics.

Based upon the studies by Pirenne (1925) and Mees (1975), Andersson (1986) argued that the sequence of fundamental changes in the world economy over the last millennium can be explained by the changing structure of logistical systems. In other words, the great structural changes of regional production, location, trade, culture and institutions are triggered by slow steady changes in the associated logistical networks. Logistical networks are those systems in space which can be used for the movement of commodities, information, people and money in association with the production or consumption of commodities. Andersson (1986) suggested a model for logistical revolutions in the world as follows:

$$\mathrm{d}y/\mathrm{d}t = F_y(y,z,x), \quad \mathrm{d}z/\mathrm{d}t = sF_z(y,z,x),$$
$$\mathrm{d}x/\mathrm{d}t = \varepsilon F_x(y,z,x),$$

where y – commodity production; z – knowledge infrastructure; x – network infrastructure; s, ε – adiabatic constants; F_x, F_z and F_y – certain specified functions of x, z and y.

By s and ε, we may distinguish speeds of the variables in the system. For instance, when s and ε are extremely small, the commodity production is a fast process in comparison with the dynamic processes of knowledge and network infrastructures. For simplicity of illustration, let us consider the following special case of the general system (e.g., Andersson and Batten, 1988):

$$\mathrm{d}y/\mathrm{d}t = -y^3/3 + ry + x, \quad \text{“fast equation”},$$
$$\mathrm{d}x/\mathrm{d}t = -\varepsilon y, \quad \text{“slow equation”}, \tag{3.1}$$

in which r is a control parameter and ε is an extremely small positive parameter. In (4.1), we neglect knowledge infrastructure. This system is a reformulation of the Van der Pol equation. It turns out that discontinuous changes in the value of y may be generated if the value of x moves slowly into critical parametric domains. Figure 1 illustrates a typical cycle.

Abrupt rises and falls in production are well documented, and may be precipitated by gradual changes in networks. It is significant to note that changes in the value of the “fast” variable may take place relatively quickly. Thus if one were to observe the state of the system just before and just

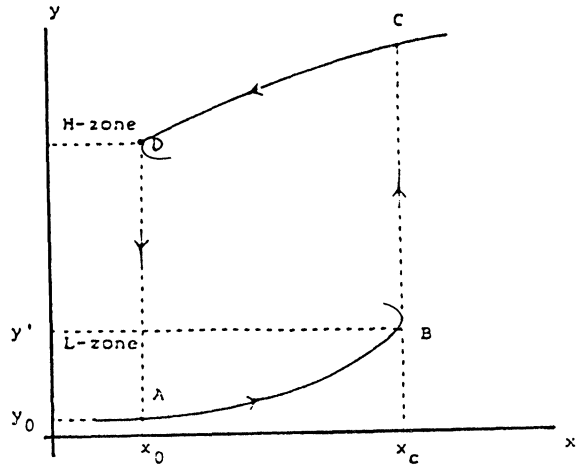


FIGURE 1 Fast production and slow infrastructure.

after the change, one may be tempted to conclude that the "slow" variable was not influential.

The slow expansion of network infrastructure, x, through investment in physical capital will follow the trajectory located in the L-zone of Fig. 1. Let the system initially be located at A. As x is changed, eventually a point b is reached beyond which the very nature of the regional production pattern changes markedly. At this point, the equilibrium loses its stability and a "phase transition" is underway.

The system is of a cyclical nature. If network improvement stops, once the H-zone is reached, and depreciation is then the dominant interactive effect, the system may follow the solution trajectory depicted on the H-zone until it finally turns to the initial state at D and then drops back down to the L-zone. This cyclical process may be referred to as divergence, because a smooth but small change in the network infrastructure capacity can cause unexpectedly large fluctuations in the equilibrium value of commodity production. This comes about through a discontinuous change of state or phase transition. The transition takes place no matter how slowly the network infrastructure increases. This implies that urban development may be triggered simply by the addition of one small but important link in the network. Slight differences in transportation conditions may result in large differences in the final production if the urban structure is located at a critical state.

Getting insight into the complexity of interregional development from this model, Andersson (1986) tried to explain the development of interregional economic relations in the world from the years around 1000 A.D. until 2000 A.D. in terms of four logistical revolutions:

I. emerging in Italy in the 11th century and ending in Northern Europe in the 16th century;
II. emerging in Italy in the 16th century and ending in Northern Europe in the 19th century;
III. emerging in England in the 18th century and ending in the developing countries, probably in the 21st century;
IV. emerging in regions of Japan, the United States, and Western Europe at the end of the 20th century.

