Hindawi Publishing Corporation
Advances in Mathematical Physics
Volume 2013, Article ID 416294, 5 pages
http://dx.doi.org/10.1155/2013/416294

Research Article

Anisotropic Bianchi Type-III Bulk Viscous Fluid Universe in Lyra Geometry

Priyanka Kumari, M. K. Singh, and Shri Ram

Department of Applied Mathematics, Institute of Technology, Banaras Hindu University, Varanasi, Uttar Pradesh 221 005, India

Correspondence should be addressed to Shri Ram; srmathitbhu@rediffmail.com

Received 22 December 2012; Accepted 10 February 2013

Academic Editor: Shao-Ming Fei

An anisotropic Bianchi type-III cosmological model is investigated in the presence of a bulk viscous fluid within the framework of Lyra geometry with time-dependent displacement vector. It is shown that the field equations are solvable for any arbitrary function of a scale factor. To get the deterministic model of the universe, we have assumed that (i) a simple power-law form of a scale factor and (ii) the bulk viscosity coefficient are proportional to the energy density of the matter. The exact solutions of the Einstein's field equations are obtained which represent an expanding, shearing, and decelerating model of the universe. Some physical and kinematical behaviors of the cosmological model are briefly discussed.

1. Introduction

After Einstein (1916) proposed his theory of general relativity which provided a geometrical description of gravitation, many physicists attempted to generalize the idea of geometrizing the gravitation to include a geometrical description of electromagnetism. One of the first attempts was made by Weyl [1] who proposed a more general theory by formulating a new kind of gauge theory involving metric tensor to geometrize gravitation and electromagnetism. But Weyl theory was criticized due to the nonintegrability of length of vector under parallel displacement. Later, Lyra [2] suggested a modification of Riemannian geometry by introducing a gauge function into the structureless manifold which removed the nonintegrability condition. This modified geometry is known as Lyra geometry. Subsequently, Sen [3] formulated a new scalar-tensor theory of gravitation and constructed an analogue of the Einstein's field equations based on Lyra geometry. He investigated that the static model with finite density in Lyra manifold is similar to the static model in Einstein's general relativity. Halford [4] has shown that the constant displacement vector field in Lyra geometry plays the role of cosmological constant in general relativity. He has also shown that the scalar-tensor treatment based in Lyra geometry predicts the same effects, within observational limits, as in Einstein's theory (Halford, [5]).

Soleng [6] has investigated cosmological models based on Lyra geometry and has shown that the constant gauge vector field either includes a creation field and be identical to Hoyle's creation cosmology (Hoyle, [7], Hoyle, and Narlikar [8, 9]) or contains a special vacuum field which together with the gauge vector term may be considered as a cosmological term. In the latter case, solutions are identical to the general relativistic cosmologies with a cosmological term.

The cosmological models based on Lyra geometry with constant and time-dependent displacement vector fields have been investigated by a number of authors, namely, Beesham [10], T. Singh and G. P. Singh [11], Chakraborty and Ghosh [12], Rahaman and Bera [13], Rahaman et al. [14, 15], Pradhan and Vishwakarma [16], Ram and Singh [17], Pradhan et al. [18, 19], Ram et al. [20], Pradhan [21, 22], Mohanty et al. [23], Bali and Chandnani [24], and so forth.

Bali and Chandnani [25] have investigated a Bianchi type-III bulk viscous dust filled universe in Lyra geometry under certain physical assumptions. Recently, V. K. Yadav and L. Yadav [26] have presented Bianchi type-III bulk viscous and barotropic perfect fluid cosmological models in Lyra's geometry with the assumption that the coefficient of viscosity of dissipative fluid is a power function of the energy density. In this paper, we investigate a Bianchi type-III universe filled with a bulk viscous fluid within the framework of Lyra geometry with time-dependent displacement vector field without assuming the barotropic equation of state for the matter field. The organization of the paper is as follows. In Section 2, we present the metric and Einstein's field equations. In Section 3, we deal with the solution of the field equations. We first show that the field equations are solvable for any arbitrary scale function. There after, we obtain exact solutions of the field equation by assuming (i) a power-law form of a scale factor and (ii) the bulk viscosity coefficient is directly proportional to the energy density of the matter. In Section 4, we discuss the physical and dynamical behaviors of the universe. Section 5 summarizes the main results presented in the paper.

