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Research Article

Abundant Exact Solition-Like Solutions to the Generalized Bretherton Equation with Arbitrary Constants

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The Riccati equation is employed to construct exact travelling wave solutions to the generalized Bretherton equation. Taking full advantage of the Riccati equation which has more new solutions, abundant new multiple solition-like solutions are obtained for the generalized Bretherton equation.

1. Introduction

Nonlinear partial differential equations (PDE) are widely chosen to describe complex phenomena in physics sciences. Searching for exact solutions to nonlinear differential equations plays more and more important role in nonlinear science. Recently, various direct methods have been proposed, such as the tanh-function method [1, 2], the Jacobi elliptic function expansion method [3, 4], the F-expansion [5–8], sine-cosine method [9, 10], and the homogeneous balance method [11–13]. Among them, the tanh-function method is improved continuously [14–17] as one of the most effectively straightforward methods for constructing exact solutions to PDEs. In the paper an extended tanh-function method is used to solve the generalized Bretherton equation with arbitrary constants.

In [18], Bretherton introduced the partial differential equation

$$u_{tt} + u_{xx} + u_{xxxx} + u - u^2 = 0 \tag{1a}$$

in time and one spatial dimension as a model of a dispersive wave system to study the resonant nonlinear interaction between three liner models. The modified Bretherton equation

$$u_{tt} + u_{xx} + u_{xxxx} + u - u^3 = 0 \tag{1b}$$

was studied by Kudryashov [19], Kudryashov et al. [20], and Berloff and Howard [21], and its travelling wave solutions were obtained.

Our aim in this paper is to investigate multiple soliton-like solutions to the generalized Bretherton equation in [22] by using the solutions to the Riccati equation:

$$u_{tt} + \alpha u_{xx} + \beta u_{xxxx} + \delta u + \gamma u^3 = 0. \tag{1c}$$

2. Multiple Soliton-Like Solutions to the Generalized Bretherton Equation

We assume the travelling wave variable

$$u(x,t) = u(\xi), \quad \xi = x - Vt, \tag{2}$$

where V is the speed of the travelling wave.

Making use of the travelling wave transformation (2), (1c) is converted into an ordinary differential equation (ODE) for $u = u(\xi)$ as follows:

$$\left(V^2 + \alpha\right) u'' + \beta u^{(4)} + \delta u + \gamma u^3 = 0. \tag{3}$$

We assume that the solutions to (3) can be expressed in the form

$$u(\xi) = \sum_{i=-M}^{M} a_i \phi^i, \tag{4}$$

where ϕ is a solution of the Riccati equation,

$$\phi' = A + B\phi + C\phi^2, \tag{5}$$

where a_i $(i = 0, \pm1, \pm2, \ldots, \pm M)$ A, B, and C are constants to be determined later, and either a_{-M} or a_M can be zero, but they cannot be zero together.

Substituting (4) into (3) together with (5) and considering the homogeneous balance between the highest-order derivative $u^{(4)}$ and the nonlinear term u^3, we obtain $M = 2$. Thus the solution to (3) takes the following form:

$$u(\xi) = a_2\phi^2 + a_1\phi + a_0 + a_{-1}\phi^{-1} + a_{-2}\phi^{-2}. \tag{6}$$

Substituting (6) with (5) into (3) and collecting all the terms of the same power of ϕ, the left-hand side of (3) is converted into another polynomial of ϕ. Setting the coefficients of ϕ^i $(i = 0, \pm1, \pm2)$ to zero yields a set of algebraic equations