(I) *The First Logistical Revolution*

Shortly after 1000 A.D., a slow but steady improvement of the transportation network started in the Mediterranean area, followed thereafter by a similar development in Northern Europe. The improvement of sailing technology and the successive opening of harbors and rivers for trade was completed by the opening up of the Champagne fairs, which were controlled by Italian tradesmen. The Southern Mediterranean trading system was integrated with the northern trading system, in particular the Hanseatic league, for the first time. Hence, the first European economic system was formed. It implied the rise to world economic dominance of a number of coastal regions of Europe: Bergen, Lubeck, Bruges, Toscana and Venice. On the basis of a new logistical network, a new integrated western European economic network was formed.

Near the beginning of the 16th century this first logistical era was drawing to a conclusion. This revolution followed a clear pattern of structural change. The cause of the revolutionary development was the successive decrease of barriers to transportation and trade. The consequences were:

1. a dramatic increase in trade over long distances;
2. increasing specialization of production and employment in Europe;
3. the emergence of new merchant towns and the growth of accessibility centers into merchant and manufacturing cities with population up to 100,000 inhabitants;
4. expansion of wealth in the hands of the merchants of these nodal towns and cities;
5. the emergence of a new class with political aspirations;
6. a massive creative expansion in a small subset of accessibility centers like Florence and Bruges;
7. The emergence of a network of centers around the coasts and along the rivers of Europe with a network ideology, most pronounced in the network of towns and cities in the so-called Hanseatic League.

(II) *The Second Logistical Revolution*

With the growing distances and volumes of trade and with increasing multiateralism of the trading houses, the need for commercial credits and reliable currencies became a pressing requirement. The need for banks had been accepted by the merchants of Venice, Florence, and Genoa during the first logistical revolution, but the demand was not pressing enough to require any major invention. With the growing volume of credits and currency movements came inevitably an interest in banking activities from monarchs, the church, and speculators. The City of Amsterdam was the first authority responding to this growing demand for efficient banking. The success of the Amsterdam bank was followed with great interest by others, and a modern central bank was *created in Britain – The Bank of England. It* was the first national state guaranteed bank with the rights to do business with money and bills of exchange, including the right to issue bank notes.

A slow and steady development of the transportation technology and the introduction of new banking techniques paved the way for a new and more extended economic system. The introduction of the Caravelle ship made it possible to extend international transportation and communication beyond the limits of the earlier economic system. Lisbon replaced the Champagne fairs as a connection for long distance trade and transportation. It became an ideal common connection point in the new network for transportation between different parts of Europe and the Americans. But Lisbon was a very unstable connection in the new international economic order. It was soon to be replaced by Antwerp of the Flanders. With the development of the new banking system and the innovations in banking developed by Amsterdam, the final stable point was shifted once more to the city of Amsterdam – the new capital of the world economy after the second logistical revolution. This revolution, different from the first one which was primarily caused by improvements to coastal transportation systems, was caused by improved possibilities of ocean transportation and by dramatically improved systems for transactions involving credits designed to serve long distance and long term trading expeditions. The consequences can be summarized as:

1. a dramatic increase in long distance multicommodity, multilateral trade;
2. an increasing specialization of production in Europe and the emergence of a state supported manufacturing system, close to new centers of economic and political activity;
3. the emergence of new metropolitan cities in which political and economic power were integrated;
4. expansion of wealth in the hands of the crowned and uncrowned heads of the new transactions and economic control system;

5. total dominance for the new class of the absolute state;
6. creative expansion of ideas in science and art in a few centers, like Amsterdam, Paris, and London;
7. the emergence of a new pattern of the international trading network with the most prominent nodes, London, Paris, and Amsterdam.

(III) *The Third Logistical Revolution*

Two innovations played a significant role in this third logistical (industrial) revolution. The former is the insight that a coordinated division of labor could liberate enormous productive activities. The latter is the idea that production techniques were not regarded as given, but were variables in space and thus powerful determinants of profits from trade. Still, this revolution was the consequence of a slow and steady change of transportation and transaction possibilities at the intercontinental and intracontinental level. Great Britain had developed a qualitatively superior transaction system based on the formation of the central Bank of England. Furthermore, Great Britain had secured successively safer and faster possibilities of North Atlantic transportation and trade. The new transportation and transaction network was to be based on an integration of Western Europe and the eastern parts of North America. It had become possible to utilize machinery equipment in plants and production units which exploited economies of scale. The cause of the third logistical revolution was an emerging awareness that coordinated division of labor between different regions of the global economic system could be very profitable, especially if new technologies could be innovated and diffused to different nodes of a vertically integrated network. The consequences were:

1. a spectacular increase of trade over long distance, especially across the North Atlantic;
2. a division of labor between regions and countries;
3. fast growth of industrial towns and cities in the vicinity of raw material bases, network junctions, and close to market agglomerations;
4. expansion of wealth of the new industrialists;
5. the emergence and fast growth of the two new classes of the industrial society: labor and capitalists;
6. adaption of the political system to suit the demands for new institutional arrangements in the labor market, and for the protection of property in other parts of the world;
7. creative expansions in science, engineering, and the arts occurring sequentially in a subset of the expanding industrial centers in Europe and North America.

