Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 236413, 14 pages
doi:10.1155/2012/236413

Research Article

Hölder Continuity of Solutions to Parametric Generalized Vector Quasiequilibrium Problems

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Received 2 October 2011; Accepted 2 December 2011

Academic Editor: Svatoslav Staněk

By using a linear scalarization method, we establish sufficient conditions for the Hölder continuity of the solution mappings to a parametric generalized vector quasiequilibrium problem with set-valued mappings. These results extend the recent ones in the recent literature, (e.g., Li et al. (2009), Li et al. (2011)). Furthermore, two examples are given to illustrate the obtained result.

1. Introduction

The vector equilibrium problem has been attracting great interest because it provides a unified model for several important problems such as vector variational inequalities, vector complementarity problems, vector optimization problems, vector min-max inequality, and vector saddle point problems. Many different types of vector equilibrium problems have been intensively studied for the past years; see, for example, [1–3] and the references therein.

It is important to derive results for parametric vector equilibrium problems concerning the properties of the solution mapping when the problems data vary. Among many desirable properties of vector equilibrium problems, the stability analysis of solutions is an essential topic in vector optimization theory and applications. In general, stability may be understood as lower (upper) semicontinuity, continuity, Lipschitz and Hölder continuity and so on. Recently, semicontinuity, especially lower semicontinuity, of solution mappings to parametric vector variational inequalities and parametric vector equilibrium problems has been intensively studied in the literature; see [4–12]. On the other hand, Hölder continuity of solutions to parametric vector equilibrium problems has also been discussed recently; see [13–22], although there are less works in the literature devoted to this property than to semicontinuity. There have been many papers devoted to discussing the local uniqueness and Hölder continuity of the solutions to parametric variational inequalities and parametric equilibrium problems; see [14–20] and the references therein. Yên [14] obtained Hölder continuity of the unique solution of a classic perturbed variational inequality by the metric projection method. Ait Mansour and Riahi [15] proved Hölder continuity of the unique solution for a parametric vector equilibrium problem under the concepts of strong monotonicity. Bianchi and Pini [16] introduced the concept of strong pseudomonotonicity and got the Hölder continuity of the unique solution of a parametric vector equilibrium problem. Bianchi and Pini [17] extend the results of [16] to vector equilibrium problems. Anh and Khanh [18] generalized the main results of [16] to the vector case and obtained Hölder continuity of the unique solutions for two classes of perturbed generalized vector equilibrium problems. Anh and Khanh [19] further discussed uniqueness and Hölder continuity of the solutions for perturbed generalized vector equilibrium problems, which improved remarkably the results in [16, 18]. Anh and Khanh [20] extended the results of [19] to the case of perturbed generalized vector quasiequilibrium problems and obtained Hölder continuity of the unique solutions.

For general perturbed vector quasiequilibrium problems, it is well known that a solution mapping is, in general, a set-valued one, but not a single-valued one. Naturally, there is a need to study Hölder continuous properties of the set-valued solution mappings. Under the Hausdorff distance and the strong quasimonotonicity, Lee et al. [21] first showed that the set-valued solution mapping for a parametric vector variational inequality is Hölder continuous. Recently, by virtue of the strong quasimonotonicity, Ait Mansour and Aussel [22] discussed Hölder continuity of set-valued solution mappings for parametric generalized variational inequalities. Li et al. [23] introduced an assumption, which is weaker than the corresponding ones of [16, 18], and established the Hölder continuity of the set-valued solution mappings for two classes of parametric generalized vector quasiequilibrium problems in general metric spaces. Li et al. [24] extended the results of [23] to perturbed generalized vector quasiequilibrium problems. Later, S. J. Li and X. B. Li [25] use a scalarization technique to obtain the Hölder continuity of the set-valued solution mappings for a parametric vector equilibrium problem in general metric spaces.

Motivated by the work reported in [21, 23, 25], this paper aims at establishing sufficient conditions for Hölder continuity of the solution sets for a class of parametric generalized vector quasiequilibrium problem ((PGVQEP), in short) with set-valued mapping, by using a linear scalarization method. The main results in this paper are different from corresponding results in [23, 24] and overcome the drawback, which requires the knowledge of detailed values of the solution mapping in a neighborhood of the point under consideration. Our main results also extend and improve the corresponding ones in [25].

The rest of the paper is organized as follows. In Section 2, we introduce the (PGVQEP) and define the solution and ξ-solution to the (PGVQEP). Then, we recall some notions and definitions which are needed in the sequel. In Section 3, we discuss Hölder continuity of the solution mapping for the (PGVQEP) and compare our main results with the corresponding ones in the recent literature. We also give two examples to illustrate that our main results are applicable.

2. Preliminaries

Throughout this paper, if not other specified, $\|\cdot\|$ and $d(\cdot,\cdot)$ denote the norm and metric in any metric space, respectively. Let $B(0,\delta)$ denote the closed ball with radius $\delta \geq 0$ and center

0 in any metric linear spaces. Let X, Λ, M, Y be metric linear spaces. Let Y^* be the topological dual space of Y. Let $C \subset Y$ be a pointed, closed, and convex cone with $\operatorname{int} C \neq \emptyset$, where $\operatorname{int} C$ denotes the interior of C. Let $C^* := \{f \in Y^* : f(y) \geq 0, \text{ for all } y \in C\}$ be the dual cone of C. Since $\operatorname{int} C \neq \emptyset$, the dual cone C^* of C has a weak* compact base. Letting $e \in \operatorname{int} C$ be given, then $B_e^* := \{\xi \in C^* : \|\xi\| = 1\}$ is a weak* compact base of C^*.