$$\begin{aligned}
&\gamma a_{-2}^3 - 120\beta a_{-2}A^4 = 0,\\
&3\gamma a_{-1}a_{-2}^2 + 24\beta a_{-1}A^4 + 96\beta a_{-2}BA^3 = 0,\\
&330\beta a_{-2}B^2A^2 + 60\beta a_{-1}BA^3 + 3\gamma a_{-1}^2a_{-2} + 6\alpha a_{-2}A^2\\
&\quad + 3\gamma a_0a_{-2}^2 + 6V^2a_{-2}A^2 = 0,\\
&40\beta a_{-1}A^3C + 3\gamma a_1a_{-2}^2 + 2V^2a_{-1}A^2 + 10V^2a_{-2}BA + 2\alpha a_{-1}A^2\\
&\quad + 50\beta a_{-1}B^2A^2 + 6\gamma a_{-1}a_0a_{-2} + 130\beta B^3A + \gamma a_{-1}^3\\
&\quad + 440\beta a_{-2}CBA^2 + 10\alpha a_{-2}BA = 0,\\
&6\gamma a_1a_{-1}a_{-2} + 60\beta a_{-1}A^2CB + 3\gamma a_2a_{-2}^2 + \delta a_{-2}\\
&\quad + 3\gamma a_{-1}^2a_0 + 3\alpha a_{-1}BA + 8V^2a_{-2}CA\\
&\quad + 8\alpha a_{-2}CA + 136\beta a_{-2}C^2A^2 + 3\gamma a_0^2a_{-2}\\
&\quad + 4\alpha a_{-2}B^2 + 4V^2a_{-2}B^2 + 16\beta a_{-2}B^4\\
&\quad + 3V^2a_{-1}BA + 15\beta a_{-1}B^3A + 232\beta a_{-2}CB^2A = 0,\\
&16\beta a_{-1}A^2C^2 + 6V^2a_{-2}CB + 2\alpha a_{-1}AC + \beta a_{-1}B^4\\
&\quad + \delta a_{-1} + 6\gamma a_2a_{-1}a_{-2} + 30\beta a_{-2}CB^3 + 3\gamma a_{-1}a_0^2\\
&\quad + 2V^2a_{-1}AC + 6\gamma a_1a_0a_{-2} + \alpha a_{-1}B^2\\
&\quad + 120\beta a_{-2}C^2AB + 6\alpha a_{-2}CB + 22\beta a_{-1}ACB^2\\
&\quad + a_{-1}V^2B^2 + 3\gamma a_1a_{-1}^2 = 0,\\
&2\alpha a_{-2}C^2 + 2\alpha a_2A^2 + 2a_2V^2A^2 + 2a_{-2}C^2 + 3\gamma a_1^2a_{-2}\\
&\quad + 3\gamma a_2a_{-1}^2 + 8\beta a_{-1}AC^2B + \delta a_0 + a_1V^2BA\\
&\quad + V^2a_{-1}BC + \alpha a_1BA + \alpha a_{-1}BC + 16\beta a_2CA^3\\
&\quad + \beta a_1B^3A + 14\beta a_2B^2A^2 + 16\beta a_{-2}C^3A + \beta a_{-1}B^3C\\
&\quad + 14\beta a_{-2}C^2B^2 + 6\gamma a_2a_0a_{-2} + 6\gamma a_1a_{-1}a_0\\
&\quad + 8\beta\alpha_1CBA^2 + \gamma a_0^3 = 0,\\
&6\gamma a_2a_{-1}a_0 + \alpha a_{-1}B^2 + 6a_2V^2BA + a_1V^2B^2\\
&\quad + 3\gamma a_1a_0^2 + 2a_1V^2CA + 2\alpha a_1CA + 3\gamma a_1^2a_{-1} + \delta a_1\\
&\quad + 6\alpha a_2BA + 22\beta a_1CB^2A + 30\beta a_2B^3A\\
&\quad + 16\beta a_1C^2A^2 + \beta a_1B^4\\
&\quad + 6\gamma a_2a_1a_{-2} + 120\beta a_2CBA^2 = 0,\\
&4\alpha a_2B^2 + 8\alpha a_2CA + 136\beta a_2C^2A^2 + 16\beta a_2B^4\\
&\quad + 3\gamma a_2a_0^2 + \delta a_2 + 3V^2BC + 3\alpha a_1CB\\
&\quad + 15\beta a_1CB^3 + 4a_2V^2B^2 + 60\beta a_1ABC^2\\
&\quad + 8a_2V^2CA + 3\gamma a_1^2a_0 + 23\beta a_2CB^2A\\
&\quad + 3\gamma a_2^2a_{-2} + 6\gamma a_2a_1a_{-1} = 0,\\
&40\beta a_1C^3A + 2\alpha a_1C^2 + 50\beta a_1C^2B^2 + 10\alpha a_2CB\\
&\quad + 6\gamma a_2a_1a_0 + \gamma a_1^3 + 10a_2V^2BC\\
&\quad + 3\gamma a_2^2a_{-1} + 130\beta a_2CB^3 + 440\beta a_2C^2BA\\
&\quad + 2a_1V^2C^2 = 0,\\
&3\gamma a_2^2a_0 + 3\gamma a_1^2a_2 + 6a_2V^2C^2 + 60\beta a_1C^3B\\
&\quad + 240\beta a_2C^3A + 330\beta a_2C^2B^2 + 6\alpha a_2C^2 = 0,\\
&3\gamma a_2^2a_1 + 24\beta a_1C^4 + 336\beta a_2C^3B = 0,\\
&\gamma a_2^3 + 120\beta a_2C^4 = 0.
\end{aligned} \tag{7}$$