(IV) *The Fourth Logistical Revolution*

We have now arrived at a period of an emerging fourth logistical revolution, associated with the growth of information processing and communication capacity as well as the growth of knowledge. This development goes hand in hand with a successive improvement of the transportation system, especially the structure and operation of the air transportation network. We will not analyze this revolution in detail because of limited space. We refer the analysis to, e.g., Andersson and Strömquist (1988). What should be emphasized is that there are differences in change speeds of different parts of infrastructure in the different logistical revolutions.

4 OPENNESS AND SOCIAL AND ECONOMIC DEVELOPMENT IN CHINA

> After a time of decay comes the turning point. The powerful light that has been banished returns. This is movement, but it is not brought about by force ... The movement is natural; arising spontaneously. For this reason the transformation of the old becomes easy. The old is discarded and

the new is introduced. Both measures accord with time; therefore, no harm results.

I. Ching

Civilization is the process of creation, diffusion and utilization of ideas. A country without new ideas is often cycling around some equilibrium point without essential social and economic structural changes. China's civilization might provide a suitable example of how the same ideas could be recycled from one generation to another. One of the main reasons that China's civilization has showed extremely permanent reviving continuity at least in last few hundred years is that she has lacked energies for creativity and has slowly changed her attitudes toward the knowledge created by other cultures. We are here not involved with historical reasons for China's unusual persistence of dynastic cycles. We are interested in showing if and how we can explain the complexity of opening the country to the outside world, applying modern dynamic theory.

There are two ways for a society to find new ideas. The one is that the society creates new ideas by cultivating talents of its own people. The other one is to introduce the ideas created by other societies. Zhang (1992) suggested a nonlinear dynamic model to provide some insight into the complexity of China's industrialization process. The system consists of the dynamics of three variables – knowledge, capital and openness. Here, openness is introduced to analyze how China may utilize the knowledge stock of the world to increase the quality of the Chinese population.

The economic system consists of one production sector. The product is either consumed by the population or invested (accumulated). The population, L ($= \exp(nt)$ with n given) is divided into two kinds labor force, L_1 ($= n_1 L$), and people at leisure, L_2 ($= n_2 L$). The output of the economy is given by the following production function: $Y = K^{\alpha}(zL_1)^{\beta}$, $\alpha > 0$, $\beta > 0$, $\alpha + \beta = 1$, where K is capital of the society and z is the average knowledge of the labor force. The capital accumulation or investment is given by

$$\mathrm{d}k/\mathrm{d}t = sF(k, z) - (\delta + n)k, \tag{4.1}$$

where $k \equiv K/L_1$, δ is the depreciation rate of capital and s is the savings rate.

We suggest the following dynamics of the openness variable, p,

$$\mathrm{d}p/\mathrm{d}t = N[\varepsilon p - \Theta p^3 + q(k, z)], \tag{4.2}$$

where N is a positive adjustment speed parameter. We interpret the term $(\varepsilon p - \Theta p^3)$ as the political forces which affect the openness of the nation. The linear term, εp, expresses the strength of the "reformers" who support the development of science and technology and learning from other cultures, and the nonlinear term Θp^3 as the power of the conservatists who are inert in opening the nation. The function, $q(k, z)$, describes factors such as knowledge and consumption per capita which affect the openness. We specify q as

$$q(k, z) = b\{c^* - n_1(1 - s)F(k, z)\},$$

where c^* is the average consumption level per capita in the developed economies which exert great influences upon developing nations, b is a given nonnegative parameter.

It should be emphasized that (4.2) is only a possible specification of the complexity of cultural openness. For instance, it is reasonable to introduce power struggle parameters, ε and Θ, as endogenous variables in the following way:

$$\mathrm{d}\varepsilon/\mathrm{d}t = f_1(p, k, z, \varepsilon, \Theta),$$
$$\mathrm{d}\Theta/\mathrm{d}t = f_2(p, k, z, \varepsilon, \Theta),$$

where f_1 and f_2 are appropriate functions. This paper only examines effects of "exogenous" changes in ε and Θ upon the system.

