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Research Article

Enhanced Symplectic Synchronization between Two Different Complex Chaotic Systems with Uncertain Parameters

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An enhanced symplectic synchronization of complex chaotic systems with uncertain parameters is studied. The traditional chaos synchronizations are special cases of the enhanced symplectic synchronization. A sufficient condition is given for the asymptotical stability of the null solution of error dynamics. The enhanced symplectic synchronization may be applied to the design of secure communication. Finally, numerical simulations results are performed to verify and illustrate the analytical results.

1. Introduction

A synchronized mechanism that enables a system to maintain a desired dynamical behavior (the goal or target) even when intrinsically chaotic has many applications ranging from biology to engineering [1–4]. Thus, it is of considerable interest and potential utility to devise control techniques capable of achieving the desired type of behavior in nonlinear and chaotic systems. Many approaches have been presented for the synchronization of chaotic systems [5–10]. There are a chaotic master system and either an identical or a different slave system. Our goal is the synchronization of the chaotic master and the chaotic slave by coupling or by other methods.

The symplectic chaos synchronization concept [11]

$$y = H(t, x, y) + F(t) \tag{1}$$

is studied, where x, y are the state vectors of the master system and of the slave system, respectively, and $F(t)$ is a given function of time in different form. The $F(t)$ may be a regular motion function or a chaotic motion function. When $H(t, x, y) + F(t) = x$ and $H(t, x, y) = x$, (1) reduces to the generalized chaos synchronization and the traditional chaos synchronization given in [1–3], respectively. In this paper, a new enhance symplectic chaos synchronization:

$$y = H(t, \dot{x}, \dot{y}, x, y) + F(t). \tag{2}$$

As numerical examples, we select hyperchaotic Chen system [12] and hyperchaotic Lorenz system [13] as the master system and the slave system, respectively.

This paper is organized as follows. In Section 2, by the Lyapunov asymptotical stability theorem, a symplectic synchronization scheme is given. In Section 3, various feedbacks of nonlinear controllers are designed for the enhanced symplectic synchronization of a hyperchaotic Chen system with uncertain parameters and a hyperchaotic Lorenz system. Numerical simulations are also given in Section 3. Finally, some concluding remarks are given in Section 4.

2. Enhanced Symplectic Synchronization Scheme

There are two different nonlinear chaotic systems. The partner A controls the partner B partially. The partner A is given by

$$\dot{x} = f(t, x, A(t)), \tag{3}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ is a state vector, $A(t) = [A_1(t), A_2(t), \ldots, A_M(t)]^T \in R^M$ is a vector of uncertain coefficients in f, and f is a vector function.

The partner B is given by

$$\dot{y} = g\left(t, y, B\left(t\right)\right), \tag{4a}$$

where $y = [y_1, y_2, \ldots, y_n]^T \in R^n$ is a state vector, $B(t) = [B_1(t), B_2(t), \ldots, B_m(t)]^T \in R^m$ is a vector of uncertain coefficients in g, and g is a vector function different from f.

After a controller $u(t)$ is added, partner B becomes

$$\dot{y} = g\left(t, y, B\left(t\right)\right) + u\left(t\right), \tag{4b}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in R^n$ is the control vector.

Our goal is to design the controller $u(t)$ so that the state vector y of the partner B asymptotically approaches $H(t, \dot{x}, \dot{y}, x, y) + F(t)$, a given function $H(t, \dot{x}, \dot{y}, x, y)$ plus a given vector function $F(t) = [F_1(t), F_2(t), \ldots, F_n(t)]^T$ which is a regular or a chaotic function. Define error vector $e(t) = [e_1, e_2, \ldots, e_n]^T$:

$$e = H\left(t, \dot{x}, \dot{y}, x, y\right) - y + F\left(t\right), \tag{5}$$

$$\lim_{t \to \infty} e = 0 \tag{6}$$

is demanded.

From (5), it is obtained that

$$\dot{e} = \frac{\partial H}{\partial t} + \nabla H^T \dot{\Psi} - \dot{y} + \dot{F}\left(t\right), \tag{7}$$

where $\dot{\Psi} = [\ddot{x} \quad \ddot{y} \quad \dot{x} \quad \dot{y}]^T$.

Using (3), (4a), and (4b), (7) can be rewritten as

$$\begin{aligned} \dot{e} = {} & \frac{\partial H}{\partial t} + \frac{\partial H}{\partial \dot{x}} \ddot{x} + \frac{\partial H}{\partial \dot{y}} \ddot{y} + \frac{\partial H}{\partial x} f\left(t, x, A\left(t\right)\right) \\ & + \frac{\partial H}{\partial y} g\left(t, y, B\left(t\right)\right) - g\left(t, y, B\left(t\right)\right) - u\left(t\right) + \dot{F}\left(t\right). \end{aligned} \tag{8}$$

Proof. A positive definite Lyapunov function $V(e)$ is chosen [14, 15] as

$$V\left(e\right) = \frac{1}{2} e^T e. \tag{9}$$

Its derivative along any solution of (8) is

$$\begin{aligned} \dot{V}\left(e\right) = e^T \Big\{ & \frac{\partial H}{\partial t} + \frac{\partial H}{\partial \dot{x}} \ddot{x} + \frac{\partial H}{\partial \dot{y}} \ddot{y} + \frac{\partial H}{\partial x} \\ & \times f\left(t, x, A\left(t\right)\right) + \frac{\partial H}{\partial y} g\left(t, y, B\left(t\right)\right) \\ & - g\left(t, y, B\left(t\right)\right) + \dot{F}\left(t\right) - u\left(t\right) \Big\}. \end{aligned} \tag{10}$$

In (10), the $u(t)$ is designed so that $\dot{V}(e) = e^T C_{n\times n} e$, where $C_{n\times n}$ is a diagonal negative definite matrix. The $\dot{V}$ is a negative definite function of e. □

Remark 1. Note that e approaches zero when time approaches infinitly, according to Lyapunov theorem of asymptotical stability. The enhanced symplectic synchronization is obtained [12, 13, 16–19].

3. Numerical Results for the Enhanced Symplectic Chaos Synchronization of Chen System with Uncertain Parameters and Hyperchaotic Lorenz System

To further illustrate the effectiveness of the controller, we select hyperchaotic Chen system and hyperchaotic Lorenz system as the master system and the slave system, respectively. Consider

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right) + x_4, \\ \dot{x}_2 &= dx_1 + cx_2 - x_1 x_3, \\ \dot{x}_3 &= -bx_3 + x_1 x_2, \\ \dot{x}_4 &= rx_4 + x_2 x_3, \end{aligned} \tag{11}$$

$$\begin{aligned} \dot{y}_1 &= a_1\left(y_2 - y_1\right) + y_4, \\ \dot{y}_2 &= b_1 y_1 - y_2 - y_1 y_3, \\ \dot{y}_3 &= -c_1 y_3 + y_1 y_2, \\ \dot{y}_4 &= d_1 y_4 - y_1 y_3, \end{aligned} \tag{12}$$

where $a, b, c, d, r, a_1, b_1, c_1$, and d_1 are parameters. The parameters of master system and slave system are chosen as $a = 31$, $b = 3.5$, $c = 11$, $d = 7.7$, $r = 0.1$, $a_1 = 11$, $b_1 = 28$, $c_1 = 2.8$, and $d_1 = 1.2$.

The controllers u_1, u_2, u_3, and u_4 are added to the four equations of (12), respectively as follows:

$$\begin{aligned} \dot{y}_1 &= a_1\left(y_2 - y_1\right) + y_4 + u_1, \\ \dot{y}_2 &= b_1 y_1 - y_2 - y_1 y_3 + u_2, \\ \dot{y}_3 &= -c_1 y_3 + y_1 y_2 + u_3, \\ \dot{y}_4 &= d_1 y_4 - y_1 y_3 + u_4. \end{aligned} \tag{13}$$

The initial values of the states of the Chen system and of the Lorenz system are taken as $x_1(0) = 11$, $x_2(0) = 13$, $x_3(0) = 12$, $x_4(0) = 12$, $y_1(0) = -11$, $y_2(0) = -13$, $y_3(0) = -12$, and $y_4(0) = -12$.