Solving (7) with the help of the symbolic computation software Maple, we obtain the following.

Case 1. One has

$$\begin{aligned}
&a_{-1} = 0, \qquad a_{-2} = 0, \qquad a_2 = 2C^2\sqrt{\frac{-30\beta}{\gamma}},\\
&a_1 = 2BC\sqrt{\frac{-30\beta}{\gamma}}, \qquad a_0 = 2AC\sqrt{\frac{-30\beta}{\gamma}},\\
&V = \pm\sqrt{-\alpha - 5B^2\beta + 20CA\beta},\\
&\delta = 4\beta\left(-8CAB^2 + B^4 + 16A^2C^2\right).
\end{aligned} \tag{8a}$$

Case 2. One has

$$\begin{aligned}
&a_{-1} = 0, \qquad a_{-2} = 0, \qquad a_2 = -2C^2\sqrt{\frac{-30\beta}{\gamma}},\\
&a_1 = -2BC\sqrt{\frac{-30\beta}{\gamma}}, \qquad a_0 = -2AC\sqrt{\frac{-30\beta}{\gamma}},
\end{aligned}$$

$$V = \pm\sqrt{-\alpha - 5B^2\beta + 20CA\beta},$$
$$\delta = 4\beta\left(-8CAB^2 + B^4 + 16A^2C^2\right), \tag{8b}$$

where A, B, and C are arbitrary constants, but C cannot be zero.

Case 3. One has

$$a_{-1} = 0, \quad a_2 = 0, \quad a_1 = 0, \quad a_{-2} = 2A^2\sqrt{\frac{30\beta}{\gamma}},$$
$$a_0 = 2AC\sqrt{\frac{30\beta}{\gamma}}, \quad B = 0,$$
$$V = \pm\sqrt{-\alpha - 60CA\beta}, \quad \delta = -16\beta A^2C^2. \tag{8c}$$

Case 4. One has

$$a_{-1} = 0, \quad a_2 = 0, \quad a_1 = 0, \quad a_{-2} = 2A^2\sqrt{\frac{30\beta}{\gamma}},$$
$$a_0 = -\frac{-15 \pm \sqrt{165}}{15}CA\sqrt{\frac{30\beta}{\gamma}}, \quad B = 0,$$
$$V = \varepsilon\sqrt{-\alpha + \left(-30 \pm 2\sqrt{165}\right)CA\beta},$$
$$\delta = -4A^2C^2\beta\left(13 \pm \sqrt{165}\right), \tag{8d}$$

where A, B, and C are arbitrary constants, but A cannot be zero.