We specify the dynamics of knowledge as follows:

$$\mathrm{d}z/\mathrm{d}t = T[gp/(1 + \sigma z) + rY + vn_1(1 - s)Y - \delta_1 z], \tag{4.3}$$

where T is a positive adjustment parameter, and g, σ, r, δ_1 and v are positive parameters. The first term, $gp/(1+\sigma z)$, implies that as the economic system becomes more open, the knowledge level tends to increase. However, effects of international interactions upon the knowledge accumulation tend to decline if knowledge of the society is already very high. Obviously the advantage of backwardness decreases with learning from leaders. The second term, rF, describes the effects of learning by doing upon the quality of labor. The third term, $vn_1(1-s)F(k,z)$, states that as people consume more, they accumulate more knowledge due to increased leisure and become physically strong due to better nutrition. The term, $\delta_1 z$, describes the possible yearly depreciation of quality.

The system consists of (4.1)–(4.3). Further explanation can be found in Zhang (1992). We now examine some special cases to illustrate the relationship among adjustment speeds, time scale and stability.

First, we consider an economy with openness, p, changing very slowly, i.e., $T=1$ and N being sufficiently small. Introducing $T^*=tN$, we can rewrite the dynamics in the form of:

$$\begin{aligned} N\mathrm{d}k/\mathrm{d}T^* &= sY-(\delta+n)k,\\ \mathrm{d}p/\mathrm{d}T^* &= \varepsilon p-\Theta p^3+q,\\ N\mathrm{d}z/\mathrm{d}T^* &= gp+r_0Y-\delta_1 z, \end{aligned} \tag{4.4}$$

in which $\sigma=0$. We assume that N is so small that we can safely let $N\,\mathrm{d}k/\mathrm{d}T^*=0$ and $N\,\mathrm{d}z/\mathrm{d}T^*=0$ in the dynamic analysis. We can show that the long-run dynamics are thus approximately given by

$$\mathrm{d}p/\mathrm{d}T^* = -\Theta p^3+(\varepsilon-b_2)p+b_1, \tag{4.5}$$

where

$$b_1\equiv bc^*,\quad b_2\equiv bn_1(1-s)F(a_1,a_2),$$
$$a_1\equiv sa_2/(\sigma+n),\quad a_2\equiv g/\{\delta_1-r_0[s/(s+n)^{\alpha}]\}.$$

We interpret the terms $(\varepsilon-b_2)$ and b_1 as measurements of progressive forces in the developing nation studied. The stationary values are found by

$$p^3+r_1p+r_2=0,$$

in which

$$r_1\equiv(b_2-\varepsilon)/\Theta,\qquad r_2\equiv -b_1/\Theta.$$

This equation has either one or three real roots. If $(-r_1/3)^3>(r_2/2)^2$, then the equation has three roots. As the left term is always positive, a necessary condition for the inequality is that $r_1<0$, i.e., $\varepsilon>b_2$. This can only be guaranteed under the conditions that the reformers are rather strong in political decisions in comparison to the anti-foreign attitudes. Otherwise, there is only one equilibrium of the system. This discussion shows that whether there is a unique p is dependent on the power of the reformers.

The boundary of the region between single and multiple solutions is determined by: $4r_1^3+27r_2^2=0$. This produces the cusp shaped curves on the control manifold – the (r_1,r_2) plane. As shown in Fig. 2, outside the cusp shaped region there is only one root and the unique equilibrium is stable; inside the region, there are three real roots – one unstable and two stable states.

PROPOSITION 4.1 *If capital and knowledge accumulation are fast variables and the openness is a slow variable, the long-run dynamics may be described by a single dynamic equation of the slow variable. In other words, capital and knowledge are "enslaved" (Haken, 1977; 1983; Zhang, 1991) by change in openness. And sudden structural changes in the long-run dynamic evolution may exist, depending upon the whole structure of interactions of economic development, knowledge growth and political struggles.*

We are now interested in the parameters ε and Θ. As $r_1=-(\varepsilon-b_2)/\Theta$, we see that for a (positive) fixed b_2, when the reformers are not strong