Let $N(\lambda_0) \subset \Lambda$ and $N(\mu_0) \subset M$ be neighborhoods of considered points λ_0 and μ_0, respectively. Let $K : X \times \Lambda \rightrightarrows X$ be a set-valued mapping, and let $F : X \times X \times M \rightrightarrows Y$ be a set-valued mapping. For each $\lambda \in N(\lambda_0)$ and $\mu \in N(\mu_0)$, consider the following parameterized generalized vector quasiequilibrium problem of finding $x_0 \in K(x_0, \lambda)$ such that

$$F(x_0, y, \mu) \subset Y \setminus -\operatorname{int} C, \quad \forall y \in K(x_0, \lambda). \tag{PGVQEP}$$

For each $\lambda \in N(\lambda_0)$ and $\mu \in N(\mu_0)$, let

$$E(\lambda) := \{x \in X \mid x \in K(x, \lambda)\}. \tag{2.1}$$

Let $S(\lambda, \mu)$ be the solution set of (PGVQEP), that is,

$$S(\lambda, \mu) := \{x \in E(\lambda) \mid F(x, y, \mu) \subset Y \setminus -\operatorname{int} C, \forall y \in K(x, \lambda)\}. \tag{2.2}$$

For each $\xi \in C^* \setminus \{0\}$, each $\lambda \in N(\lambda_0)$ and $\mu \in N(\mu_0)$, let $S_\xi(\lambda, \mu)$ denote the set of ξ-solution set to (PGVQEP), that is,

$$S_\xi(\lambda, \mu) := \left\{x \in E(\lambda) : \inf_{z \in F(x,y,\mu)} f(z) \geq 0, \forall y \in K(x, \lambda)\right\}. \tag{2.3}$$

Special Case

(i) When $K(x, \lambda) = K(\lambda)$, that is, K does not depend on x, the (PGVQEP) reduces to the parametric generalized vector equilibrium problem (PGVEP) considered by Li et al. [23].

(ii) If $F : X \times X \times M \to \mathbb{R}$, the (PGVQEP) collapses to the quasiequilibrium problem (QEP) considered by Anh and Khanh [26].

(iii) If $K(x, \lambda) = K(\lambda)$ and F is a vector-valued mapping, that is, $F : X \times X \times M \to Y$, the (PGVQEP) reduce to the parametric Ky Fan inequality (PKI) considered by S. J. Li and X. B. Li [25].

Now we recall some basic definitions and their properties which are needed in this paper.

Definition 2.1 (classical notion). A set-valued mapping $G : M \rightrightarrows X$ is said to be $\ell \cdot \alpha$-Hölder continuous at μ_0 if there is a neighborhood $U(\mu_0)$ of μ_0 such that, for all $\mu_1, \mu_1 \in U(\mu_0)$,

$$G(\mu_1) \subseteq G(\mu_2) + \ell B(0, d^\alpha(\mu_1, \mu_2)), \tag{2.4}$$

where $\ell \geq 0$ and $\alpha > 0$.

Definition 2.2. A set-valued mapping $G : X \times \Lambda \rightrightarrows Y$ is said to be $(\ell_1 \cdot \alpha_1, \ell_2 \cdot \alpha_2)$-Hölder continuous at (x_0, λ_0) if and only if there exists neighborhoods $N(x_0)$ of x_0 and $N(\mu_0)$ of μ_0 such that, for all $x_1, x_2 \in N(x_0)$, for all $\lambda_1, \lambda_2 \in N(\lambda_0)$,

$$G(x_1, \lambda_1) \subseteq G(x_2, \lambda_2) + (\ell_1 d^{\alpha_1}(x_1, x_2) + \ell_2 d^{\alpha_2}(\lambda_1, \lambda_2))B(0,1), \tag{2.5}$$

where $\ell_1, \ell_2 \geq 0$ and $\alpha_1, \alpha_2 > 0$.

Definition 2.3 (see [25]). A set-valued mapping $G : M \rightrightarrows Y$ is said to be $(\ell \cdot \alpha)$-Hölder continuous with respect to $e \in \operatorname{int} C$ at μ_0 if and only if there exists neighborhoods $N(\mu_0)$ of μ_0 such that, for all $\mu_1, \mu_2 \in N(\mu_0)$,

$$G(\mu_1) \subseteq G(\mu_2) + \ell d^{\alpha}(\mu_1, \mu_2)[-e, e], \tag{2.6}$$

where $\ell \geq 0$, $\alpha > 0$ and $[-e, e] = \{x : x \in e - C, x \in -e + C\}$.

Definition 2.4. Let $F : X \times X \times \Lambda \rightrightarrows Y$ be a set-valued mapping with nonempty values; $F(x, \cdot, \mu)$ is called C-like convex on $A(\lambda)$ if and only if for any $x_1, x_2 \in X$ and any $t \in [0,1]$, there exists $x_3 \in X$ such that

$$tF(x, x_1, \lambda) + (1 - t)F(x, x_2, \lambda) \subset F(x, x_3, \lambda) + C. \tag{2.7}$$

Remark 2.5. If for each $\mu \in N(\mu_0)$ and each $x \in E(N(\lambda_0))$, $F(x, \cdot, \mu)$ is C-like convex on $E(N(\lambda_0))$, then $F(x, E(N(\lambda_0)), \mu) + C$ is a convex set.