Case 5. One has

$$a_{-1} = 0, \quad a_2 = 0, \quad a_1 = 0, \quad a_{-2} = -2A^2\sqrt{\frac{30\beta}{\gamma}},$$
$$a_0 = -2AC\sqrt{\frac{30\beta}{\gamma}}, \quad B = 0,$$
$$V = \pm\sqrt{-\alpha - 60CA\beta}, \quad \delta = -16BA^2C^2. \tag{8e}$$

Case 6. One has

$$a_{-1} = 0, \quad a_2 = 0, \quad a_1 = 0, \quad a_{-2} = -2A^2\sqrt{\frac{30\beta}{\gamma}},$$
$$a_0 = \frac{\left(-15 \pm \sqrt{165}\right)CA}{15}\sqrt{\frac{30\beta}{\gamma}}, \quad B = 0,$$
$$V = \varepsilon\sqrt{-\alpha + \left(-30 \pm 2\sqrt{165}\right)CA\beta},$$
$$\delta = -4A^2C^2\beta\left(13 \pm \sqrt{165}\right), \tag{8f}$$

where A, B, and C are arbitrary constants, while C cannot be zero in Cases 1 and 2 and A cannot be zero in Cases 3–6. ε is an arbitrary element of $\{-1, 1\}$.

Substituting ((8a), (8b), (8c), (8d), (8e), (8f)) into (6) respectively and taking advantage of solitions to (5), we can find the following solutions which contain multiple soliton-like and triangular periodic solutions for the generalized Bretherton equation.

When $\delta = 4\beta$,

$$u_{1a} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\coth\left(x + \varepsilon\sqrt{-\alpha - 5\beta}t\right) \pm \operatorname{csch}\left(x + \varepsilon\sqrt{-\alpha - 5\beta}t\right)\right)^2 - \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9a}$$

$$u_{1b} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\frac{\tanh\left(x + \varepsilon\sqrt{-\alpha - 5\beta}t\right)}{\left(1 \pm \operatorname{sech}\left(x + \varepsilon\sqrt{-\alpha - 5\beta}t\right)\right)}\right)^2 - \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9b}$$

$$u_{1c} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\tan\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right) \pm \sec\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right)\right)^2 + \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9c}$$

$$u_{1d} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\csc\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right) - \cot\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right)\right)^2 + \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9d}$$

$$u_{1e} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\frac{\tan\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right)}{\left(1 \pm \sec\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right)\right)}\right)^2 + \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9e}$$

$$u_{1f} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\cot\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right) \pm \csc\left(x + \varepsilon\sqrt{-\alpha + 5\beta}t\right)\right)^2 + \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9f}$$

$$u_{1g} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\sec\left(x+\varepsilon\sqrt{-\alpha+5\beta}t\right) - \tan\left(x+\varepsilon\sqrt{-\alpha+5\beta}t\right)\right)^2 + \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9g}$$

$$u_{1h} = \theta\sqrt{\frac{-15\beta}{2\gamma}}\left(\frac{\cot\left(x+\varepsilon\sqrt{-\alpha+5\beta}t\right)}{\left(1\pm\csc\left(x+\varepsilon\sqrt{-\alpha+5\beta}t\right)\right)}\right)^2 + \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{9h}$$

when $\delta = 64\beta$,

$$u_{2a} = 2\theta\sqrt{\frac{-30\beta}{\gamma}}\tanh^2\left(x\pm\sqrt{-\alpha-20\beta}t\right) - 2\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{10a}$$

$$u_{2b} = 2\theta\sqrt{\frac{-30\beta}{\gamma}}\coth^2\left(x\pm\sqrt{-\alpha-20\beta}t\right) - 2\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{10b}$$

$$u_{2c} = 2\theta\sqrt{\frac{-30\beta}{\gamma}}\tan^2\left(x\pm\sqrt{-\alpha+20\beta}t\right) + 2\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{10c}$$