2. Metric and Field Equations

The diagonal form of the Bianchi type-III space-time is considered in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2\alpha x} dy^2 - C^2 dz^2, \quad (1)$$

where α is a constant and A, B, and C are functions of cosmic time t.

Einstein's field equations in normal gauge for Lyra's geometry given by Sen [3] are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \frac{3}{2}\left(\phi_\mu \phi_\nu - \frac{1}{2} g_{\mu\nu} \phi_\alpha \phi^\alpha\right) = -T_{\mu\nu}, \quad (2)$$

where ϕ_μ is the displacement vector field defined as $\phi_\mu = (0, 0, 0, \beta(t))$. The energy-momentum tensor $T_{\mu\nu}$ for bulk viscous fluid is given by

$$T_{\mu\nu} = (\rho + \overline{p}) u_\mu u_\nu - \overline{p} g_{\mu\nu}, \quad (3)$$

where ρ is the energy density and u^ν is the four-velocity vector of the fluid satisfying $u_\mu u^\nu = 1$. The effective pressure $\overline{p}$ is given by

$$\overline{p} = p - \xi u^\mu_{;\mu}, \quad (4)$$

where p is the isotropic pressure, β is the gauge function, and ξ is the bulk viscosity coefficient.

For the metric (1), the field equations (2) together with (3) and (4) lead to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\overline{p} - \frac{3}{4}\beta^2, \quad (5)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} = -\overline{p} - \frac{3}{4}\beta^2, \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -\overline{p} - \frac{3}{4}\beta^2, \quad (7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{\alpha^2}{A^2} = \rho + \frac{3}{4}\beta^2, \quad (8)$$

$$\alpha\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \quad (9)$$

where an overhead dot denotes differentiation with respect to time t. Here, we have used the geometrized unit in which $8\pi G = 1$ and $c = 1$.

The energy conservation equation $T^\nu_{\mu;\nu} = 0$ leads to

$$\dot{\rho} + (\rho + \overline{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \quad (10)$$

The conservation of left-hand side of (2) yields (Bali and Chandnani [25])

$$\beta\dot{\beta} + \beta^2\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \quad (11)$$

The physical quantities that are important in cosmology are proper volume V, average scale factor R, expansion scalar θ, shear scalar σ, and Hubble parameter H. For the metric (1), they have the following form:

$$V = R^3 = ABC, \quad (12)$$

$$\theta = u^\mu_{;\mu} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (13)$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \quad (14)$$

$$H = \frac{\dot{R}}{R}. \quad (15)$$

An important observational quantity in cosmology is the deceleration parameter q which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2}. \quad (16)$$

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to a standard decelerating model, whereas negative sign indicates inflation.

3. Exact Solutions

We now solve the field equations (5)–(11) by using the method developed by Mazumdar [27] and further used by Verma and Ram [28].

From (9), we obtain

$$A = kB, \quad (17)$$

where k is an integration constant. Without loss of generality, we take $k = 1$. Using (17), (5)–(11) reduce to

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\overline{p} - \frac{3}{4}\beta^2, \quad (18)$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\alpha^2}{B^2} = -\overline{p} - \frac{3}{4}\beta^2, \quad (19)$$

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{B^2} = \rho + \frac{3}{4}\beta^2, \quad (20)$$

$$\dot{\rho} + (\rho + \overline{p})\left(\frac{2\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0, \quad (21)$$

$$\frac{\dot{\beta}}{\beta} + \left(\frac{2\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0. \quad (22)$$

From (18) and (19), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{B^2} = 0. \quad (23)$$