3. Main Results

In this section, we mainly discuss the Hölder continuity of the solution mappings to (PGVQEP).

Lemma 3.1. *Suppose that $N(\lambda_0), N(\mu_0)$ are the given neighborhoods of λ_0, μ_0, respectively.*

(a) *If for each $x, y \in E(N(\lambda_0))$, $F(x, y, \cdot)$ is $m_1 \cdot \gamma_1$-Hölder continuous with respect to $e \in \operatorname{int} C$ at $\mu_0 \in M$, then for any $\xi \in B_e^*$, the function $\varphi_\xi(x, y, \cdot) = \inf_{z \in F(x,y,\cdot)} \xi(z)$ is $m_1 \cdot \gamma_1$-Hölder continuous at μ_0.*

(b) *If for each $x \in E(N(\lambda_0))$ and $\mu \in N(E(\mu_0))$, $F(x, \cdot, \mu)$ is $m_2 \cdot \gamma_2$-Hölder continuous with respect to $e \in \operatorname{int} C$ on $E(N(\lambda_0))$, then for each $\xi \in B_e^*$, $\varphi_\xi(x, \cdot, \mu) = \inf_{z \in F(x,\cdot,\mu)} \xi(z)$ is also $m_2 \cdot \gamma_2$-Hölder continuous on $E(N(\lambda_0))$.*

Proof. (a) By assumption, there exists a neighborhood $N(\mu_0)$ of μ_0, such that for all $\mu_1, \mu_2 \in N(\mu_0)$, for all $x, y \in E(N(\lambda_0)) : x \neq y$,

$$F(x, y, \mu_1) \subset F(x, y, \mu_2) + m_1 d^{\gamma_1}(\mu_1, \mu_2)[-e, e]. \tag{3.1}$$

So, for any $z_1 \in F(x, y, \mu_1)$, there exist $z_2 \in F(x, y, \mu_2)$ and $e_0 \in [-e, e]$ such that

$$z_1 = z_2 + m_1 d^{\gamma_1}(\mu_1, \mu_2) e_0. \tag{3.2}$$

Then, by the linearity of ξ, we have

$$\xi(z_1) - \xi(z_2) = m_1 d^{\gamma_1}(\mu_1, \mu_2)\xi(e_0). \tag{3.3}$$

It follows from $\xi(e) = 1, e_0 \in [-e, e]$, and the structure of $[-e, e]$ that

$$\xi(e_0) \geq -1. \tag{3.4}$$

Therefore, (3.3) and (3.4) together yield that

$$-m_1 d^{\gamma_1}(\mu_1, \mu_2) \leq \xi(z_1) - \xi(z_2). \tag{3.5}$$

Since z_1 is arbitrary and $\xi(z_2) \geq \inf_{z\in F(x,y,\mu_2)}\xi(z)$, we have

$$-m_1 d^{\gamma_1}(\mu_1, \mu_2) \leq \inf_{z\in F(x,y,\mu_1)} \xi(z) - \inf_{z\in F(x,y,\mu_2)} \xi(z). \tag{3.6}$$

Due to the symmetry between μ_1 and μ_2, the same estimate is also valid, that is,

$$-m_1 d^{\gamma_1}(\mu_1, \mu_2) \leq \inf_{z\in F(x,y,\mu_2)} \xi(z) - \inf_{z\in F(x,y,\mu_1)} \xi(z). \tag{3.7}$$

Thus, it follows (3.6) and (3.7) that

$$\left| \inf_{z\in F(x,y,\mu_1)} \xi(z) - \inf_{z\in F(x,y,\mu_2)} \xi(z) \right| = \left|\varphi_\xi(x,y,\mu_1) - \varphi_\xi(x,y,\mu_2)\right| \leq m_1 d^{\gamma_1}(\mu_1, \mu_2) \tag{3.8}$$

and the proof is completed.

(b) As the proof of (b) is similar to (a), we omit it. Then the proof is completed. □

Lemma 3.2. *If for each $\mu \in N(\mu_0)$ and each $x \in E(N(\lambda_0))$, $F(x, \cdot, \mu)$ is C-like convex on $E(N(\lambda_0))$, that is, $F(x, E(N(\lambda_0)), \mu) + C$ is a convex set, then*

$$S(\lambda, \mu) = \bigcup_{\xi\in C^*\setminus 0} S_\xi(\lambda, \mu) = \bigcup_{\xi\in B_e^*} S_\xi(\lambda, \mu). \tag{3.9}$$

Proof. In a similar way to the proof of Lemma 3.1 in [8], with suitable modifications, we can obtain the conclusion. □

Theorem 3.3. *Assume that for each $\xi \in B_e^*$, the ξ-solution set for* (PGVQEP) *exists in a neighborhood $N(\lambda_0)\times N(\mu_0)$ of the considered point $(\lambda_0, \mu_0) \in \Lambda\times M$. Assume further that the following conditions hold.*

(i) *$K(\cdot,\cdot)$ is $(\ell_1 \cdot \alpha_1, \ell_2 \cdot \alpha_2)$-Hölder continuous in $E(N(\lambda_0)) \times N(\mu_0)$.*

(ii) *For each $x, y \in E(N(\lambda_0))$, $F(x, y, \cdot)$ is $m_1 \cdot \gamma_1$-Hölder continuous with respect to $e \in \operatorname{int} C$ at $\mu_0 \in M$.*

(iii) *For each* $x \in E(N(\lambda_0))$ *and* $\mu \in N(E(\mu_0))$, $F(x,\cdot,\mu)$ *is* $m_2 \cdot \gamma_2$*-Hölder continuous with respect to* $e \in \operatorname{int} C$ *on* $E(N(\lambda_0))$.