Case 1 (a symplectic synchronization). We take $F_1(t) = x_4^3(t)$, $F_2(t) = x_1^3(t)$, $F_3(t) = x_2^3(t)$, and $F_4(t) = x_3^3(t)$. They are chaotic functions of time. $H_i(x, y, t) = -x_i^2 y_i$ $(i = 1, 2, 3, 4)$ are given. By (6), we have

$$\begin{gathered} \lim_{t \to \infty} e_i = \lim_{t \to \infty} \left(-x_i^2 y_i - y_i + x_j^3\right) = 0, \\ i = 1, 2, 3, 4; \quad j = \begin{cases} 4, & i = 1, \\ i - 1, & i \neq 1. \end{cases} \end{gathered} \tag{14}$$

From (7), we have

$$\begin{gathered} \dot{e}_i = -2\dot{x}_i x_i y_i - x_i^2 \dot{y}_i - \dot{y}_i + 3\dot{x}_j x_j^2, \\ i = 1, 2, 3, 4; \quad j = \begin{cases} 4, & i = 1, \\ i - 1, & i \neq 1. \end{cases} \end{gathered} \tag{15}$$

Equation (8) can be expressed as

$$\begin{aligned}\dot{e}_1 = &-2y_1x_1\left[a\left(x_2-x_1\right)+x_4\right]\\&-\left(1+x_1^2\right)\times\left[a_1\left(y_2-y_1\right)+y_4\right]\\&-u_1+3x_4^2\left(rx_4+x_2x_3\right),\\\dot{e}_2 = &-2y_2x_2\left(dx_1+cx_2-x_1x_3\right)\\&-\left(1+x_2^2\right)\times\left(b_1y_1-y_2-y_1y_3\right)\\&-u_2+3x_1^2\left[a\left(x_2-x_1\right)+x_4\right],\\\dot{e}_3 = &-2y_3x_3\left(-bx_3+x_1x_2\right)\\&-\left(1+x_3^2\right)\times\left(-c_1y_3+y_1y_2\right)\\&-u_3+3x_2^2\left(dx_1+cx_2-x_1x_3\right),\\\dot{e}_4 = &-2y_4x_4\left(rx_4+x_2x_3\right)\\&-\left(1+x_4^2\right)\times\left(d_1y_4-y_1y_3\right)\\&-u_4+3x_3^2\left(-bx_3+x_1x_2\right),\end{aligned}\tag{16}$$

where $e_1 = -x_1^2y_1 - y_1 + x_4^3$, $e_2 = -x_2^2y_2 - y_2 + x_1^3$, $e_3 = -x_3^2y_3 - y_3 + x_2^3$, and $e_4 = -x_4^2y_4 - y_4 + x_3^3$.

Choose a positive definite Lyapunov function as

$$V\left(e_1,e_2,e_3,e_4\right)=\frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_4^2\right).\tag{17}$$

Its time derivative along any solution of (16) is

$$\begin{aligned}\dot{V} = e_1&\left\{-2y_1x_1\left[a\left(x_2-x_1\right)+x_4\right]\right.\\&-\left(1+x_1^2\right)\times\left[a_1\left(y_2-y_1\right)+y_4\right]-u_1\\&\left.+3x_4^2\left(rx_4+x_2x_3\right)\right\}\\+e_2&\left\{-2y_2x_2\left(dx_1+cx_2-x_1x_3\right)\right.\\&-\left(1+x_2^2\right)\times\left(b_1y_1-y_2-y_1y_3\right)\\&\left.-u_2+3x_1^2\left[a\left(x_2-x_1\right)+x_4\right]\right\}\\+e_3&\left\{-2y_3x_3\left(-bx_3+x_1x_2\right)\right.\\&-\left(1+x_3^2\right)\times\left(-c_1y_3+y_1y_2\right)-u_3\\&\left.+3x_2^2\left(dx_1+cx_2-x_1x_3\right)\right\}\\+e_4&\left\{-2y_4x_4\left(rx_4+x_2x_3\right)\right.\\&-\left(1+x_4^2\right)\times\left(d_1y_4-y_1y_3\right)\\&\left.-u_4+3x_3^2\left(-bx_3+x_1x_2\right)\right\}.\end{aligned}\tag{18}$$

According to (10), we get the controller

$$\begin{aligned}u_1 = &-2y_1x_1\left[a\left(x_2-x_1\right)+x_4\right]+3x_4^2\left(rx_4+x_2x_3\right)\\&-\left(1+x_1^2\right)\times\left[a_1\left(y_2-y_1\right)+y_4\right]+e_1,\\u_2 = &-2y_2x_2\left(dx_1+cx_2-x_1x_3\right)+3x_1^2\left[a\left(x_2-x_1\right)+x_4\right]\\&-\left(1+x_2^2\right)\times\left(b_1y_1-y_2-y_1y_3\right)+e_2,\\u_3 = &-2y_3x_3\left(-bx_3+x_1x_2\right)+3x_2^2\left(dx_1+cx_2-x_1x_3\right)\\&+\left(1+x_3^2\right)\times\left(c_1y_3-y_1y_2\right)+e_3,\\u_4 = &-2y_4x_4\left(rx_4+x_2x_3\right)+3x_3^2\left(-bx_3+x_1x_2\right)\\&-\left(1+x_4^2\right)\times\left(d_1y_4-y_1y_3\right)+e_4.\end{aligned}\tag{19}$$

Equation (18) becomes

$$\dot{V}=-\left(e_1^2+e_2^2+e_3^2+e_4^2\right)<0,\tag{20}$$

which is negative definite. The Lyapunov asymptotical stability theorem is satisfied. The symplectic synchronization of the Chen system and the Lorenz system is achieved. The numerical results are shown in Figures 1, 2, and 3. After 1 second, the motion trajectories enter a chaotic attractor.

Case 2 (a symplectic synchronization with uncertain parameters). The master Chen system with uncertain variable parameters is

$$\begin{aligned}\dot{x}_1 &= a\left(t\right)\left(x_2-x_1\right)+x_4,\\\dot{x}_2 &= d\left(t\right)x_1+c\left(t\right)x_2-x_1x_3,\\\dot{x}_3 &= -b\left(t\right)x_3+x_1x_2,\\\dot{x}_4 &= r\left(t\right)x_4+x_2x_3,\end{aligned}\tag{21}$$

where $a(t)$, $b(t)$, $c(t)$, $d(t)$, and $r(t)$ are uncertain parameters. In simulation, we take

$$\begin{gathered}a\left(t\right)=a\left(1+k_1\sin\omega_1t\right),\qquad b\left(t\right)=b\left(1+k_2\sin\omega_2t\right),\\c\left(t\right)=c\left(1+k_3\sin\omega_3t\right),\qquad d\left(t\right)=d\left(1+k_4\sin\omega_4t\right),\\r\left(t\right)=r\left(1+k_5\sin\omega_5t\right),\end{gathered}\tag{22}$$

where k_1, k_2, k_3, k_4, k_5, ω_1, ω_2, ω_3, ω_4, and ω_5 are constants. Take $k_1 = 0.3$, $k_2 = 0.5$, $k_3 = 0.2$, $k_4 = 0.4$, $k_5 = 0.6$, $\omega_1 = 13$, $\omega_2 = 17$, $\omega_3 = 19$, $\omega_4 = 23$, and $\omega_5 = 29$. So, (21) is chaotic system, shown in Figure 4.