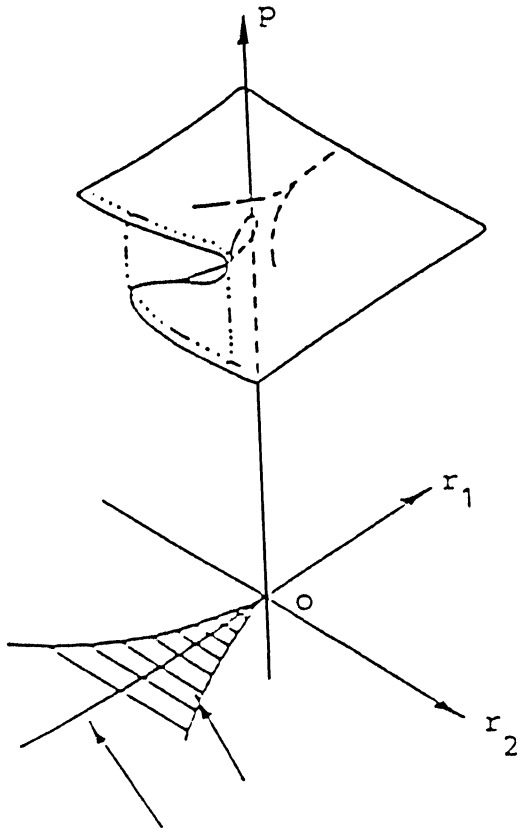


FIGURE 2 The cusp catastrophe.

there is unique stationary state. As the power of the reformers increases to such a degree that r_1 becomes negative, the situations become more complicated. There is the possibility of multiple equilibria. Now, we consider a specific political struggle: when Θ is shifted, ε is changed in such a way that r_1 keeps fixed during the study period. Although it is hard to exactly interpret the specified policies, the assumption simply implies that as the power of the conservatists increases, the power of the reformers will also increase, and vice versa. It should be noted that these specific assumptions are not essential for our analytical results.

Under this assumption, we obtain the relations between the opening policy and the power of the conservatives as shown in Fig. 3.

With respect to the power of the conservatives, there are sudden changes in the openness. Outside the interval $[\Theta_1, \Theta_2]$, there is a unique p for each value of Θ. However, if $\Theta_1 < \Theta < \Theta_2$, there are three equilibria, two stable and one unstable. Consider a possible case of the dynamics. When the economy is just opened, the power of the conservatives begins to increase, i.e., Θ increases toward Θ_1 from the right side. The economy becomes increasingly open as Θ continuously changes. When Θ arrives at the critical point, Θ_1, there are sudden increases in communication and trade between the economy under consideration and the rest of the world. Near such a point, there is structural change. The consumption per capita, capital per capita, and average knowledge increase during a very short period.

However, as such increased communications and improved living conditions may "cause" the officials to become corrupt and may introduce some "undesirable things" from other countries, the power of the conservatives may either increase or decrease. If the conservatives continuously become weaker, the country will become opener and there are no sudden changes, at least in a local sense. However, if the conservatives become stronger after the sudden change, the country becomes more isolated. When the conservatives get so much power that the parameter

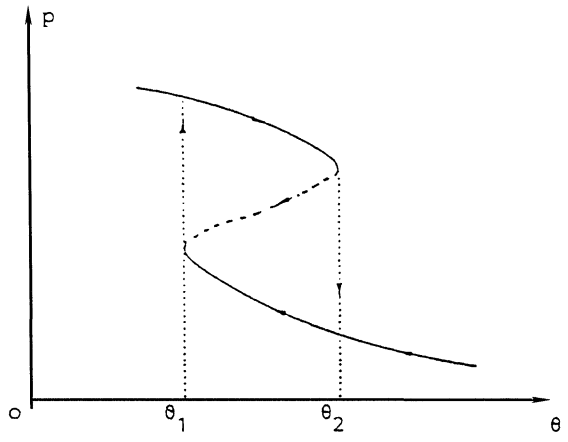


FIGURE 3 Structural changes.

value reaches Θ_2, there is a sudden change again. The nation suddenly becomes more isolated and it is impossible for scientists and entrepreneurs to interact with other countries. Near such points, there are sudden decreases in consumption and capital per capita and the average knowledge. A miserable day for the masses of the population is coming again.

Another case is rather suggestive. It is meaningful to examine what will happen to the system if both the openness and learning processes are very slow, i.e., N and T are very small. Let $T = N$, $\sigma = 0$, $a > 0$, $b > 0$, and s be constant, and introduce $T^* = tN$. Then, we can rewrite the system in the form of:

$$\begin{aligned} dp/dT^* &= \varepsilon p - \Theta p^3 - b_2 z + b_1, \\ dz/dT^* &= \mu p + s_2 z, \end{aligned} \tag{4.6}$$

where

$$s_1 \equiv [s/(\delta + n)]^{1/\beta}, \quad b_1 \equiv bc^*, \quad b_2 \equiv bn_1(1-s)s_1^{\alpha},$$
$$s_2 \equiv r_0 s_1^{\alpha} - \delta_1.$$

According to the celebrated Hopf bifurcation theorem, we can prove the following proposition.