(iv) *for all* $\xi \in B_e^*, \mu \in N(\mu_0)$, *for all* $x, y \in E(N(\lambda_0)) : x \neq y$, *there exists two constants* $h > 0$ *and* $\beta > 0$ *such that*

$$hd^{\beta}(x,y) \leq d\left(\inf_{z\in F(x,y,\mu)} \xi(z), \mathbb{R}_+\right) + d\left(\inf_{z\in F(y,x,\mu)} \xi(z), \mathbb{R}_+\right). \tag{3.10}$$

(v) $\alpha_1\gamma_2 = \beta$ *and* $h > 2m_2\ell_1^{\gamma_2}$.

Then, for any $\bar{\xi} \in B_e^*$, *there exists open neighborhoods* $N(\bar{\xi})$ *of* $\bar{\xi}$, $N_{\bar{\xi}}(\lambda_0)$ *of* λ_0 *and* $N_{\bar{\xi}}(\mu_0)$ *of* μ_0, *such that the* ξ*-solution set* $S_\xi(\cdot,\cdot)$ *on* $N(\bar{\xi}) \times N_{\bar{\xi}}(\lambda_0) \times N_{\bar{\xi}}(\mu_0)$ *satisfies the following Hölder condition: for all* $\xi \in N(\bar{\xi})$, *for all* $(\lambda_1,\mu_1),(\lambda_2,\mu_2) \in N_{\bar{\xi}}(\lambda_0) \times N_{\bar{\xi}}(\mu_0)$,

$$d\left(x^{\xi}(\lambda_1,\mu_1), x^{\xi}(\lambda_2,\mu_2)\right) \leq \left(\frac{m_1}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\gamma_1/\beta}(\mu_1,\mu_2) + \left(\frac{2m_2\ell_2^{\gamma_2}}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2), \tag{3.11}$$

where $x^{\xi}(\lambda_i,\mu_i) \in S_\xi(\lambda_i,\mu_i), i = 1,2$.

Proof. Let $(\lambda_1,\mu_1),(\lambda_2,\mu_2) \in N_{\bar{\xi}}(\lambda_0) \times N_{\bar{\xi}}(\mu_0)$ be arbitrarily given. For all $\xi \in B_e^*$, $x, y \in X$, and $\mu \in M$, we set $\varphi_\xi(x,y,\cdot) := \inf_{z\in F(x,y,\cdot)} \xi(z)$ for the sake of convenient statement in the sequel. We prove that (3.11) holds by the following three steps.

Step 1. We first show that, for all $x^{\xi}(\lambda_1,\mu_1) \in S_\xi(\lambda_1,\mu_1)$, for all $x^{\xi}(\lambda_1,\mu_2) \in S_\xi(\lambda_1,\mu_2)$,

$$d\left(x^{\xi}(\lambda_1,\mu_1), x^{\xi}(\lambda_1,\mu_2)\right) \leq \left(\frac{m_1}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\gamma_1/\beta}(\mu_1,\mu_2). \tag{3.12}$$

Obviously, if $x^{\xi}(\lambda_1,\mu_1) = x^{\xi}(\lambda_1,\mu_2)$, we have that (3.12) holds. So we suppose $x^{\xi}(\lambda_1,\mu_1) \neq x^{\xi}(\lambda_1,\mu_2)$. Since $x^{\xi}(\lambda_1,\mu_1) \in K(x^{\xi}(\lambda_1,\mu_1),\lambda_1), x^{\xi}(\lambda_1,\mu_2) \in K(x^{\xi}(\lambda_1,\mu_2),\lambda_1)$, and by the Hölder continuity of $K(\cdot,\lambda_1)$, there exists $x_1 \in K(x^{\xi}(\lambda_1,\mu_1),\lambda_1)$ and $x_2 \in K(x^{\xi}(\lambda_1,\mu_2),\lambda_1)$ such that

$$\begin{aligned} &d(x^{\xi}(\lambda_1,\mu_1),x_2) \leq \ell_1 d^{\alpha_1}(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)),\\ &d(x^{\xi}(\lambda_1,\mu_2),x_1) \leq \ell_1 d^{\alpha_1}(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)). \end{aligned} \tag{3.13}$$

Since $x^{\xi}(\lambda_1,\mu_1), x^{\xi}(\lambda_1,\mu_2)$ are ξ-solutions to (PGVQEP) at parameters $(\lambda_1,\mu_1),(\lambda_1,\mu_2)$, respectively, we obtain

$$\begin{aligned} &\varphi_\xi(x^{\xi}(\lambda_1,\mu_1),x_1,\mu_1) \geq 0,\\ &\varphi_\xi(x^{\xi}(\lambda_1,\mu_2),x_2,\mu_2) \geq 0. \end{aligned} \tag{3.14}$$