We take $F_1(t) = x_4^3(t)$, $F_2(t) = x_1^3(t)$, $F_3(t) = x_2^3(t)$, and $F_4(t) = x_3^3(t)$. They are chaotic functions of time. $H_i(x, y, t) = -x_i^2 y_i$ $(i = 1, 2, 3, 4)$ are given. By (6), we have

$$\begin{gathered}\lim_{t\to\infty}e_i=\lim_{t\to\infty}\left(-x_i^2y_i-y_i+x_j^3\right)=0,\\i=1,2,3,4;\ \ j=\begin{cases}4, & i=1,\\i-1, & i\neq 1.\end{cases}\end{gathered}\tag{23}$$

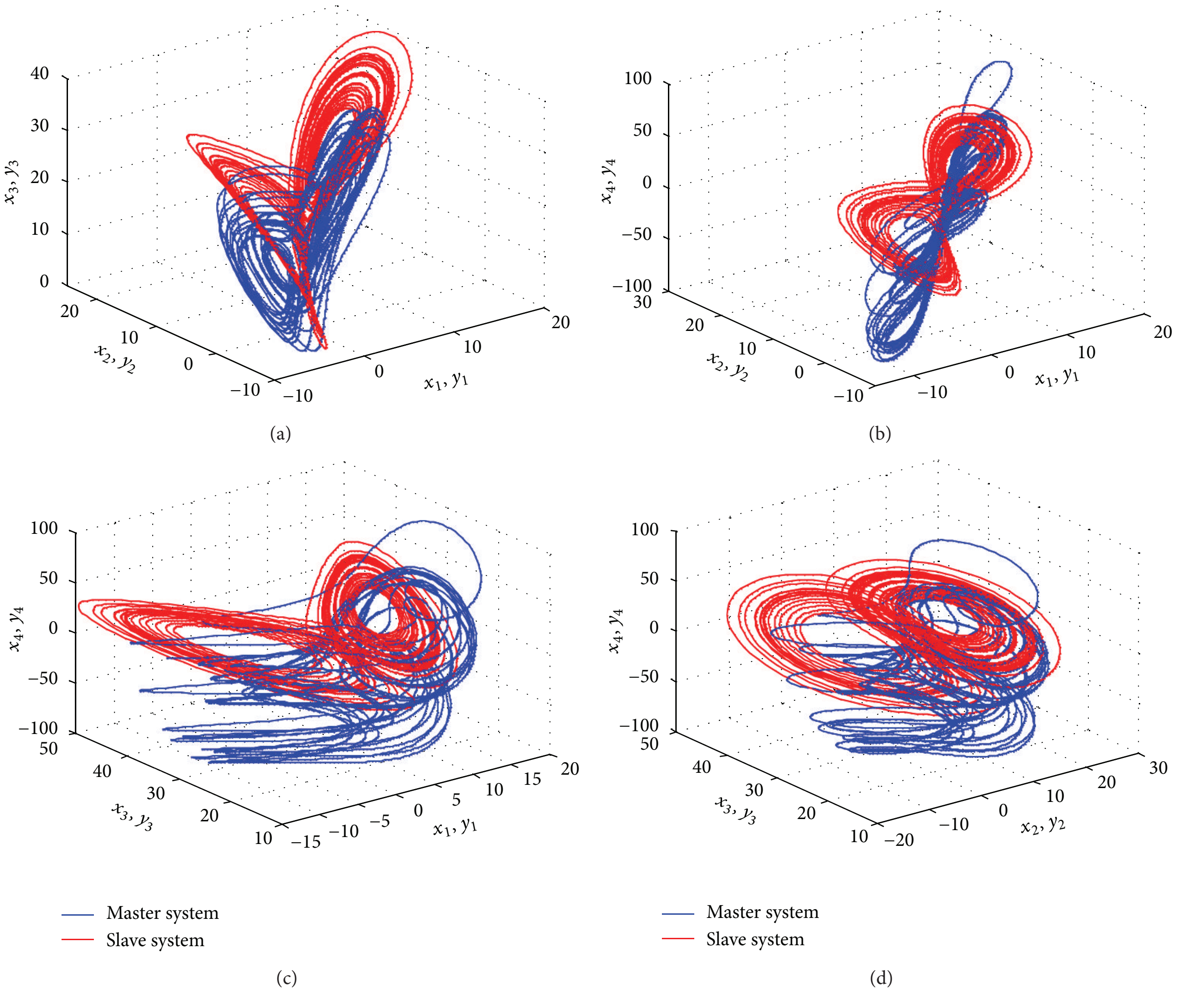

FIGURE 1: Projections of phase portrait for master system (11) and slave system (12).

From (7), we have

$$\dot{e}_i = -2\dot{x}_i x_i y_i - x_i^2 \dot{y}_i - \dot{y}_i + 3\dot{x}_j x_j^2,$$
$$i = 1, 2, 3, 4;\quad j = \begin{cases} 4, & i = 1, \\ i - 1, & i \neq 1. \end{cases} \tag{24}$$

Equation (8) can be expressed as

$$\begin{aligned}
\dot{e}_1 = {} & -2y_1 x_1 \left[a(t)(x_2 - x_1) + x_4\right] \\
& - \left(1 + x_1^2\right) \times \left[a_1 (y_2 - y_1) + y_4 + u_1\right] \\
& + 3x_4^2 \left(r(t) x_4 + x_2 x_3\right), \\
\dot{e}_2 = {} & -2y_2 x_2 \left(d(t) x_1 + c(t) x_2 - x_1 x_3\right) \\
& - \left(1 + x_2^2\right) \times \left(b_1 y_1 - y_2 - y_1 y_3 + u_2\right) \\
& + 3x_1^2 \left[a(t)(x_2 - x_1) + x_4\right], \\
\dot{e}_3 = {} & -2y_3 x_3 \left(-b(t) x_3 + x_1 x_2\right) \\
& - \left(1 + x_3^2\right) \times \left(-c_1 y_3 + y_1 y_2 + u_3\right) \\
& + 3x_2^2 \left(d(t) x_1 + c(t) x_2 - x_1 x_3\right), \\
\dot{e}_4 = {} & -2y_4 x_4 \left(r(t) x_4 + x_2 x_3\right) \\
& - \left(1 + x_4^2\right) \times \left(d_1 y_4 - y_1 y_3 + u_4\right) \\
& + 3x_3^2 \left(-b(t) x_3 + x_1 x_2\right),
\end{aligned} \tag{25}$$

where $e_1 = -x_1^2 y_1 - y_1 + x_4^3$, $e_2 = -x_2^2 y_2 - y_2 + x_1^3$, $e_3 = -x_3^2 y_3 - y_3 + x_2^3$, and $e_4 = -x_4^2 y_4 - y_4 + x_3^3$.

Choose a positive definite Lyapunov function as

$$V(e_1, e_2, e_3, e_4) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right). \tag{26}$$