This is an equation involving two unknown functions B and C which will admit solution if one of them is a known function of t. To get a physically realistic model, Bali and Chandanani [25] and V. K. Yadav and L. Yadav [26] have assumed a supplementary condition $B = C^n$ between the metric potentials B and C. This condition is based on the physical assumption that the shear scalar σ is proportional to the expansion scalar θ. In order to obtain a simple but physically realistic solution, we make the mathematical assumption that

$$C = t^n, \quad (24)$$

where n is positive real number. For this, we first show that (23) is solvable for an arbitrary choice of C. Multiplying (23) by B^2C, we get

$$BC\ddot{B} - B\dot{B}\dot{C} + C\dot{B}^2 - B^2\ddot{C} = \alpha^2 C. \quad (25)$$

This can be written in the following form:

$$\frac{d}{dt}\left(-B^2\dot{C} + BC\dot{B}\right) = \alpha^2 C. \quad (26)$$

The first integral of (26) is

$$-B^2\dot{C} + BC\dot{B} = \alpha^2\left(\int C\,dt + k_1\right), \quad (27)$$

where k_1 is a constant of integration. We can write (27) in the following form:

$$\frac{d}{dt}\left(B^2\right) - \frac{2\dot{C}}{C}B^2 = F(t), \quad (28)$$

where

$$F(t) = \frac{2\alpha^2}{C}\left(\int C\,dt + k_1\right). \quad (29)$$

The general solution of (28) is given by

$$B^2 = C^2\left(\int \frac{F(t)}{C^2}dt + k_2\right), \quad (30)$$

where k_2 is a constant of integration. Note that in this case the solution of Einstein's field equations reduces to the integration of (30) if C is explicitly known function of t. Making use of (24) in (29) and (30), we obtain the general solution B^2 of the form

$$B^2 = \frac{\alpha^2 t^2}{1-n^2} + \frac{2k_1\alpha^2 t^{1-n}}{1-3n} + k_2 t^{2n}. \quad (31)$$

For further discussion of the solutions, we take $k_1 = k_2 = 0$. Then,

$$B^2 = \frac{\alpha^2 t^2}{1-n^2}, \quad n \neq 1. \quad (32)$$

Hence, the metric of our solutions can be written in the form

$$ds^2 = dt^2 - \frac{\alpha^2 t^2}{1-n^2}\left(dx^2 + e^{-2\alpha x}dy^2\right) - t^{2n}dz^2, \quad (33)$$

where $0 < n < 1$.

4. Some Physical and Kinematical Properties of the Model

The model (33) represents an anisotropic Bianchi type-III cosmological universe filled with a bulk viscous fluid in the framework of Lyra geometry.

From (22), (24), and (32), the gauge function β has the expression given by

$$\beta = \frac{c_1\left(1-n^2\right)}{\alpha^2}t^{-(n+2)}, \quad (34)$$

where c_1 is an integration constant. Using (24), (32), and (34) into (18) and (20), we obtain

$$\overline{p} = -\left[\frac{n^2}{t^2} + \frac{3c_1^2\left(1-n^2\right)^2}{4\alpha^4 t^{2n+4}}\right], \quad (35)$$

$$\rho = \frac{n(n+2)}{t^2} - \frac{3c_1^2\left(1-n^2\right)^2}{4\alpha^4 t^{2n+4}}. \quad (36)$$

It is clear that, given $\xi(t)$, we can determine isotropic pressure p. In most of the investigations involving bulk viscosity, $\xi(t)$ is assumed to be simple power function of energy density (Weinberg, [29]):

$$\xi(t) = \xi_0\rho^m, \quad (37)$$

where ξ_0 and $m > 0$ are constants. The case $m = 1$ has been considered by Murphy [30] which corresponds to a radiating fluid. We take $m = 1$ so that

$$\xi = \xi_0\rho. \quad (38)$$

From (4), (13), (36), (37), and (38), we obtain

$$p = \frac{\xi_0 n(n+2)^2}{t^3} - \frac{n^2}{t^2} - \frac{3c_1^2\left(1-n^2\right)^2}{4\alpha^4 t^{2n+4}}\left[1 + \frac{\xi_0(n+2)}{t}\right]. \tag{39}$$

Clearly, the viscosity contributes significantly to the isotropic pressure of the fluid.