By virtue of (iv), we get that

$$\begin{aligned} hd^{\beta}\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)\right) \le{}& d\left(\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_1,\mu_1),\mu_1\right),R_+\right) \\ &+ d\left(\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2),\mu_1\right),R_+\right), \end{aligned} \tag{3.15}$$

which together with (3.14) yields that

$$\begin{aligned} hd^{\beta}\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)\right) \le{}& \left|\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_1,\mu_1),\mu_1\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x_2,\mu_2\right)\right| \\ &+\left|\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2),\mu_1\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_1),x_1,\mu_1\right)\right| \\ \le{}& \left|\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_1,\mu_1),\mu_1\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_1,\mu_1),\mu_2\right)\right| \\ &+\left|\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_1,\mu_1),\mu_2\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x_2,\mu_2\right)\right| \\ &+\left|\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2),\mu_1\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_1),x_1,\mu_1\right)\right|. \end{aligned} \tag{3.16}$$

Then, from Lemma 3.1, (3.13), we have

$$\begin{aligned} &hd^{\beta}\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)\right) \\ &\quad\le m_1 d^{\gamma_1}(\mu_1,\mu_2)+m_2 d^{\gamma_2}\left(x^{\xi}(\lambda_1,\mu_2),x_1\right)+m_2 d^{\gamma_2}\left(x^{\xi}(\lambda_1,\mu_1),x_2\right) \\ &\quad\le m_1 d^{\gamma_1}(\mu_1,\mu_2)+2m_2\ell^{\gamma_2}d^{\alpha_1\gamma_2}\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)\right). \end{aligned} \tag{3.17}$$

The assumption (v) yields that

$$d\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)\right) \le \left(\frac{m_1}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\gamma_1/\beta}(\mu_1,\mu_2). \tag{3.18}$$

Hence, we have that (3.12) holds.

Step 2. Now we show that, for all $x^{\xi}(\lambda_1,\mu_2)\in S_{\xi}(\lambda_1,\mu_2)$, for all $x^{\xi}(\lambda_2,\mu_2)\in S_{\xi}(\lambda_2,\mu_2)$,

$$d\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right) \le \left(\frac{2m_2\ell_2^{\gamma_2}}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2). \tag{3.19}$$

Obviously, we only need to prove that (3.19) holds when $x^{\xi}(\lambda_1,\mu_2)\neq x^{\xi}(\lambda_2,\mu_2)$. By virtue of assumption (i), there exists $x_1'\in K(x^{\xi}(\lambda_2,\mu_2),\lambda_1)$ and $x_2'\in K(x^{\xi}(\lambda_1,\mu_2),\lambda_2)$ such that

$$\begin{aligned} d\left(x^{\xi}(\lambda_2,\mu_2),x_1'\right)&\leq \ell_2 d^{\alpha_2}(\lambda_1,\lambda_2),\\ d\left(x^{\xi}(\lambda_1,\mu_2),x_2'\right)&\leq \ell_2 d^{\alpha_2}(\lambda_1,\lambda_2). \end{aligned}\tag{3.20}$$

By the Hölder continuity of $K(\cdot,\cdot)$, there exists $x_1''\in K(x^{\xi}(\lambda_1,\mu_2),\lambda_1)$ and $x_2''\in K(x^{\xi}(\lambda_2,\mu_2),\lambda_2)$ such that

$$\begin{aligned} d(x_1',x_1'')&\leq \ell_1 d^{\alpha_1}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right),\\ d(x_2',x_2'')&\leq \ell_1 d^{\alpha_1}(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)). \end{aligned}\tag{3.21}$$

From the definition of ξ-solution for (PGVQEP), we have

$$\begin{aligned} \phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x_1'',\mu_2\right)&\geq 0,\\ \phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x_2'',\mu_2\right)&\geq 0. \end{aligned}\tag{3.22}$$

From assumptions (ii)–(iv), (3.22), and Lemma 3.1, we have

$$\begin{aligned} &hd^{\beta}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right)\\ &\quad\leq d\left(\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2),\mu_2\right),R_+\right)+d\left(\phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x^{\xi}(\lambda_1,\mu_2),\mu_2\right),R_+\right)\\ &\quad\leq d\left(\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x(\lambda_2,\mu_2),\mu_2\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x_1'',\mu_2\right)\right)\\ &\qquad+d\left(\phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x^{\xi}(\lambda_1,\mu_2),\mu_2\right)-\phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x_2'',\mu_2\right)\right)\\ &\quad\leq d\left(\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x(\lambda_2,\mu_2),\mu_2\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x_1',\mu_2\right)\right)\\ &\qquad+d\left(\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x_1',\mu_2\right)-\phi_{\xi}\left(x^{\xi}(\lambda_1,\mu_2),x_1'',\mu_2\right)\right)\\ &\qquad+d\left(\phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x^{\xi}(\lambda_1,\mu_2),\mu_2\right)-\phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x_2',\mu_2\right)\right)\\ &\qquad+d\left(\phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x_2',\mu_2\right)-\phi_{\xi}\left(x^{\xi}(\lambda_2,\mu_2),x_2'',\mu_2\right)\right)\\ &\quad\leq m_2 d^{\gamma_2}(x(\lambda_2,\mu_2),x_1')+m_2 d^{\gamma_2}(x_1',x_1'')+m_2 d^{\gamma_2}(x(\lambda_1,\mu_2),x_2')+m_2 d^{\gamma_2}(x_2',x_2''). \end{aligned}\tag{3.23}$$