PROPOSITION 4.2 *Under appropriate conditions (see, Zhang, 1992), when the power of the reformers increases, social cycles appear in the system.*

The cycles can be illustrated as in Fig. 4.

The system oscillates around the stationary state (p_0, z_0). Let us begin the movement at point D. Knowledge tends to increase near D. Since the knowledge level is increased, there are increases in production and consumption. Near such a state, the nation tends to become opener, which results in further expansion of knowledge. When the system arrives at A, the conservatives become so strong that it is impossible to increase the openness of the economy; thus the nation becomes more isolated. The situation is continued. When the conservatives have increased their power, the knowledge level does not seem to decrease rapidly. Instead, knowledge is increased until the system arrives at B. This also implies that just after the nation becomes isolated, production and consumption will not decrease. Economic conditions are further improved because of the improved knowledge. After B, the knowledge level decreases and the nation continues to be further isolated. During the period $B-C$, the nation may assume a very pessimistic outlook. However, after the social conditions tend to worsen, the effects of the conservatives begin to weaken. The nation becomes open again. However, even when the nation is open, knowledge cannot be increased very rapidly. It will still take a long time for the effects of the opening policy to be recognized. It is after D that the masses may become a little more optimistic because everything seems to improve, although the deeply thoughtful reformers might not be so happy about such mere short-run improvements.

A question naturally arises whether such a mathematical analysis can explain actual cycles of opening and isolation in modern China. We think that the model can be used to illustrate the problem qualitatively.

An important fact about China's opening is that during the period of 1644 to 1911 China was controlled by the Manchus who were a non-Chinese people of different habitat, language,

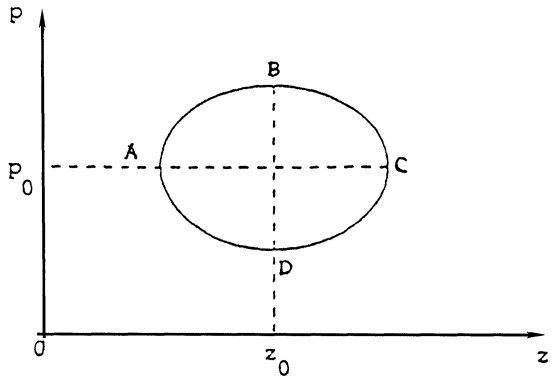


FIGURE 4 The social cycles after the reformers become stronger.

and culture. The notice of this factor may help us to account for why China was much slower to adapt Western cultures than, e.g., Japan. It was quite "rational" for the Manchu rulers to prefer ruling power (even in the name of being Chinese) to industrialization. It was obvious that industrialization implied the loss of power of the Manchu ruling. During this period, there were very complicated dynamic interactions between the opening speed, knowledge diffusion, cultural conflicts, economic development and political (and racial) struggles (e.g., Fairbank, 1983; Reischauer and Fairbank, 1965; Seagrave, 1985).

The end of the Manchus monarchy in 1912 marked the beginning of a prolonged crisis of authority and central power in China. Two foreign circumstances also complicated the decade after 1912. First, the imperialist powers became absorbed in World War I and industry in China had a breathing space in which to develop in relative freedom from the pressure of foreign commercial competition. Second, the World War provided Japan an opportunity for political aggression. During this period, the two important parties for the near future of China – the Kuomintang in 1912 and the Communist party in 1921 – were formed. It should be remarked that the early Kuomintang and Communist leaders differed in that the former leaders were mainly city men; the latter were from rural areas. The social backgrounds and life experiences (in a broad sense, accumulated knowledge) might have affected the choice of their ideologies.

The world was basically split by two ideologies – communism and capitalism. China, like many other countries during that period, was confronted with a puzzling choice. During this chaotic period, China was relatively open to the outside world. Struggles for power, foreign invasions (in particular, Japanese ones), and complicated processes of understanding and interpreting social and economic implications of different (non-Chinese) ideologies such as socialism and capitalism resulted in the separation of Chinese into two different political and economic structures – the mainland China (under the communist party) and Taiwan (under the Kuomintang). The mainland China became isolated because of, except many other reasons, her emphasis upon social and political stability rather than upon economic efficiency; while Taiwan carried out a series of political and economic reforms before she could take off to go into trajectories of modern industrialized civilization.

Since the revolution of 1949, mainland China has experienced long cycles of isolation from the world. Then, from the end of the 1970s, there were significant openings to the international community. Many Chinese students were going abroad, international business began investing in China, and a web of international linkages were developed.