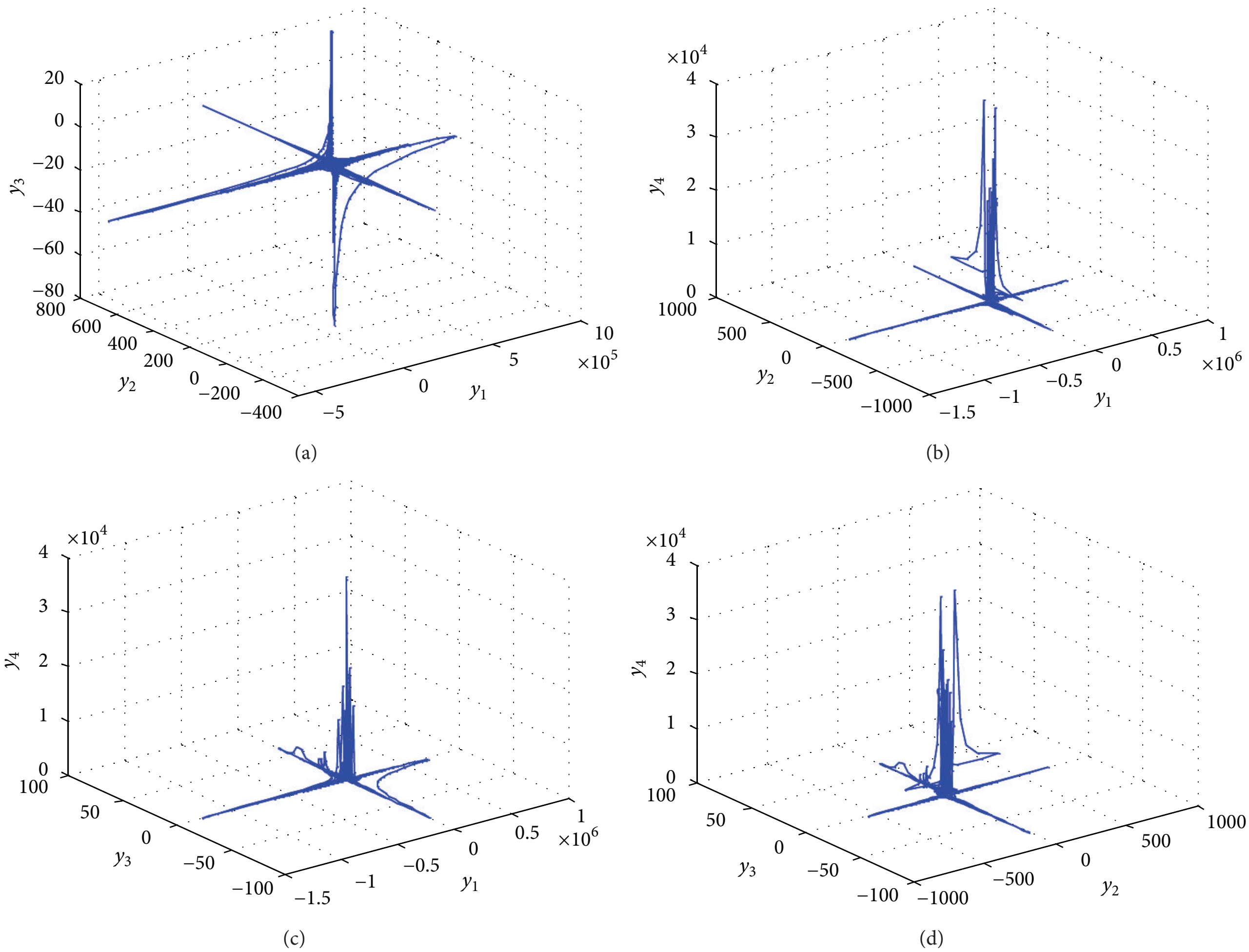

FIGURE 2: Projections of the phase portrait for chaotic system (13) of Case 1.

Its time derivative along any solution of (25) is

$$\begin{aligned}
\dot{V} = {} & e_1 \{-2y_1x_1 [a(t)(x_2 - x_1) + x_4] \\
& - (1 + x_1^2) \times [a_1(y_2 - y_1) + y_4] - u_1 \\
& + 3x_4^2 (r(t)x_4 + x_2x_3)\} \\
& + e_2 \{-2y_2x_2 (d(t)x_1 + c(t)x_2 - x_1x_3) \\
& - (1 + x_2^2) \times (b_1y_1 - y_2 - y_1y_3) - u_2 \\
& + 3x_1^2 [a(t)(x_2 - x_1) + x_4]\} \\
& + e_3 \{-2y_3x_3 (-b(t)x_3 + x_1x_2) \\
& - (1 + x_3^2) \times (-c_1y_3 + y_1y_2) - u_3 \\
& + 3x_2^2 (d(t)x_1 + c(t)x_2 - x_1x_3)\} \\
& + e_4 \{-2y_4x_4 (r(t)x_4 + x_2x_3) \\
& - (1 + x_4^2) \times (d_1y_4 - y_1y_3) \\
& - u_4 + 3x_3^2 (-b(t)x_3 + x_1x_2)\}.
\end{aligned} \tag{27}$$

According to (10), we get the controller

$$\begin{aligned}
u_1 = {} & -2y_1x_1 [a(t)(x_2 - x_1) + x_4] \\
& + 3x_4^2 (r(t)x_4 + x_2x_3) \\
& - (1 + x_1^2) [a_1(y_2 - y_1) - y_4] + e_1, \\
u_2 = {} & -2y_2x_2 (d(t)x_1 + c(t)x_2 - x_1x_3) \\
& + 3x_1^2 [a(t)(x_2 - x_1) + x_4] \\
& - (1 + x_2^2)(b_1y_1 - y_2 - y_1y_3) + e_2, \\
u_3 = {} & -2y_3x_3 (-b(t)x_3 + x_1x_2) \\
& + 3x_2^2 (d(t)x_1 + c(t)x_2 - x_1x_3) \\
& + (1 + x_3^2)(c_1y_3 - y_1y_2) + e_3, \\
u_4 = {} & -2y_4x_4 (r(t)x_4 + x_2x_3) \\
& + 3x_3^2 (-b(t)x_3 + x_1x_2) \\
& - (1 + x_4^2)(d_1y_4 - y_1y_3) + e_4.
\end{aligned} \tag{28}$$

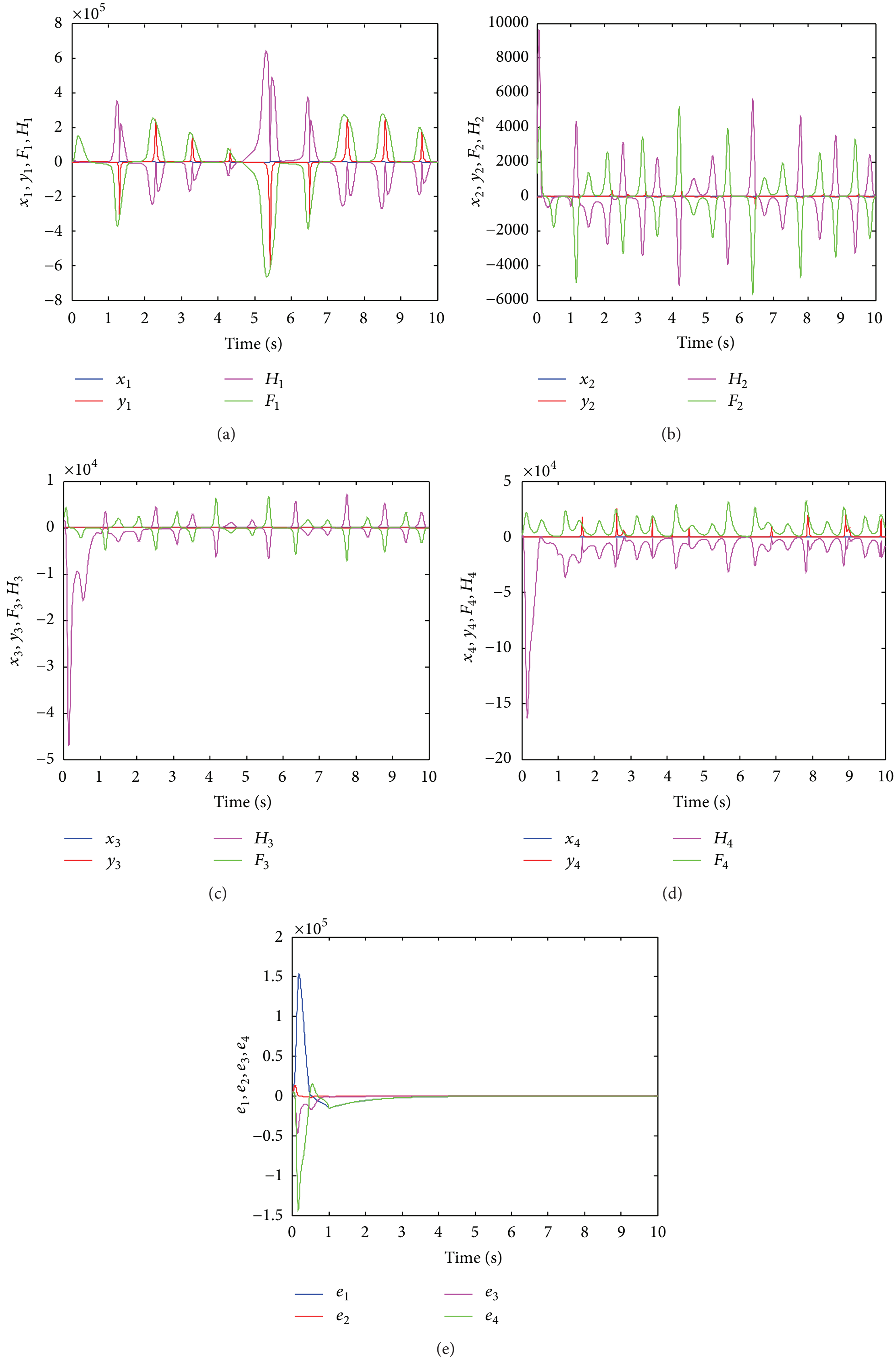

FIGURE 3: Time histories of states, state errors, F_1, F_2, F_3, F_4, H_1, H_2, H_3, and H_4 for Case 1.