The physical and kinematical parameters of the model (33) have the following expressions:

$$\begin{aligned} V = R^3 &= \frac{\alpha^2}{1-n^2}t^{n+2}, \\ \theta &= \frac{n+2}{t}, \\ \sigma &= \frac{1-n}{\sqrt{3}t}, \\ H &= \frac{n+2}{3t}, \\ q &= \frac{1-n}{2+n}. \end{aligned} \tag{40}$$

The deceleration parameter is positive which shows the decelerating behavior of the cosmological model. It is worthwhile to mention the work of Vishwakarma [31], where he has shown that the decelerating model is also consistent with recent CMB observations model by WNAP, as well as with the high-redshift supernovae Ia data including 1997ff at $Z = 1.755$.

We observe that the spatial volume V is zero at $t = 0$, and it increases with the cosmic time. This means that the model starts expanding with a big bang at $t = 0$. All the physical and kinematical parameters ρ, p, θ, and σ diverge at this initial singularity. The physical and kinematical parameters are well defined and are decreasing functions for $0 < t < \infty$, and ultimately tend to zero for large time. The gauge function $\beta(t)$ and bulk viscosity coefficient $\xi(t)$ are infinite at the beginning and gradually decrease as time increases and ultimately tend to zero at late times. Since $\sigma/\theta = (1-n)/\sqrt{3}(n+2) = \text{const}$, the anisotropy in the universe is maintained throughout the passage of time.

5. Conclusions

We have presented an anisotropic Bianchi type-III cosmological model in the presence of a bulk viscous fluid within the framework of Lyra's geometry with time-dependent displacement vector. The model describes an expanding, shearing, and decelerating universe with a big-bang singularity at $t = 0$. All the physical and kinematical parameters start off with extremely large values, which continue to decrease with the expansion of the universe and ultimately tend to zero for large time. As ρ tends to zero as t tends to infinity, the model would essentially give an empty space-time for large time.

Acknowledgment

The authors are extremely grateful to the referee for his valuable suggestions.

References

[1] H. Weyl, *Sitzungsberichte Der Preussischen Akademie Der Wissenschaften*, 1918.

[2] G. Lyra, "Über eine modifikation der riemannschen geometrie," *Mathematische Zeitschrift*, vol. 54, pp. 52–64, 1951.

[3] D. K. Sen, "A static cosmological model," *Zeitschrift für Physik C*, vol. 149, pp. 311–323, 1957.

[4] W. D. Halford, "Cosmological theory based Lyra's geometry," *Australian Journal of Physics*, vol. 23, pp. 863–869, 1970.

[5] W. D. Halford, "Scalar-tensor theory of gravitation in a Lyra manifold," *Journal of Mathematical Physics*, vol. 13, no. 11, pp. 1699–1703, 1972.

[6] H. H. Soleng, "Cosmologies based on Lyra's geometry," *General Relativity and Gravitation*, vol. 19, no. 12, pp. 1213–1216, 1987.

[7] F. Hoyle, "A new model for the expanding universe," *Monthly Notices of the Royal Astronomical Society*, vol. 108, pp. 372–382, 1948.

[8] F. Hoyle and J. V. Narlikar, "On the avoidance of singularities in C-field cosmology," *Proceedings of the Royal Society A*, vol. 278, pp. 465–478, 1964.

[9] F. Hoyle and J. V. Narlikar, "Time symmetric electrodynamics and the arrow of time in cosmology," *Proceedings of the Royal Society A*, vol. 277, pp. 1–23, 1964.

[10] A. Beesham, "FLRW cosmological models in Lyra's manifold with time dependent displacement field," *Australian Journal of Physics*, vol. 41, no. 6, pp. 833–842, 1988.

[11] T. Singh and G. P. Singh, "Lyra's geometry and cosmology: a review," *Fortschritte der Physik*, vol. 41, no. 8, pp. 737–764, 1993.

[12] S. Chakraborty and A. Ghosh, "Generalized scalar tensor theory in four and higher dimension," *International Journal of Modern Physics D*, vol. 9, no. 5, pp. 543–549, 2000.

[13] F. Rahaman and J. Bera, "Higher-dimensional cosmological model in Lyra geometry," *International Journal of Modern Physics D*, vol. 10, no. 5, pp. 729–733, 2001.