New and different questions arose with the events of the summer of 1989. But perhaps, due to more than 15 years' economic reforms and openness, China has already accumulated certain levels of knowledge and capital stocks that had made a complete isolation impossible. To what degree China will be further opened to the outside world and adapted to modern civilization, as shown in our mathematical model, is strongly related to knowledge and capital of the society, not to say political struggles among different groups of people and international political and economic environment.

As observed in Howell (1993), the opening policy has followed a spiral pattern since the economic reform started in 1978. This study may be considered as a theoretical explanation for the existence of opening policy cycles. The observed dynamics consists of a number of cycles, each cycle having an upswing and a downswing. The upswing periods were characterized by policy innovation, introduction of more open policies, and a more rapid pace of establishment and implementation; while the downswings periods by slowing-downs in implementation, more restrict policies and less favorable political climate for opening. The long-run dynamics contained an increamental spiral evolution of the policy as

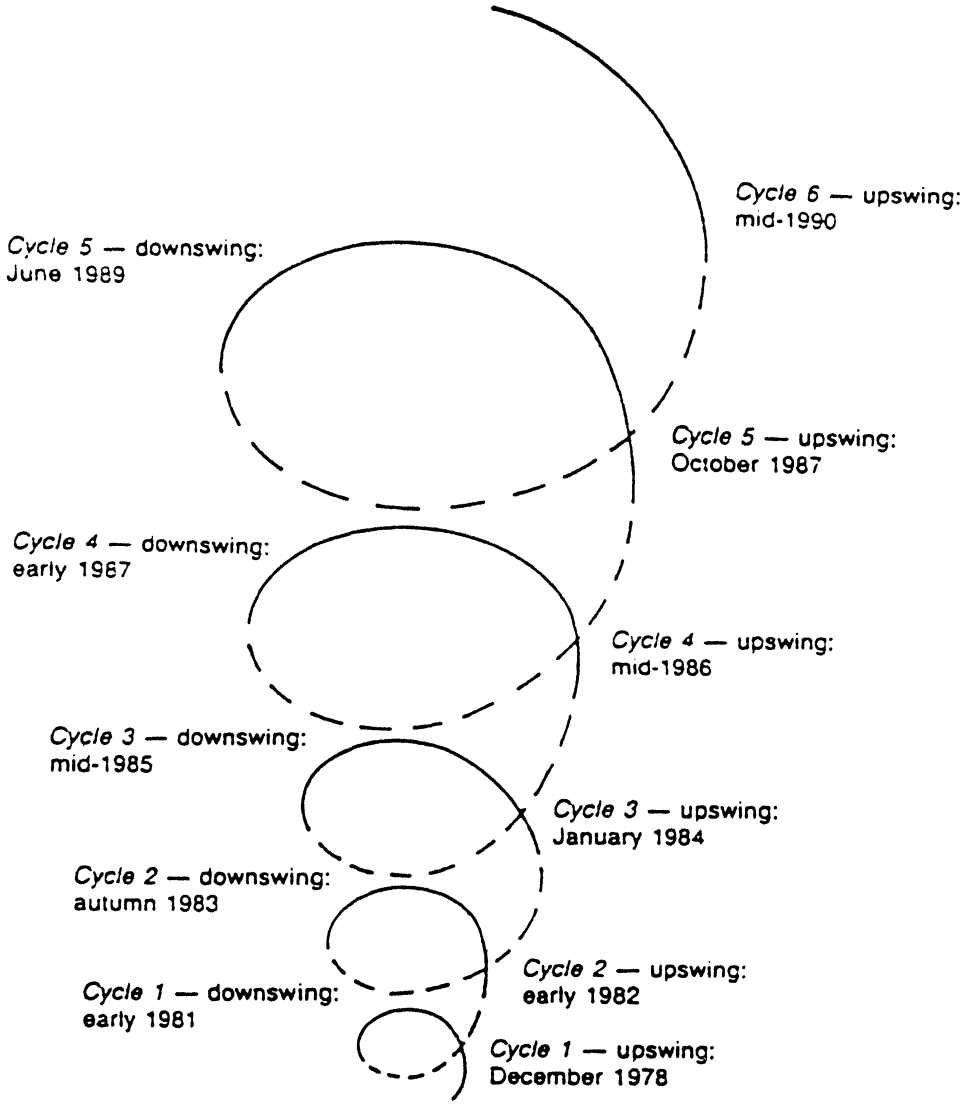


FIGURE 5 The cycles of the open policy.

shown in Fig. 5 (source: Howell, 1993). We see that this spiral dynamics can be produced by a nonlinear interdependence between rational knowledge, economic growth, and political struggle. The theoretical results from our model are qualitatively corresponsive to Howell's observations. The interdependence between political and economic conditions is illustrated in Fig. 6 (source: Howell, 1993). It must be emphasized that cycles may be simplified behavior of the actual dynamics. The system may exhibit more complicated patterns. Howell observes that it is not easy to determine precisely the relative weight of political and economic factors. Political factors such as the leadership struggle between the reformers and ultra-leftists in the early 1980s are significant in determining the dynamics. In other cycles, economic factors may be driving forces.