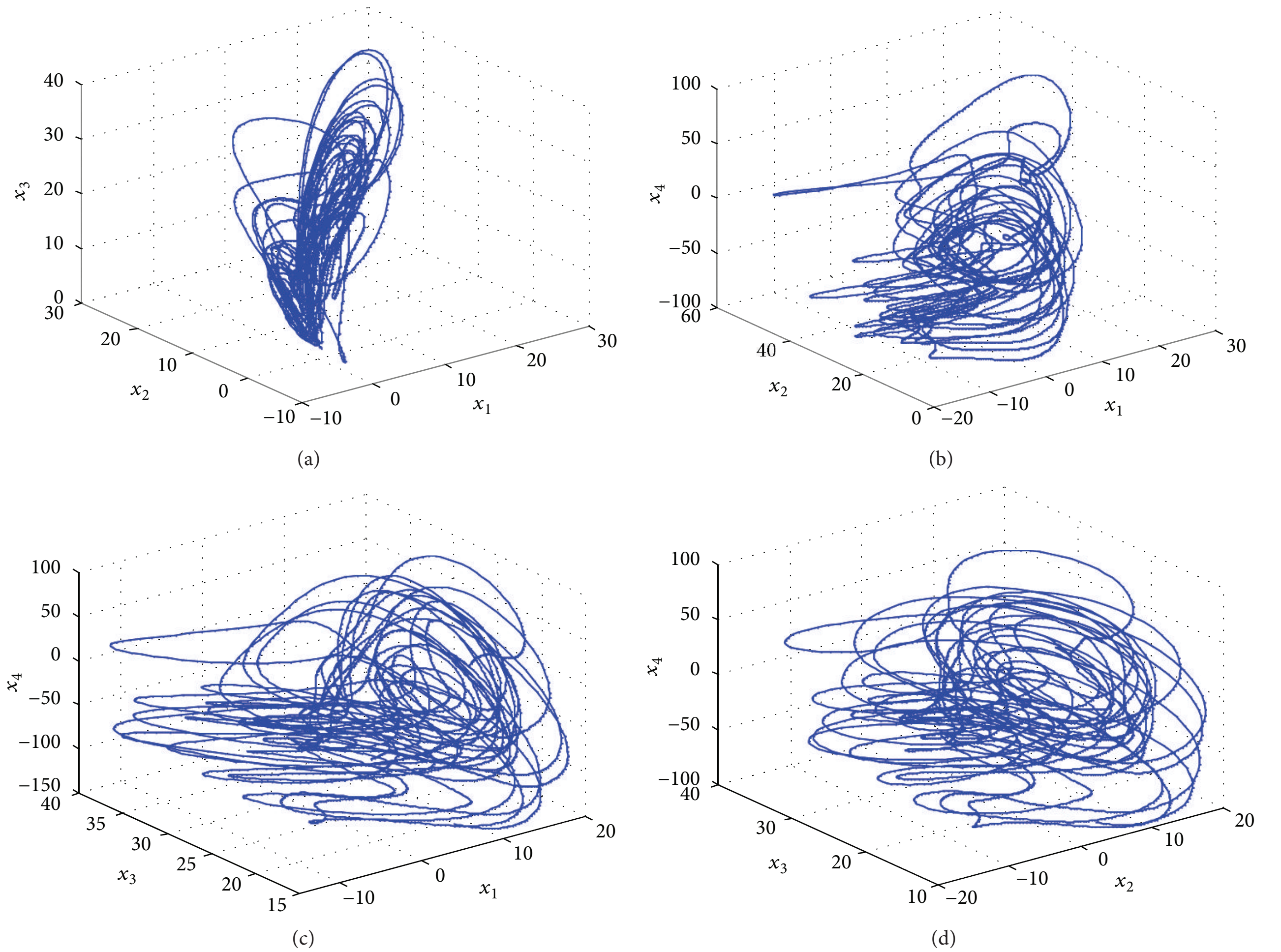

Figure 4: Projections of the phase portrait for chaotic system (21).

Equation (27) becomes

$$\dot{V} = -\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) < 0, \tag{29}$$

which is negative definite. The Lyapunov asymptotical stability theorem is satisfied. The symplectic synchronization of the Chen system with uncertain parameters and the Lorenz system is achieved. The numerical results are shown in Figures 5 and 6. After 1 second, the motion trajectories enter a chaotic attractor.

Case 3 (an enhanced symplectic synchronization with uncertain parameters). We take $F_1(t) = x_4^3(t)$, $F_2(t) = x_1^3(t)$, $F_3(t) = x_2^3(t)$, and $F_4(t) = x_3^3(t)$. They are chaotic functions of time. $H_i(\dot{x}, \dot{y}, x, y, t) = -x_i^2 y_i - \dot{x} - K\dot{y}$ ($i = 1, 2, 3, 4$) are given. The K value is 0.0001. By (6), we have

$$\lim_{t\to\infty} e_i = \lim_{t\to\infty}\left(-x_i^2 y_i - \dot{x}_i - K\dot{y}_i - y_i + x_j^3\right) = 0, \qquad i = 1, 2, 3, 4;\;\; j = \begin{cases} 4, & i = 1, \\ i-1, & i \neq 1. \end{cases} \tag{30}$$

From (7) we have

$$\dot{e}_i = -2x_i\dot{x}_i y_i - x_i^2\dot{y}_i - \ddot{x}_i - K\ddot{y}_i - \dot{y}_i + 3\dot{x}_j x_j^2, \qquad i = 1, 2, 3, 4;\;\; j = \begin{cases} 4, & i = 1, \\ i-1, & i \neq 1. \end{cases} \tag{31}$$

Equation (8) can be expressed as

$$\begin{aligned}
\dot{e}_1 &= -2y_1x_1\left[a(t)(x_2 - x_1) + x_4\right] \\
&\quad - \left(1 + x_1^2\right) \times \left[a_1(y_2 - y_1) + y_4 + u_1\right] \\
&\quad - \ddot{x}_1 - K\ddot{y}_1 + 3x_4^2\left(r(t)x_4 + x_2x_3\right), \\
\dot{e}_2 &= -2y_2x_2\left(d(t)x_1 + c(t)x_2 - x_1x_3\right) \\
&\quad - \left(1 + x_2^2\right) \times \left(b_1y_1 - y_2 - y_1y_3 + u_2\right) \\
&\quad - \ddot{x}_2 - K\ddot{y}_2 + 3x_1^2\left[a(t)(x_2 - x_1) + x_4\right], \\
\dot{e}_3 &= -2y_3x_3\left(-b(t)x_3 + x_1x_2\right) \\
&\quad - \left(1 + x_3^2\right) \times \left(-c_1y_3 + y_1y_2 + u_3\right)
\end{aligned}$$

