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Research Article

Global Behavior for a Strongly Coupled Predator-Prey Model with One Resource and Two Consumers

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We consider a strongly coupled predator-prey model with one resource and two consumers, in which the first consumer species feeds on the resource according to the Holling II functional response, while the second consumer species feeds on the resource following the Beddington-DeAngelis functional response, and they compete for the common resource. Using the energy estimates and Gagliardo-Nirenberg-type inequalities, the existence and uniform boundedness of global solutions for the model are proved. Meanwhile, the sufficient conditions for global asymptotic stability of the positive equilibrium for this model are given by constructing a Lyapunov function.

1. Introduction

The principle of competitive exclusion asserts that two or more consumer species cannot coexist indefinitely upon a single limiting resource, which dates back to the pioneering work of Volterra [1] in the 1920s. Subsequently, Ayala [2] in 1969 demonstrated experimentally that two species of Drosophila can coexist upon a single limiting resource. Ayala's experiments have received much attention (see the comprehensive survey by Cantrell and Cosner [3]). Schoener [4] in 1976 found that intraspecific interference among consumers may lead to coexistence of multiple consumer species upon a single resource. To examine more closely the implications of feeding interference among conspecific consumers on

consumer-resource dynamics, Cantrell et al. [5] in 2004 proposed the following predator-prey system:

$$\begin{aligned}
\frac{du}{dt} &= ru\left(1-\frac{u}{K}\right)-\frac{auv}{1+bu}-\frac{Auw}{1+Bu+Dw},\\
\frac{dv}{dt} &= v\left(-l+\frac{eu}{1+bu}\right),\\
\frac{dw}{dt} &= w\left(-L+\frac{Eu}{1+Bu+Dw}\right),
\end{aligned} \tag{1.1}$$

where $r, a, b, e, l, A, B, D, E, L$, and K are positive constants, $u(t)$ represents the density of the limiting resource at time t, $v(t)$ and $w(t)$ denote two consumers species. The first consumer species feeds upon the resource according to the Holling II functional response, while the second consumer species feeds upon the resource following the Beddington-DeAngelis functional response, and they compete for the common resource. For more details on the backgrounds about this system see [5].

The system (1.1) has a positive equilibrium $\widetilde{E}=(\tilde{u},\tilde{v},\tilde{w})$ under the suitable conditions, where

$$\tilde{u}=\frac{l}{e-bl},\qquad \tilde{v}=\frac{e}{al}\left\{r\tilde{u}\left(1-\frac{\tilde{u}}{K}\right)-\frac{A}{DE}[(E-BL)\tilde{u}-L]\right\},\qquad \tilde{w}=\frac{(E-BL)\tilde{u}-L}{DL}. \tag{1.2}$$

The Jacobian matrix of the system (1.1) at $\widetilde{E}$ can be written as

$$J\left(\widetilde{E}\right)=\begin{pmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}, \tag{1.3}$$

where

$$\begin{aligned}
&a_{11}=\tilde{u}\left[-\frac{r\tilde{u}}{K}+\frac{ab\tilde{v}}{(1+b\tilde{u})^2}+\frac{AB\tilde{w}}{(1+B\tilde{u}+D\tilde{w})^2}\right],\qquad a_{12}=-\frac{a\tilde{u}}{1+b\tilde{u}}<0,\\
&a_{13}=-\frac{A\tilde{u}(1+B\tilde{u})}{(1+B\tilde{u}+D\tilde{w})^2}<0,\qquad a_{21}=\frac{e\tilde{v}}{(1+b\tilde{u})^2}>0,\qquad a_{31}=\frac{E\tilde{w}(1+D\tilde{w})}{(1+B\tilde{u}+D\tilde{w})^2}>0,\\
&a_{33}=-\frac{DE\tilde{u}\tilde{w}}{(1+B\tilde{u}+D\tilde{w})^2}<0,\qquad a_{22}=a_{23}=a_{32}=0.
\end{aligned} \tag{1.4}$$

The following results were proved in [5]:

(1) the system (1.1) is dissipative;

(2) the positive equilibrium $\widetilde{E}$ of (1.1) is locally stable if $-(a_{11}+a_{33})>0$ and $a_{11}(a_{11}a_{33}-a_{13}a_{31}-a_{12}a_{21})-a_{33}(a_{11}a_{33}-a_{31}a_{31})<0$; and

(3) the positive equilibrium $\widetilde{E}$ of (1.1) is globally stable if $\max\{b,B\}(K-\tilde{u})<1$.