[14] F. Rahaman, S. Chakraborty, and J. Bera, "Inhomogeneous cosmological model in Lyra geometry," *International Journal of Modern Physics D*, vol. 11, no. 9, pp. 1501–1504, 2002.

[15] F. Rahaman, N. Begum, G. Bag, and B. C. Bhui, "Cosmological models with negative constant deceleration parameter in Lyra geometry," *Astrophysics and Space Science*, vol. 299, no. 3, pp. 211–218, 2005.

[16] A. Pradhan and A. K. Vishwakarma, "A new class of LRS Bianchi type-I cosmological models in Lyra geometry," *Journal of Geometry and Physics*, vol. 49, no. 3-4, pp. 332–342, 2004.

[17] S. Ram and P. Singh, "Anisotropic cosmological models of Bianchi types III and IV in Lyra's geometry," *International Journal of Theoretical Physics*, vol. 31, no. 12, pp. 2095–2102, 1992.

[18] A. Pradhan, V. K. Yadav, and I. Chakrabarty, "Bulk viscous FRW cosmology in Lyra geometry," *International Journal of Modern Physics D*, vol. 10, no. 3, pp. 339–349, 2001.

[19] A. Pradhan, V. Rai, and S. Otarod, "Plane symmetric inhomogeneous bulk viscous domain wall in Lyra geometry," *Fizika B*, vol. 15, pp. 57–70, 2006.

[20] S. Ram, M. Zeyauddin, and C. P. Singh, "Anisotropic Bianchi type V perfect fluid cosmological models in Lyra's geometry," *Journal of Geometry and Physics*, vol. 60, no. 11, pp. 1671–1680, 2010.

[21] A. Pradhan, "Cylindrically symmetric viscous fluid universe in Lyra geometry," *Journal of Mathematical Physics*, vol. 50, Article ID 022501, 13 pages, 2009.

[22] A. Pradhan, "Thick domain walls in lyra geometry with bulk viscosity," *Communications in Theoretical Physics*, vol. 51, no. 2, pp. 378–384, 2009.

[23] G. Mohanty, K. L. Mahanta, and R. R. Sahoo, "Non-existence of five dimensional perfect fluid cosmological model in Lyra manifold," *Astrophysics and Space Science*, vol. 306, no. 4, pp. 269–272, 2006.

[24] R. Bali and N. K. Chandnani, "Bianchi type V barotropic perfect fluid cosmological model in Lyra geometry," *International Journal of Theoretical Physics*, vol. 48, no. 5, pp. 1523–1533, 2009.

[25] R. Bali and N. K. Chandnani, "Bianchi type-III bulk viscous dust filled universe in Lyra geometry," *Astrophysics and Space Science*, vol. 318, no. 3-4, pp. 225–230, 2008.

[26] V. K. Yadav and L. Yadav, "Bianchi type-III bulk viscous and barotropic perfect fluid cosmological models in Lyra's geometry," *International Journal of Theoretical Physics*, vol. 50, no. 5, pp. 1382–1394, 2011.

[27] A. Mazumder, "Solutions of LRS Bianchi I space-time filled with a perfect fluid," *General Relativity and Gravitation*, vol. 26, no. 3, pp. 307–310, 1994.

[28] M. K. Verma and S. Ram, "Bulk viscous Bianchi type-III cosmological model with time-dependent G and Λ," *International Journal of Theoretical Physics*, vol. 49, no. 4, pp. 693–700, 2010.

[29] S. Weinberg, *Gravitational and Cosmology*, Wiley, New York, NY, USA, 1972.

[30] G. L. Murphy, "Big-bang model without singularities," *Physical Review D*, vol. 8, no. 12, pp. 4231–4233, 1973.

[31] R. G. Vishwakarma, "Is the present expansion of the Universe really accelerating?" *Monthly Notices of the Royal Astronomical Society*, vol. 345, no. 2, pp. 545–551, 2003.


Advances in Operations Research
Advances in Decision Sciences
Mathematical Problems in Engineering
Algebra
Journal of Probability and Statistics
The Scientific World Journal
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Differential Equations
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
International Journal of Stochastic Analysis
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
Journal of Applied Mathematics
Journal of Optimization