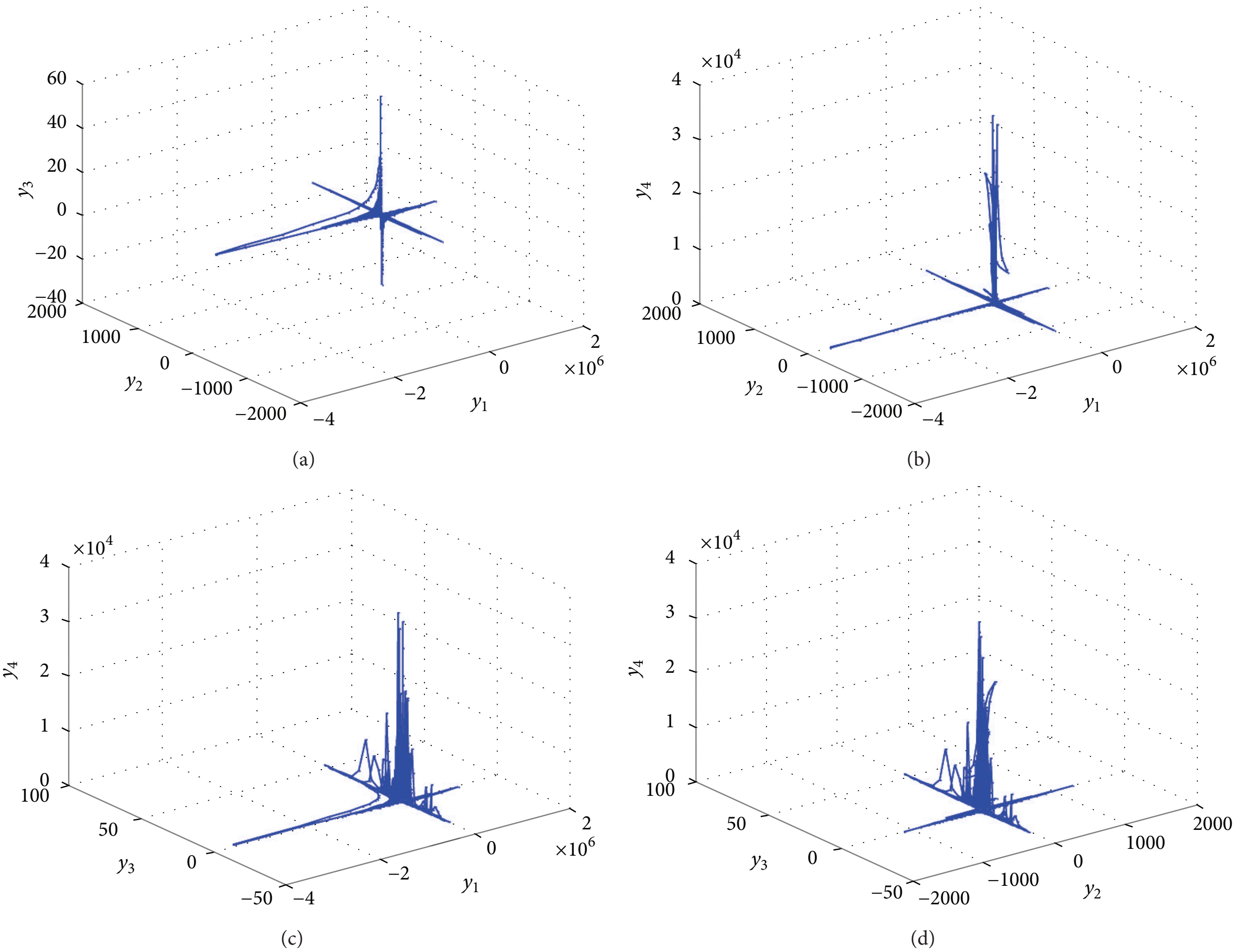

FIGURE 5: Projections of the phase portrait for chaotic system (13) of Case 2.

$$
\begin{aligned}
&\quad - \ddot{x}_3 - K\ddot{y}_3 + 3x_2^2\left(d(t)x_1 + c(t)x_2 - x_1x_3\right),\\
\dot{e}_4 &= -2y_4x_4\left(r(t)x_4 + x_2x_3\right)\\
&\quad - \left(1 + x_4^2\right) \times \left(d_1y_4 - y_1y_3 + u_4\right)\\
&\quad - \ddot{x}_4 - K\ddot{y}_4 + 3x_3^2\left(-b(t)x_3 + x_1x_2\right),
\end{aligned}
\tag{32}
$$

where $e_1 = -x_1^2y_1 - y_1 - \dot{x}_1 - K\dot{y}_1 + x_4^3$, $e_2 = -x_2^2y_2 - y_2 - \dot{x}_2 - K\dot{y}_2 + x_1^3$, $e_3 = -x_3^2y_3 - y_3 - \dot{x}_3 - K\dot{y}_3 + x_2^3$, and $e_4 = -x_4^2y_4 - y_4 - \dot{x}_4 - K\dot{y}_4 + x_3^3$.

Choose a positive definite Lyapunov function as

$$
V\left(e_1, e_2, e_3, e_4\right) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right).
\tag{33}
$$

Its time derivative along any solution of (32) is

$$
\begin{aligned}
\dot{V} &= e_1\Big\{-2y_1x_1\left[a(t)\left(x_2 - x_1\right) + x_4\right]\\
&\quad - \left(1 + x_1^2\right) \times \left[a_1\left(y_2 - y_1\right) + y_4\right] + u_1\\
&\quad - \ddot{x}_1 - K\ddot{y}_1 + 3x_4^2\left(r(t)x_4 + x_2x_3\right)\Big\}\\
&\quad + e_2\Big\{-2y_2x_2\left(d(t)x_1 + c(t)x_2 - x_1x_3\right)\\
&\quad - \left(1 + x_2^2\right) \times \left(b_1y_1 - y_2 - y_1y_3\right) + u_2\\
&\quad - \ddot{x}_2 - K\ddot{y}_2 + 3x_1^2\left[a(t)\left(x_2 - x_1\right) + x_4\right]\Big\}\\
&\quad + e_3\Big\{-2y_3x_3\left(-b(t)x_3 + x_1x_2\right)\\
&\quad - \left(1 + x_3^2\right) \times \left(-c_1y_3 + y_1y_2\right) + u_3\\
&\quad - \ddot{x}_3 - K\ddot{y}_3 + 3x_2^2\left(d(t)x_1 + c(t)x_2 - x_1x_3\right)\Big\}\\
&\quad + e_4\Big\{-2y_4x_4\left(r(t)x_4 + x_2x_3\right)\\
&\quad - \left(1 + x_4^2\right) \times \left(d_1y_4 - y_1y_3\right) + u_4\\
&\quad - \ddot{x}_4 - K\ddot{y}_4 + 3x_3^2\left(-b(t)x_3 + x_1x_2\right)\Big\}.
\end{aligned}
\tag{34}
$$