$$u_{2d} = 2\theta\sqrt{\frac{-30\beta}{\gamma}}\cot^2\left(x\pm\sqrt{-\alpha+20\beta}t\right) + 2\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{10d}$$

$$u_{2e} = 8\theta\sqrt{\frac{-30\beta}{\gamma}}\left(\frac{\tan\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)}{\left(1\pm\tan\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)\right)}\right)^2 \mp 8\theta\sqrt{\frac{-30\beta}{\gamma}}\frac{\tan\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)}{\left(1\pm\tan\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)\right)} + 4\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{10e}$$

$$u_{2f} = 8\theta\sqrt{\frac{-30\beta}{\gamma}}\left(\frac{\cot\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)}{\left(1\pm\cot\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)\right)}\right)^2 \mp 8\theta\sqrt{\frac{-30\beta}{\gamma}}\frac{\cot\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)}{\left(1\pm\cot\left(x+\varepsilon\sqrt{-\alpha+20\beta}t\right)\right)} + 4\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{10f}$$

when $\delta = 1024\beta$,

$$u_{3a} = 32\theta\sqrt{\frac{-30\beta}{\gamma}}\left(\frac{\tanh\left(x\pm\sqrt{-\alpha-80\beta}t\right)}{\left(1+\tanh^2\left(x\pm\sqrt{-\alpha-80\beta}t\right)\right)}\right)^2 - 8\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{11a}$$

$$u_{3b} = 32\theta\sqrt{\frac{-30\beta}{\gamma}}\left(\frac{\tan\left(x\pm\sqrt{-\alpha+80\beta}t\right)}{\left(1-\tan^2\left(x\pm\sqrt{-\alpha+80\beta}t\right)\right)}\right)^2 + 8\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{11b}$$

$$u_{3c} = 32\theta\sqrt{\frac{-30\beta}{\gamma}}\left(\frac{\cot\left(x\pm\sqrt{-\alpha+80\beta}t\right)}{\left(1-\cot^2\left(x\pm\sqrt{-\alpha+80\beta}t\right)\right)}\right)^2 + 8\theta\sqrt{\frac{-30\beta}{\gamma}}, \tag{11c}$$

when $\delta = 0$,

$$u_4 = \frac{\pm 2\sqrt{-30\beta/\gamma}}{\left(\left(x+\varepsilon\sqrt{-\alpha}t\right)^2+c_0\right)^2}. \tag{12}$$

When $\delta = -\beta$,

$$u_{5a} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\coth\left(x+\varepsilon\sqrt{-\alpha+15\beta}t\right) \pm \operatorname{csch}\left(x+\varepsilon\sqrt{-\alpha+15\beta}t\right)\right)^{-2} - \theta\sqrt{\frac{15\beta}{2\gamma}}, \tag{13a}$$

$$u_{5b} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\frac{\tanh\left(x+\varepsilon\sqrt{-\alpha+15\beta}t\right)}{\left(1\pm\operatorname{sech}\left(x+\varepsilon\sqrt{-\alpha+15\beta}t\right)\right)}\right)^{-2} - \theta\sqrt{\frac{15\beta}{2\gamma}}, \tag{13b}$$

$$u_{5c} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\tan\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right) \pm \sec\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right)\right)^{-2} + \theta\sqrt{\frac{15\beta}{2\gamma}}, \tag{13c}$$

$$u_{5d} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\frac{\tan\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right)}{\left(1\pm\sec\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right)\right)}\right)^{-2} + \theta\sqrt{\frac{15\beta}{2\gamma}}, \tag{13d}$$

$$u_{5e} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\cot\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right) \pm \csc\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right)\right)^{-2} + \theta\sqrt{\frac{15\beta}{2\gamma}}, \tag{13e}$$

$$u_{5f} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\sec\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right) - \tan\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right)\right)^{-2} + \theta\sqrt{\frac{15\beta}{2\gamma}}, \tag{13f}$$