Rescaling the system (1.1) such that

$$\frac{u}{K}\longmapsto u,\quad v\longmapsto v,\quad w\longmapsto w,\quad rt\longmapsto t,\quad \frac{a}{r}\longmapsto a,\quad bK\longmapsto b,\quad \frac{A}{r}\longmapsto A,\quad BK\longmapsto B,$$
$$D\longmapsto D,\quad \frac{l}{r}\longmapsto l,\quad \frac{(eK)}{l}\longmapsto e,\quad \frac{L}{r}\longmapsto L,\quad \frac{(EK)}{L}\longmapsto E \tag{1.5}$$

yields

$$\begin{aligned}\frac{du}{dt}&=u(1-u)-\frac{auv}{1+bu}-\frac{Auw}{1+Bu+Dw},\\ \frac{dv}{dt}&=lv\left(-1+\frac{eu}{1+bu}\right),\\ \frac{dw}{dt}&=Lw\left(-1+\frac{Eu}{1+Bu+Dw}\right).\end{aligned} \tag{1.6}$$

The corresponding weakly coupled reaction-diffusion system for (1.6) is as follows:

$$\begin{aligned}&u_t=d_1\Delta u+u(1-u)-\frac{auv}{1+bu}-\frac{Auw}{1+Bu+Dw},\quad x\in\Omega,\ t>0,\\ &v_t=d_2\Delta v+lv\left(-1+\frac{eu}{1+bu}\right),\quad x\in\Omega,\ t>0,\\ &w_t=d_3\Delta w+Lw\left(-1+\frac{Eu}{1+Bu+Dw}\right),\quad x\in\Omega,\ t>0,\\ &\partial_\nu u=\partial_\nu v=\partial_\nu w=0,\quad x\in\partial\Omega,\ t>0,\\ &u(x,0)=u_0(x),\qquad v(x,0)=v_0(x),\qquad w(x,0)=w_0(x),\quad x\in\Omega,\end{aligned} \tag{1.7}$$

where $\Omega\subset\mathbb{R}^N$ is a bounded smooth domain, ν is the outward unit normal vector of the boundary $\partial\Omega,\partial_\nu=\partial/\partial\nu$. The constants d_1,d_2, and d_3, called diffusion coefficients, are positive, and $u_0(x),v_0(x)$, and $w_0(x)$ are nonnegative functions which are not identically zero. The system (1.7) has a constant positive steady-state solution $E^*=(u^*,v^*,w^*)$ if and only if

$$e>b,\quad E-B>e-b,\quad DE(e-b-1)-A(e-b)[(E-B)-(e-b)]>0, \tag{1.8}$$

where

$$\begin{aligned}u^*&=\frac{1}{e-b},\qquad v^*=\frac{e\{DE(e-b-1)-A(e-b)[(E-B)-(e-b)]\}}{aDE(e-b)^2},\\ w^*&=\frac{(E-B)-(e-b)}{D(e-b)}.\end{aligned} \tag{1.9}$$

In [6], Hei and Yu proved the following main results.

(1) The equilibrium (u^*, v^*, w^*) of (1.7) is locally asymptotically stable if (1.8) and

$$DE^2[b(e-b-1)-e]+A(e-b)(Be-bE)[(E-B)-(e-b)]<0 \tag{1.10}$$

hold.

(2) Let d_2^* and d_3^* be fixed positive constants which satisfy $d_2^*\mu_1 \geq l(e-b-1)/(1+b)$ and $d_3^*\mu_1 \geq L(E-B-1)/(1+B)$.

Then there exists a positive constant D_1, such that (1.7) has no nonconstant positive solution if $d_1 > D_1$, $d_2 \geq d_2^*$ and $d_3 \geq d_3^*$, where $0=\mu_0<\mu_1<\mu_2<\cdots$ are the eigenvalues of the operator $-\Delta$ on Ω with the homogeneous Neumann boundary condition.