By virtue of (3.20)–(3.21) and (3.23), we can get

$$hd^{\beta}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right)\leq m_2\ell_2^{\gamma_2}d^{\alpha_2\gamma_2}(\lambda_1,\lambda_2)+m_2\ell_1^{\gamma_2}d^{\alpha_1\gamma_2}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right)$$
$$+m_2\ell_2^{\gamma_2}d^{\alpha_2\gamma_2}(\lambda_1,\lambda_2)+m_2\ell_1^{\gamma_2}d^{\alpha_1\gamma_2}\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right). \tag{3.24}$$

Therefore, it follows from (v) that

$$d\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right)\leq\left(\frac{2m_2\ell_2^{\gamma_2}}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta}d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2) \tag{3.25}$$

and the conclusion (3.19) holds.

Step 3. Finally, by the arbitrariness of $x^{\xi}(\lambda_1,\mu_1)\in S_{\xi}(\lambda_1,\mu_1)$, $x^{\xi}(\lambda_1,\mu_2)\in S_{\xi}(\lambda_1,\mu_2)$, $x^{\xi}(\lambda_2,\mu_2)\in S_{\xi}(\lambda_2,\mu_2)$, (3.12) and (3.19), we can easily get that

$$d\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_2,\mu_2)\right)\leq d\left(x^{\xi}(\lambda_1,\mu_1),x^{\xi}(\lambda_1,\mu_2)\right)+d\left(x^{\xi}(\lambda_1,\mu_2),x^{\xi}(\lambda_2,\mu_2)\right)$$
$$\leq\left(\frac{m_1}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta}d^{\gamma_1/\beta}(\mu_1,\mu_2)+\left(\frac{2m_2\ell_2^{\gamma_2}}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta}d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2) \tag{3.26}$$

and the conclusion (3.11) holds. This completes the proof. □

Remark 3.4. Theorem 3.3 generalizes Lemma 3.3 in S. J. Li and X. B. Li [25] from vector-valued version to set valued version. Moreover, the assumption (H_4) of Lemma 3.3 in [25] is removed.

Now, we give an example to illustrate that Theorem 3.3 is applicable under the case that the mapping F is set valued.

Example 3.5. Let $X=Y=R,\Lambda=M=[0,1],C=\mathbb{R}_+$ and $e=3/2\in\operatorname{int}C$. Let $K:X\times M\rightrightarrows Y$ be defined by

$$K(x,\lambda)=\left[\frac{\lambda^2+x}{16},1\right] \tag{3.27}$$

and $F:X\times X\times M\rightrightarrows Y$ a set-valued mapping defined by

$$F(x,y,\lambda)=\left[(1+\lambda)\left(x+\frac{1}{2}\right)(y-x),28-2x^{3/2}\right]. \tag{3.28}$$

Then, $E(\lambda)=[\lambda^2/15,1]$. Consider that $\lambda_0=0.5$ and $N(\lambda_0)=\Lambda$. Direct computation shows that $E(\Lambda)=E(N(\lambda_0))=[0,1]$.

It can be checked that $K(\cdot,\cdot)$ is $((1/16)\cdot 1,(3/2)\cdot 1)$-Hölder continuous in $E(N(\lambda_0)) \times N(\mu_0)$; for all $x,y \in E(N(\lambda_0))$, $F(x,y,\cdot)$ is $6\sqrt{2}.1$-Hölder continuous with respect to $e = 3/2 \in \operatorname{int} C$ at $\lambda_0 \in M$; for each $x \in E(N(\lambda_0))$ and $\lambda \in N(E(\lambda_0))$, $F(x,\cdot,\lambda)$ is 3.1-Hölder continuous with respect to $e \in \operatorname{int} C$ on $E(N(\lambda_0))$. Here $\ell_1 = 1/16, \alpha_1 = 1, \ell_2 = 3/2, \alpha_2 = 1, m_1 = 6\sqrt{2}, \gamma_1 = 1, m_2 = 3, \gamma_2 = 1$. Take $\beta = 1$ and $h = 1/2$, for any $\xi \in B_e^*$ and for all $x,y \in E(N(\lambda_0)) : x \neq y$, we have

$$hd^{\beta}(x,y) \le d\left(\inf_{z\in F(x,y,\mu)} \xi(z), R_+\right) + d\left(\inf_{z\in F(y,x,\mu)} \xi(z), R_+\right) \tag{3.29}$$

and also have $\alpha_1\gamma_2 = \beta$ and $h > 2m_2\ell_1^{\gamma_2} = 3/8$. Hence, all assumptions of Theorem 3.3 hold, and thus it is valid.