$$u_{5g} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\frac{\cot\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right)}{\left(1\pm\csc\left(x+\varepsilon\sqrt{-\alpha-15\beta}t\right)\right)}\right)^{-2} + \theta\sqrt{\frac{-15\beta}{2\gamma}}, \tag{13g}$$

when $\delta = -16\beta$,

$$u_{6a} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\tanh^{-2}\left(x\pm\sqrt{-\alpha+60\beta}t\right) - 2\theta\sqrt{\frac{30\beta}{\gamma}}, \tag{14a}$$

$$u_{6b} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\coth^{-2}\left(x\pm\sqrt{-\alpha+60\beta}t\right) - 2\theta\sqrt{\frac{30\beta}{\gamma}}, \tag{14b}$$

$$u_{6c} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\tan^{-2}\left(x\pm\sqrt{-\alpha-60\beta}t\right) + 2\theta\sqrt{\frac{30\beta}{\gamma}}, \tag{14c}$$

$$u_{6d} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\cot^{-2}\left(x\pm\sqrt{-\alpha-60\beta}t\right) + 2\theta\sqrt{\frac{30\beta}{\gamma}}, \tag{14d}$$

when $\delta = -64\beta$,

$$u_{7a} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\left(\frac{\tanh\left(x\pm\sqrt{-\alpha-240\beta}t\right)}{\left(1-\tanh^2\left(x\pm\sqrt{-\alpha-240\beta}t\right)\right)}\right)^{-2} + 8\theta\sqrt{\frac{30\beta}{\gamma}}, \tag{15a}$$

$$u_{7b} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\left(\frac{\tanh\left(x\pm\sqrt{-\alpha+240\beta}t\right)}{\left(1+\tanh^2\left(x\pm\sqrt{-\alpha+240\beta}t\right)\right)}\right)^{-2} - 8\theta\sqrt{\frac{30\beta}{\gamma}}, \tag{15b}$$

$$u_{7c} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\left(\frac{\cot\left(x\pm\sqrt{-\alpha-240\beta}t\right)}{\left(1-\cot^2\left(x\pm\sqrt{-\alpha-240\beta}t\right)\right)}\right)^{-2} + 8\theta\sqrt{\frac{30\beta}{\gamma}}, \tag{15c}$$

when $\delta = -(13\pm\sqrt{165})\beta$,

$$u_{8a} = \theta\sqrt{\frac{15\beta}{2\gamma}} \times\left(\coth\left(x+\varepsilon\sqrt{-\alpha-\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta}t\right) \pm \operatorname{csch}\left(x+\varepsilon\sqrt{-\alpha-\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta}t\right)\right)^{-2} + \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16a}$$

$$u_{8b} = \theta\sqrt{\frac{15\beta}{2\gamma}} \times\left(\frac{\tanh\left(x+\varepsilon\sqrt{-\alpha-(1/4)\left(-30+2\sigma\sqrt{165}\right)\beta}t\right)}{\left(1\pm\operatorname{sech}\left(x+\varepsilon\sqrt{-\alpha-(1/4)\left(-30+2\sigma\sqrt{165}\right)\beta}t\right)\right)}\right)^{-2} + \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16b}$$

$$u_{8c} = \theta\sqrt{\frac{15\beta}{2\gamma}}\left(\tan\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta}t\right) \pm\sec\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta}t\right)\right)^{-2} - \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16c}$$

$$u_{8d} = \theta\sqrt{\frac{15\beta}{2\gamma}} \times\left(\csc\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta}t\right)\right.$$

$$- \cot\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\Bigg)^{-2} - \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16d}$$

$$u_{8e} = \theta\sqrt{\frac{15\beta}{2\gamma}} \times\left(\frac{\tan\left(x+\varepsilon\sqrt{-\alpha+(1/4)\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)}{\left(1\pm\sec\left(x+\varepsilon\sqrt{-\alpha+(1/4)\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\right)}\right)^{-2} - \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16e}$$