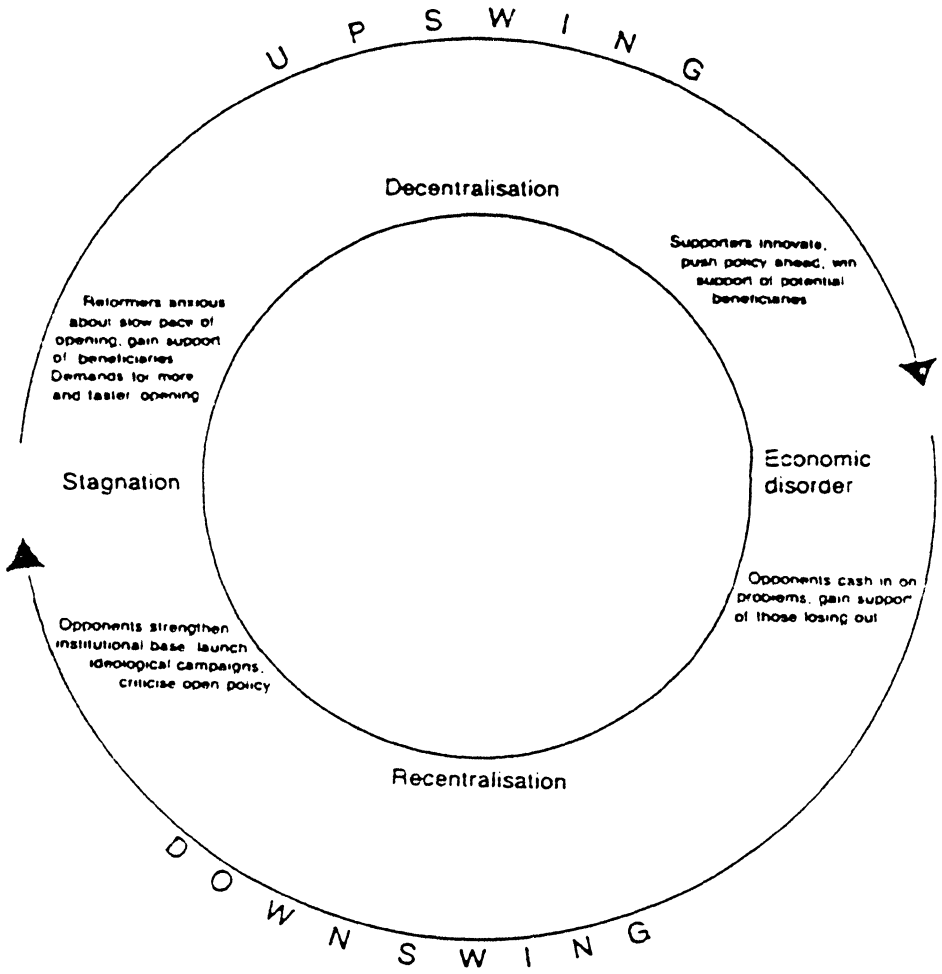


FIGURE 6 Political and economic cycles.

5 CONCLUDING REMARKS

This study examined the significance of distinguishing speeds of changes and time scales of dynamic analysis for understanding the complexity of social and economic changes. We found that the new visions about changes by modern nonlinear theory may provide deep insights into the complexity of human societies. First, we provided a brief introduction to nonlinear theory. Second, we discussed possible differences in change speeds of different social and economic variables under different cultural circumstances in different times. Third, we illustrated the ideas of the preceding sections by explaining the development of inter-regional economic relations in the world from the years around 1000 A.D until 2000 in the term of Andersson's four logistical revolutions. Finally, we utilized Zhang's economic development model for Chinese economy to provide some insight into the complicated interactions among capital and knowledge accumulation and economic and cultural opening to the outside world.

The models represented in this paper are described by differential equations. We may develop

the basic ideas in the form of discrete processes. Dynamic systems described as discrete processes and continuous processes show different complexity. Chaos may occur in one-dimensional mapping; while one cannot observe chaos in a one-dimensional differential equation system. In the literature of social sciences there are many dynamic systems described as discrete processes. Analytical methods for investigating properties of such systems are referred to, for instance (de Melo and van Srien, 1991; Sonis, 1993).

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