Theorem 3.6. *Assume that for each $\xi \in B_e^*$, the ξ-solution set for* (PGVQEP) *exists in a neighborhood $N(\lambda_0)\times N(\mu_0)$ of the considered point $(\lambda_0,\mu_0) \in \Lambda\times M$. Assume further that the following conditions hold:*

(i) *$K(\cdot,\cdot)$ is $(\ell_1\cdot\alpha_1, \ell_2\cdot\alpha_2)$-Hölder continuous in $E(N(\lambda_0)) \times N(\mu_0)$;*

(ii) *for each $x,y \in E(N(\lambda_0))$, $F(x,y,\cdot)$ is $m_1\cdot\gamma_1$-Hölder continuous with respect to $e \in \operatorname{int} C$ at $\mu_0 \in M$;*

(iii) *for each $x \in E(N(\lambda_0))$ and $\mu \in N(E(\mu_0))$, $F(x,\cdot,\mu)$ is $m_2\cdot\gamma_2$-Hölder continuous with respect to $e \in \operatorname{int} C$ on $E(N(\lambda_0))$;*

(iv) *for all $\xi \in B_e^*, \mu \in N(\mu_0)$, for all $x,y \in E(N(\lambda_0))(x \neq y)$, there exist two constants $h > 0$ and $\beta > 0$ such that*

$$hd^{\beta}(x,y) \le d\left(\inf_{z\in F(x,y,\mu)} \xi(z), R_+\right) + d\left(\inf_{z\in F(y,x,\mu)} \xi(z), R_+\right); \tag{3.30}$$

(v) *for all $x \in E(N(\lambda_0))$, for all $\mu \in N(\mu_0)$, $F(x,\cdot,\mu)$ is C-like convex on $E(N(\lambda_0))$;*

(vi) *$\alpha_1\gamma_2 = \beta$ and $h > 2m_2\ell_1^{\gamma_2}$.*

Then there exist neighborhoods $\widetilde{N}(\lambda_0)$ of λ_0 and $\widetilde{N}(\mu_0)$ of μ_0, such that the solution set $S(\cdot,\cdot)$ on $\widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$ is nonempty and satisfies the following Hölder continuous condition, for all $(\lambda_1,\mu_1),(\lambda_2,\mu_2) \in \widetilde{N}(\lambda_0)\times\widetilde{N}(\mu_0)$:

$$\begin{aligned} S(\lambda_1,\mu_1) &\subset S(\lambda_2,\mu_2) \\ &+ \left(\left(\frac{m_1}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\gamma_1/\beta}(\mu_1,\mu_2) + \left(\frac{2m_2\ell_2^{\gamma_2}}{h-2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2)\right) B(0,1). \end{aligned} \tag{3.31}$$

Proof. Since the system of $\{N'(\bar{\xi})\}_{\bar{\xi}\in B_e^*}$, which are given by Theorem 3.3, is an open covering of the weak* compact set B_e^*, there exist a finite number of points (ξ_i) $(i = 1, 2, \ldots, n.)$ from B_e^* such that

$$B_e^* \subset \bigcup_{i=1}^{n} N'(\xi_i). \tag{3.32}$$

Hence, let $\widetilde{N}(\lambda_0) = \bigcap_{i=1}^{n} N'_{\xi_i}(\lambda_0)$ and $\widetilde{N}(\mu_0) = \bigcap_{i=1}^{n} N'_{\xi_i}(\mu_0)$. Then $\widetilde{N}(\lambda_0)$ and $\widetilde{N}(\mu_0)$ are desired neighborhoods of λ_0 and μ_0, respectively. Indeed, let $(\lambda, \mu) \in \widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$ be given arbitrarily. For any $\xi \in B_e^*$, by virtue of (3.32), there exists $i_0 \in \{1, 2, \ldots, n\}$ such that $\xi \in N'(\xi_{i_0})$. From the construction of the neighborhoods $\widetilde{N}(\lambda_0)$ and $\widetilde{N}(\mu_0)$, one has

$$(\lambda, \mu) \in N'_{\xi_{i_0}}(\lambda_0) \times N'_{\xi_{i_0}}(\mu_0). \tag{3.33}$$

Then, from the assumption of existence for ξ-solution set and Lemma 3.2, $S(\lambda, \mu) = \bigcup_{\xi\in B_e^*} S_\xi(\lambda, \mu)$ is nonempty.

Now, we show that (3.31) holds. Indeed, taking any $(\lambda_1, \mu_1), (\lambda_2, \mu_2) \in \widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$, we need to show that for any $x_1 \in S(\lambda_1, \mu_1)$, there exists $x_2 \in S(\lambda_2, \mu_2)$ satisfying

$$d(x_1, x_2) \leq \left(\frac{m_1}{h - 2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\gamma_1/\beta}(\mu_1, \mu_2) + \left(\frac{2m_2\ell_2^{\gamma_2}}{h - 2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1, \lambda_2). \tag{3.34}$$

Since $x_1 \in S(\lambda_1, \mu_1) = \bigcup_{\xi\in B_e^*} S_\xi(\lambda_1, \mu_1)$, there exists $\widehat{\xi} \in B_e^*$ such that

$$x_1 = x^{\widehat{\xi}}(\lambda_1, \mu_1) \in S_{\widehat{\xi}}(\lambda_1, \mu_1). \tag{3.35}$$

It follows from (3.32) that there exists $i_0 \in \{1, 2, \ldots, n\}$ such that $\widehat{\xi} \in N'(\xi_{i_0})$. Thus, by the construction of the neighborhoods $\widetilde{N}(\lambda_0)$ and $\widetilde{N}(\mu_0)$, we have

$$(\lambda_1, \mu_1), (\lambda_2, \mu_2) \in N_{\xi_{i_0}}(\lambda_0) \times N_{\xi_{i_0}}(\mu_0). \tag{3.36}$$

Obviously, thanks to Theorem 3.3, we have

$$d\left(x^{\widehat{\xi}}(\lambda_1, \mu_1), x^{\widehat{\xi}}(\lambda_2, \mu_2)\right) \leq \left(\frac{m_1}{h - 2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\gamma_1/\beta}(\mu_1, \mu_2) + \left(\frac{2m_2\ell_2^{\gamma_2}}{h - 2m_2\ell_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1, \lambda_2). \tag{3.37}$$

Let $x_2 = x^{\widehat{\xi}}(\lambda_2, \mu_2)$. Then, (3.34) holds, and the proof is complete. □

Remark 3.7. Theorem 3.6 generalizes, and improves the corresponding results of S. J. Li and X. B. Li [25] in the following three aspects.