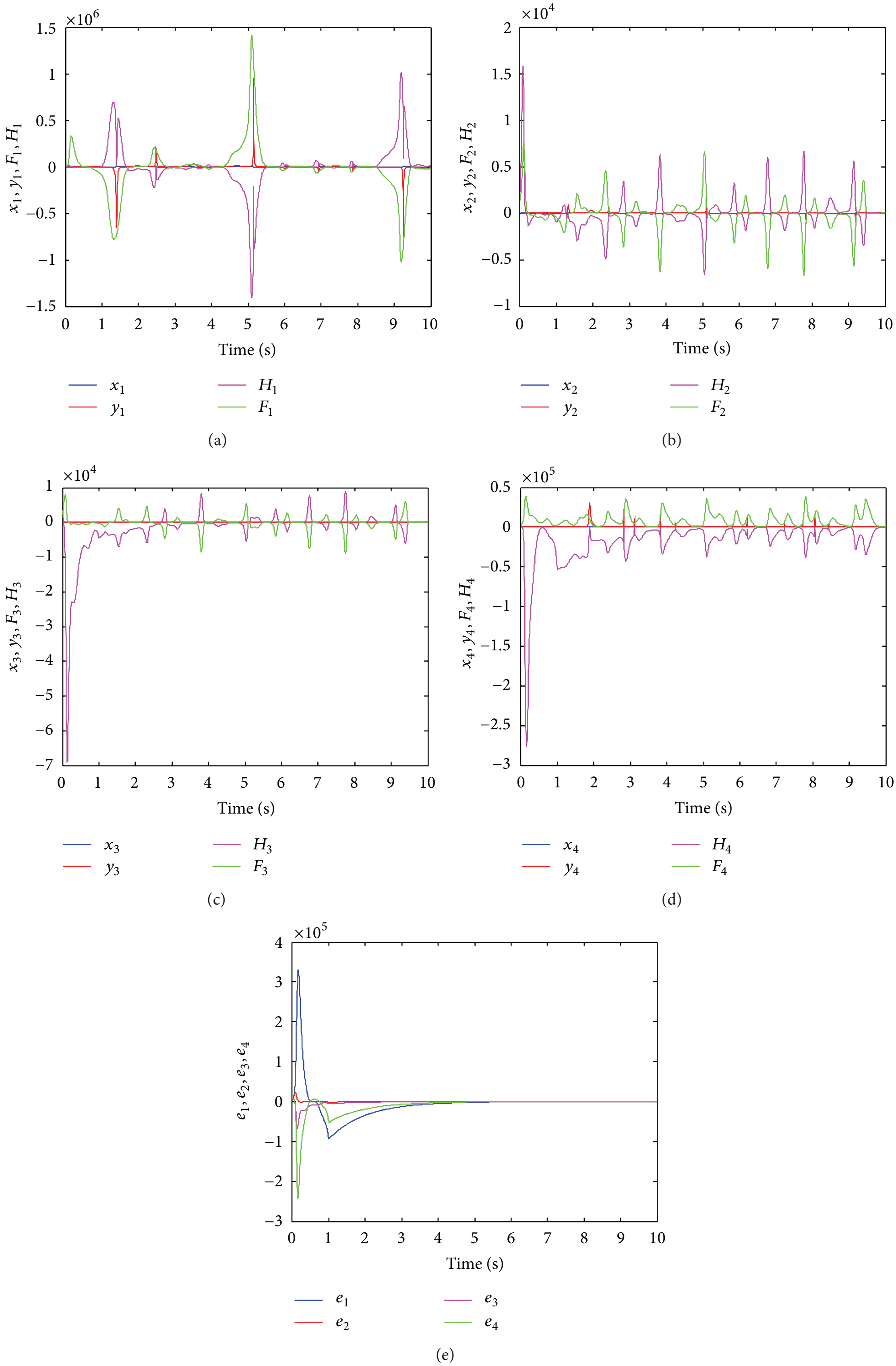

FIGURE 6: Time histories of states, state errors, F_1, F_2, F_3, F_4, H_1, H_2, H_3, and H_4 for Case 2.

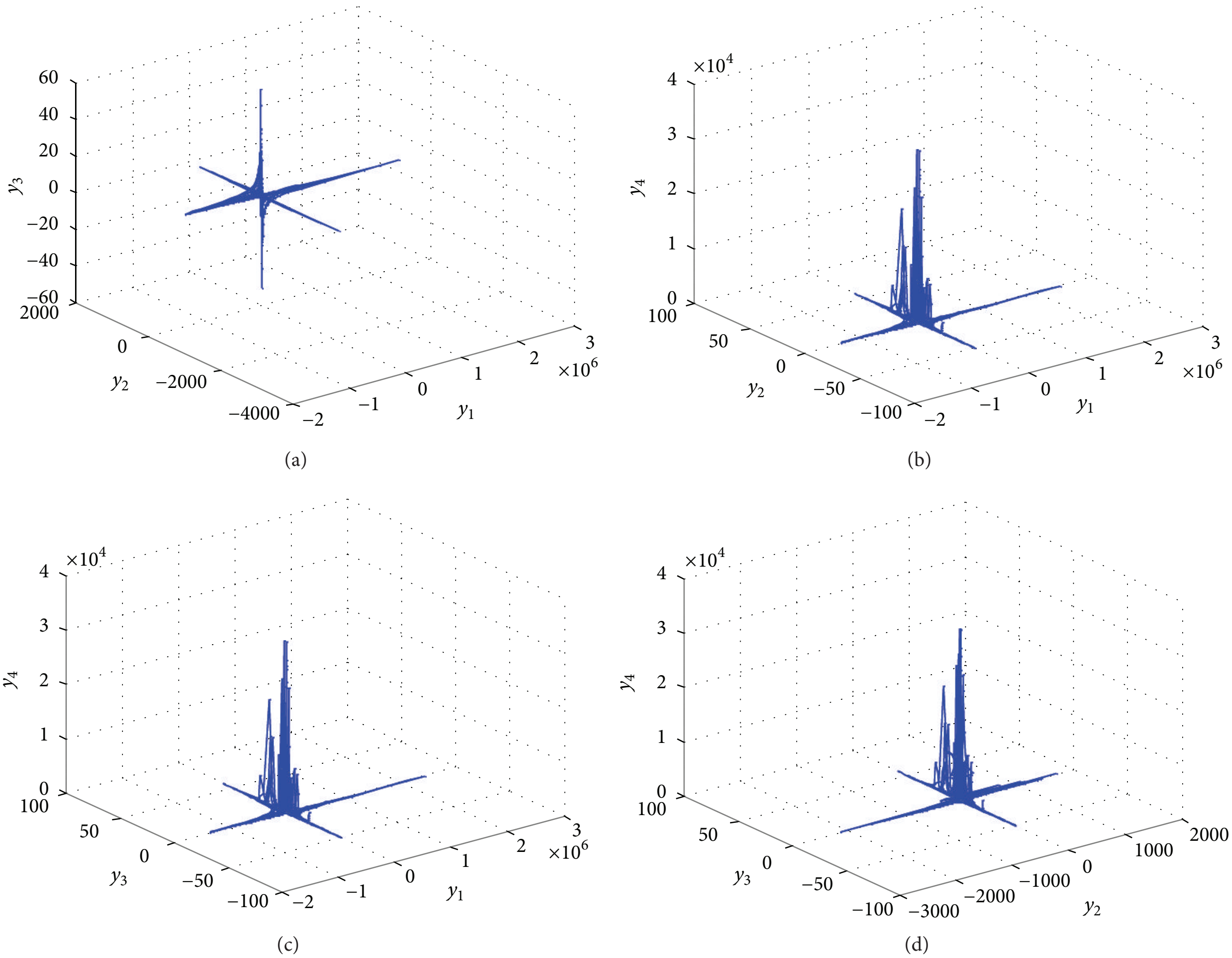

Figure 7: Projections of the phase portrait for chaotic system (13) of Case 3.

According to (10), we get the controller

$$
\begin{aligned}
u_1 = {} & -2y_1x_1\left[a(t)(x_2-x_1)+x_4\right] \\
& -\ddot{x}_1-K\ddot{y}_1+3x_4^2\left(r(t)x_4+x_2x_3\right) \\
& -\left(1+x_1^2\right)\left[a_1(y_2-y_1)+y_4\right]+e_1, \\
u_2 = {} & -2y_2x_2\left(d(t)x_1+c(t)x_2-x_1x_3\right) \\
& -\ddot{x}_2-K\ddot{y}_2+3x_1^2\left[a(t)(x_2-x_1)+x_4\right] \\
& -\left(1+x_2^2\right)\left(b_1y_1-y_2-y_1y_3\right)+e_2, \\
u_3 = {} & -2y_3x_3\left(-b(t)x_3+x_1x_2\right) \\
& -\ddot{x}_3-K\ddot{y}_3+3x_2^2\left(d(t)x_1+c(t)x_2-x_1x_3\right) \\
& +\left(1+x_3^2\right)\left(c_1y_3-y_1y_2\right)+e_3, \\
u_4 = {} & -2y_4x_4\left(r(t)x_4+x_2x_3\right)-\ddot{x}_4 \\
& -K\ddot{y}_4+3x_3^2\left(-b(t)x_3+x_1x_2\right) \\
& -\left(1+x_4^2\right)\left(d_1y_4-y_1y_3\right)+e_4.
\end{aligned}
\tag{35}
$$

Equation (34) becomes

$$
\dot{V}=-\left(e_1^2+e_2^2+e_3^2+e_4^2\right)<0, \tag{36}
$$

which is negative definite. The Lyapunov asymptotical stability theorem is satisfied. The enhanced symplectic synchronization of the Chen system with uncertain parameters and the Lorenz system is achieved. The numerical results are shown in Figures 7 and 8. After 1 second, the motion trajectories enter a chaotic attractor.