(3) Let d_i $(i=1,3)$ be fixed positive constants. Assume that $a_{11}^*>0$,

$$\min\left\{\frac{4d_2-4eld_1-ed_2}{4d_2}, \frac{[D(E-B)-A(E-B-1)]}{D(E-B)}\right\} > \frac{1}{e-b} \tag{1.11}$$

and (1.8) hold. Furthermore, assume that one of the following conditions is satisfied:

(i) $a_{11}^*a_{33}^*-a_{31}^*a_{13}^*<0$, $\check{\mu}\in(\mu_n,\mu_{n+1})$ for some $n\geq 1$, and the sum $\sigma_n=\sum_{i=1}^{n}\dim E(\mu_i)$ is odd;

(ii) $a_{11}^*a_{33}^*-a_{31}^*a_{13}^*>0$, $\widehat{\mu}\in(\mu_k,\mu_{k+1})$, $\check{\mu}\in(\mu_n,\mu_{n+1})$ for some $n\geq k\geq 1$, and the sum $\sigma_n=\sum_{i=k+1}^{n}\dim E(\mu_i)$ is odd.

Then there exists a positive constant D_2, such that (1.7) has at least one nonconstant positive solution if $d_2 \geq D_2$, where

$$\begin{aligned}
a_{11}^* &= -u^*+\frac{abv^*}{e^2u^*}+\frac{ABw^*}{E^2u^*},\\
a_{33}^* &= -\frac{DELu^*w^*}{(1+Bu^*+Dw^*)^2},\\
a_{13}^* &= -\frac{Aw^*}{Eu^*},\\
a_{31}^* &= \frac{ELw^*(1+Dw^*)}{(1+Bu^*+Dw^*)^2},\\
\check{\mu} &= \frac{a_{11}^*d_3+a_{33}^*d_1+\sqrt{(a_{11}^*d_3-a_{33}^*d_1)^2+4a_{31}^*a_{13}^*d_1d_3}}{2d_1d_3},\\
\widehat{\mu} &= \frac{a_{11}^*d_3+a_{33}^*d_1-\sqrt{(a_{11}^*d_3-a_{33}^*d_1)^2+4a_{31}^*a_{13}^*d_1d_3}}{2d_1d_3}
\end{aligned} \tag{1.12}$$

and $E(\mu_i)$ is the eigenspace corresponding to μ_i in $H^1(\Omega)$.

(4) The bifurcation of nonconstant positive solutions for (1.7) was studied.

In recent years, the SKT type cross-diffusion systems have attracted the attention of a great number of investigators and have been successfully developed on the theoretical

backgrounds. The above work mainly concentrate on (1) the instability and stability induced by cross-diffusion, and the existence of nonconstant positive steady-state solutions [7–14]; (2) the global existence of strong solutions [15–23]; (3) the global existence of weak solutions based on semidiscretization or finite element approximation [24–30]; and (4) the dynamical behaviors [18, 19, 31, 32], and so forth. The corresponding SKT type cross-diffusion system for (1.7) is as follows:

$$\begin{aligned}
&u_t = \Delta\left(d_1 u + \alpha_{11}u^2 + \alpha_{12}uv + \alpha_{13}uw\right) + u(1-u) - \frac{auv}{1+bu} - \frac{Auw}{1+Bu+Dw}, \quad x \in \Omega,\ t > 0,\\
&v_t = \Delta\left(d_2 v + \alpha_{21}uv + \alpha_{22}v^2 + \alpha_{23}vw\right) + lv\left(-1 + \frac{eu}{1+bu}\right), \quad x \in \Omega,\ t > 0,\\
&w_t = \Delta\left(d_3 w + \alpha_{31}uw + \alpha_{32}vw + \alpha_{33}w^2\right) + Lw\left(-1 + \frac{Eu}{1+Bu+Dw}\right), \quad x \in \Omega,\ t > 0,\\
&\partial_\nu u = \partial_\nu v = \partial_\nu w = 0, \quad x \in \partial\Omega,\ t > 0,\\
&u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), \qquad w(x,0) = w_0(x), \quad x \in \Omega,
\end{aligned} \tag{1.13}$$

where $\alpha_{ij}(i,j = 1,2,3)$ are positive constants, $\alpha_{ii}(i = 1,2,3)$ are referred as self-diffusion pressures, and $\alpha_{ij}(i,j = 1,2,3,\ i \neq j)$ are cross-diffusion pressures. The self-diffusion implies the movement of individuals from a higher to lower concentration region. Cross-diffusion expresses the population fluxes of one species due to the presence of the other species. The value of cross-diffusion coefficient may be positive, negative, or zero. The positive cross-diffusion coefficient denotes the movement of the species in the direction of lower concentration of another species and negative cross-diffusion coefficient denotes that one species tends to diffuse in the direction of higher concentration of another species (e.g., [33]).