(i) The vector-valued mapping $F(x,y,\mu)$ is extended to set-valued, and the parametric vector equilibrium problem is extended to the parametric vector quasiequilibrium problem.

(ii) The assumption (H_4) of Theorem 3.1 in [25] is removed.

(iii) The C-convexity of $F(x,\cdot,\mu)$ (see Theorem 3.1 in [25]) is extended to C-convexlikeness.

In addition, it is easy to see that the assumption (iv) of Theorem 3.6 is different form the assumption (H_1) of Theorem 3.1 in S. J. Li and X. B. Li [25].

Moreover, we also can see that the obtained result extends the ones of [23]. Now, we give the following example to illustrate the case.

Example 3.8. Let $X = Y = \mathbb{R}, \Lambda = M = [0,1], C = \mathbb{R}_+$, and $e = \sqrt{2}/2 \in \operatorname{int} C$. Let $K : X \times M \rightrightarrows Y$ be defined by $K(x,\lambda) = [\lambda^2, 1]$, and let $F : X \times X \times M \rightrightarrows Y$ be a set-valued mapping defined by

$$F(x,y,\lambda) = \left[\left(\frac{3}{4} + 2\lambda\right)(y+3)(x-y), 20 - |x|^{1/2}\right]. \tag{3.38}$$

Consider that $\lambda_0 = 0.5$ and $N(\lambda_0) = \Lambda$. Then, $E(\lambda) = [\lambda^2, 1]$ and $E(\Lambda) = E(N(\lambda_0)) = [0,1]$.

Obviously, $K(\cdot,\cdot)$ is $(0.1, \sqrt{2}\cdot 1)$-Hölder continuous in $E(N(\lambda_0)) \times N(\mu_0)$; for all $x, y \in E(N(\lambda_0))$, $F(x,y,\cdot)$ is $7\sqrt{2}\cdot 1$-Hölder continuous with respect to $e = \sqrt{2}/2 \in \operatorname{int} C$ at $\lambda_0 \in M$; for each $x \in E(N(\lambda_0))$ and $\lambda \in N(E(\lambda_0))$, $F(x,\cdot,\lambda)$ is $9\sqrt{5}\cdot 1$-Hölder continuous with respect to $e \in \operatorname{int} C$ on $E(N(\lambda_0))$. Here $\ell_1 = 0, \alpha_1 = 1, \ell_2 = \sqrt{2}, \alpha_2 = 1, m_1 = 7\sqrt{2}, \gamma_1 = 1, m_2 = 9\sqrt{5}, \gamma_2 = 1$. Take $\beta = 1$ and $h = 3/4$, for any $\xi \in B_e^*$ and for all $x, y \in E(N(\lambda_0))(x \neq y)$, we have

$$hd^{\beta}(x,y) \leq d\left(\inf_{z\in F(x,y,\mu)} \xi(z), \mathbb{R}_+\right) + d\left(\inf_{z\in F(y,x,\mu)} \xi(z), \mathbb{R}_+\right) \tag{3.39}$$

and also have $\alpha_1\gamma_2 = \beta$ and $h = 3/4 > 2m_2\ell_1^{\gamma_2}$. Therefore, all assumptions of Theorem 3.3 hold, and thus it is applicable.

However, the assumption (ii) of Theorem 3.1 (or (ii') of Theorem 4.1) in [23] does not hold. In fact, for any $\lambda \in \Lambda$, for any $h > 0$ and $\beta > 0$, there exists $y_0 = 0 \in E(N(\lambda_0)) \setminus S_{\xi}(\lambda,\mu)$ such that

$$\begin{aligned} F(y_0,\overline{x},\lambda) + hB(0,d^{\beta}(\overline{x},y_0)) &= \left[-\left(\frac{3}{4} + 2\lambda\right)(\overline{x}+3)\overline{x}, 20 - |\overline{x}|^{1/2}\right] + hB(0,d^{\beta}(0,\overline{x})) \\ &\not\subseteq -\mathbb{R}_+ \end{aligned} \tag{3.40}$$

for all $\overline{x} \in S_{\xi}(\lambda,\mu)$. Thus, Theorems 3.1 and 4.1 in Li et al. [23] are not applicable.

Acknowledgments

The author would like to thank the anonymous referees for valuable comments and suggestions, which helped to improve the paper. This work was supported by the Natural

Science Foundation of China (no. 10831009. 11001287), the Natural Science Foundation Project of ChongQing (no. CSTC, 2010BB9254. 2011AC6104), and the Research Grant of Chongqing Key Laboratory of Operations and System Engineering.

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