$$u_{8f} = \theta\sqrt{\frac{15\beta}{2\gamma}} \times\left(\cot\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta t}\right) \pm\csc\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\right)^{-2} - \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16f}$$

$$u_{8g} = \theta\sqrt{\frac{15\beta}{2\gamma}} \times\left(\sec\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta t}\right) - \tan\left(x+\varepsilon\sqrt{-\alpha+\frac{1}{4}\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\right)^{-2} - \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16g}$$

$$u_{8h} = \theta\sqrt{\frac{15\beta}{2\gamma}} \times\left(\frac{\cot\left(x+\varepsilon\sqrt{-\alpha+(1/4)\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)}{\left(1\pm\csc\left(x+\varepsilon\sqrt{-\alpha+(1/4)\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\right)}\right)^{-2} - \frac{1}{4}\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{16h}$$

when $\delta = -4(13 \pm \sqrt{165})\beta$,

$$u_{9a} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\tanh^{-2}\left(x+\varepsilon\sqrt{-\alpha-\left(-30+2\sigma\sqrt{165}\right)\beta t}\right) + \theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{17a}$$

$$u_{9b} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\coth^{-2}\left(x+\varepsilon\sqrt{-\alpha-\left(-30+2\sigma\sqrt{165}\right)\beta t}\right) + \theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{17b}$$

$$u_{9c} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\tan^{-2}\left(x+\varepsilon\sqrt{-\alpha+\left(-30+2\sigma\sqrt{165}\right)\beta t}\right) - \theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{17c}$$

$$u_{9d} = 2\theta\sqrt{\frac{30\beta}{\gamma}}\cot^{-2}\left(x+\varepsilon\sqrt{-\alpha+\left(-30+2\sigma\sqrt{165}\right)\beta t}\right) - \theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{17d}$$

when $\delta = -64(13 \pm \sqrt{165})\beta$,

$$u_{10a} = 2\theta\sqrt{\frac{30\beta}{\gamma}} \times\left(\frac{\tanh\left(x+\varepsilon\sqrt{-\alpha-4\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)}{\left(1+\tanh^2\left(x+\varepsilon\sqrt{-\alpha-4\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\right)}\right)^{-2} + 4\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{18a}$$

$$u_{10b} = 2\theta\sqrt{\frac{30\beta}{\gamma}} \times\left(\frac{\tanh\left(x+\varepsilon\sqrt{-\alpha+4\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)}{\left(1-\tanh^2\left(x+\varepsilon\sqrt{-\alpha+4\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\right)}\right)^{-2} - 4\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{18b}$$

$$u_{10c} = 2\theta\sqrt{\frac{30\beta}{\gamma}} \times \left(\frac{\cot\left(x+\varepsilon\sqrt{-\alpha+4\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)}{\left(1-\cot^2\left(x+\varepsilon\sqrt{-\alpha+4\left(-30+2\sigma\sqrt{165}\right)\beta t}\right)\right)}\right)^{-2} - 4\theta\sqrt{\frac{30\beta}{\gamma}}\left(-1+\sigma\frac{\sqrt{165}}{15}\right), \tag{18c}$$

where ε, θ, and σ are arbitrary elements of $\{-1, 1\}$.

3. Conclusion

In this paper, we have used solutions to the Riccati equation to solve the generalized Bretherton equation with arbitrary constants and obtained abundant new multiple solition-like and triangular periodic solutions. It is significant to observe practical denotation of the obtained solutions, so the obtained solutions involving arbitrary constants in this paper have potential applications in dispersive wave systems to research for resonant nonlinear interactions.