4. Conclusions

We achieve the novel enhanced symplectic synchronization of a Chen system with uncertain parameters, and a Lorenz system is obtained by the Lyapunov asymptotical stability

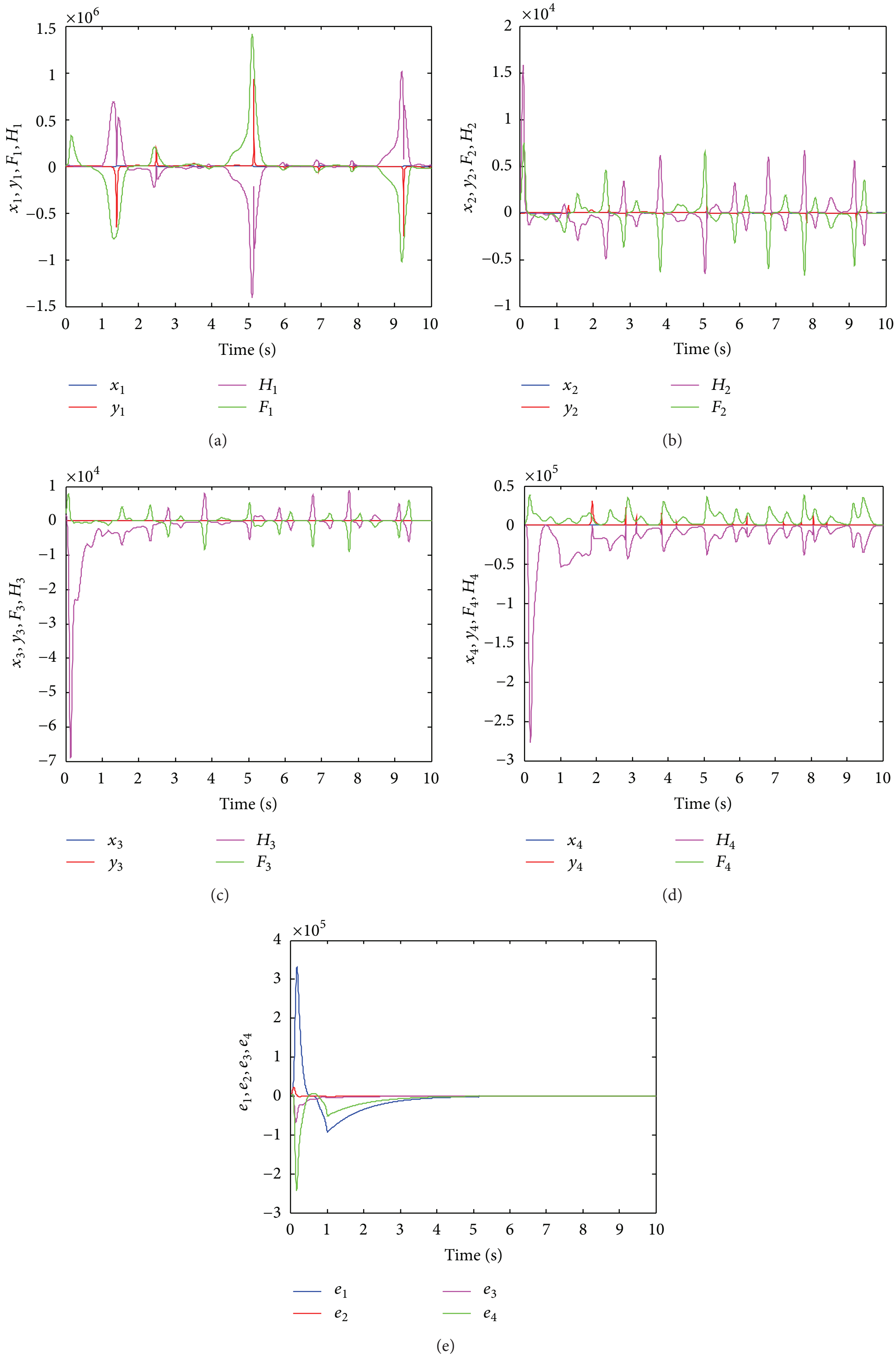

FIGURE 8: Time histories of states, state errors, F_1, F_2, F_3, F_4, H_1, H_2, H_3, and H_4 for Case 3.

theorem. All the theoretical results are verified by numerical simulations to demonstrate the effectiveness of the three cases of proposed synchronization schemes. The enhanced symplectic synchronization of chaotic systems with uncertain parameters can be used to increase the security of secret communication.

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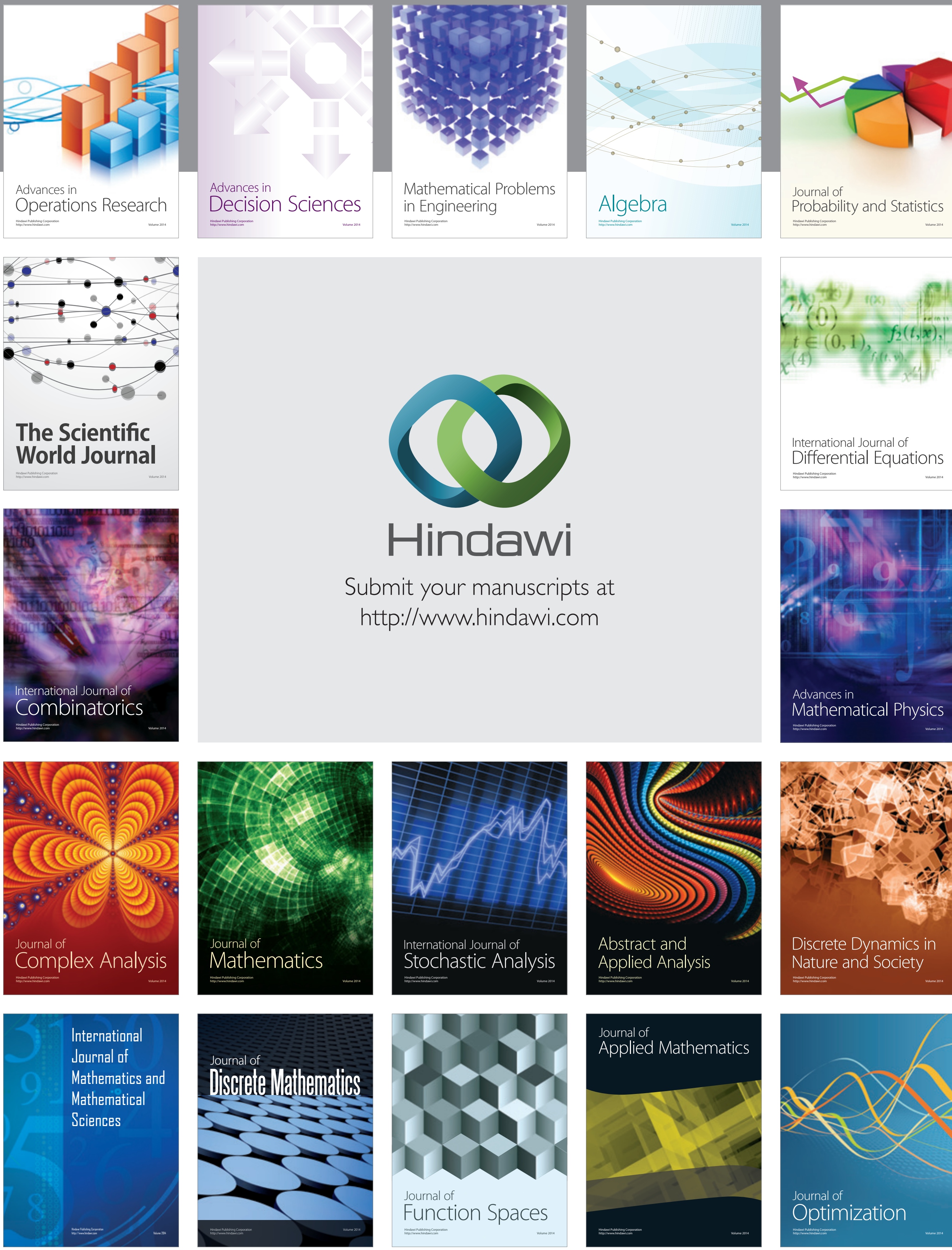
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