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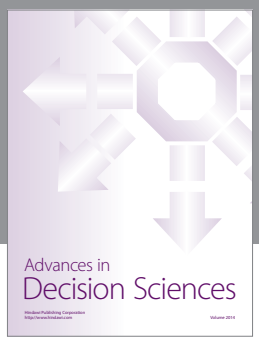

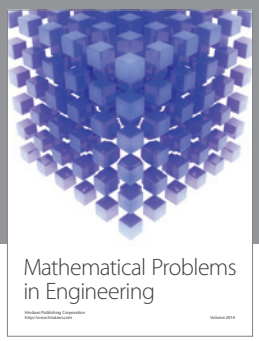

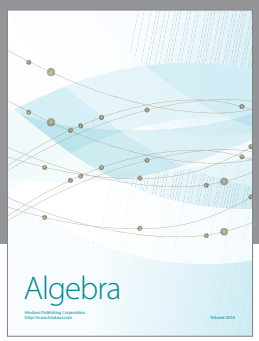

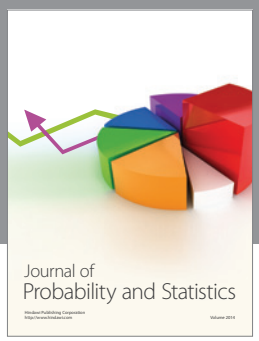

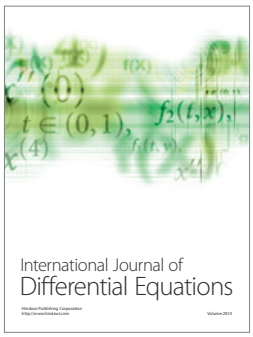

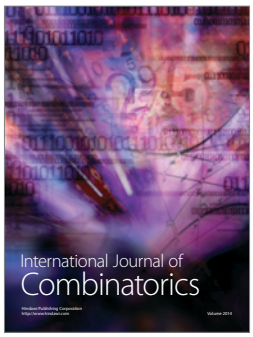

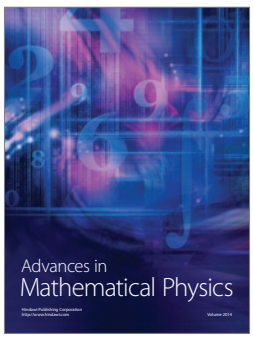

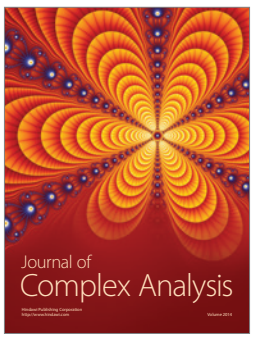

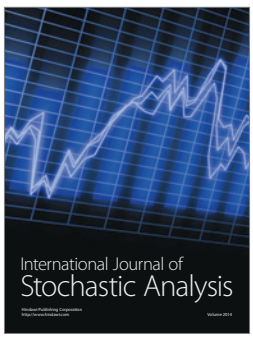

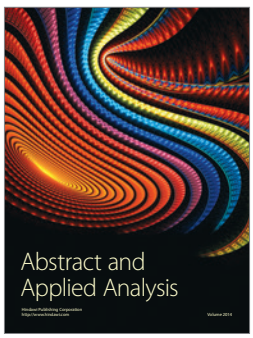

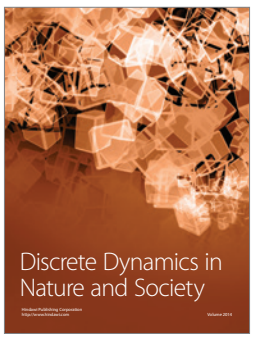

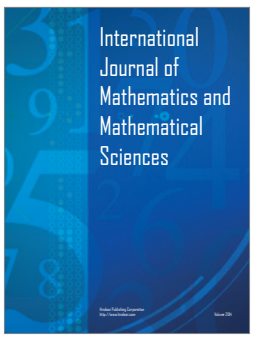

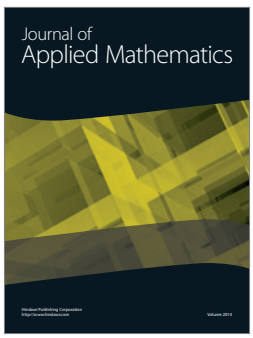

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