The local existence of solutions for the system (1.13) is an immediate consequence of a series of important papers [34–36] by Amann. Roughly speaking, if $u_0(x), v_0(x)$, and $w_0(x)$ in $W_p^1(\Omega)$ with $p > n$, then (1.13) has a unique nonnegative solution $u, v, w \in C([0,T), W_p^1(\Omega)) \cap C^\infty((0,T), C^\infty(\Omega))$, where $T \in (0,\infty]$ is the maximal existence time for the solution. If the solution (u,v,w) satisfies the estimate

$$\sup\left\{\|u(\cdot,t)\|_{W_p^1(\Omega)}, \|v(\cdot,t)\|_{W_p^1(\Omega)}, \|w(\cdot,t)\|_{W_p^1(\Omega)} : 0 < t < T\right\} < \infty, \tag{1.14}$$

then $T = +\infty$. Moreover, if $u_0(x), v_0(x), w_0(x) \in W_p^2(\Omega)$, then $u, v, w \in C([0,\infty), W_p^2(\Omega))$.

For the following SKT system

$$\begin{aligned}
&u_t = d_1\Delta\left[\left(1 + \alpha v + \gamma u\right)u\right] + au(1 - u - cv), \quad x \in \Omega,\ t > 0,\\
&v_t = d_2\Delta\left[(1 + \delta v)v\right] + bv(1 - du - v), \quad x \in \Omega,\ t > 0,\\
&\partial_\nu u = \partial_\nu v = 0, \quad x \in \partial\Omega,\ t > 0,\\
&u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), \quad x \in \Omega,
\end{aligned} \tag{P}$$

Yamada in [23] proposed four open problems:

(1) the global existence of solutions of (P) in the case $\delta > 0$ and the space dimension $N \geq 6$;

(2) the global existence in the case $\gamma = 0$;

(3) in order to study the asymptotic behavior of u, v as $t \to \infty$ need to establish the uniform boundedness of global solutions; and

(4) the global existence of solutions for the following full SKT system:

$$\begin{aligned}
&u_t = d_1\Delta\left[\left(1+\alpha v+\gamma u\right)u\right] + au(1-u-cv), \quad x\in\Omega,\ t>0,\\
&v_t = d_2\Delta\left[\left(1+\beta u+\delta v\right)v\right] + bv(1-du-v), \quad x\in\Omega,\ t>0,\\
&\partial_\nu u = \partial_\nu v = 0, \quad x\in\partial\Omega,\ t>0,\\
&u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), \quad x\in\Omega
\end{aligned} \tag{1.15}$$

with $\alpha, \gamma, \beta, \delta > 0$.

Very few global existence results for (1.13) are known. The main purpose of this paper is to establish the uniform boundedness of global solutions for the system (1.13) in one space dimension. For convenience, we consider the following system:

$$\begin{aligned}
&u_t = \left(d_1u+\alpha_{11}u^2+\alpha_{12}uv+\alpha_{13}uw\right)_{xx} + u(1-u) - \frac{auv}{1+bu} - \frac{Auw}{1+Bu+Dw}, \quad 0<x<1,\ t>0,\\
&v_t = \left(d_2v+\alpha_{21}uv+\alpha_{22}v^2+\alpha_{23}vw\right)_{xx} + lv\left(-1+\frac{eu}{1+bu}\right), \quad 0<x<1,\ t>0,\\
&w_t = \left(d_3w+\alpha_{31}uw+\alpha_{32}vw+\alpha_{33}w^2\right)_{xx} + Lw\left(-1+\frac{Eu}{1+Bu+Dw}\right), \quad 0<x<1,\ t>0,\\
&u_x(x,t) = v_x(x,t) = w_x(x,t) = 0, \quad x = 0,1,\ t>0,\\
&u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), \qquad w(x,0) = w_0(x), \quad 0<x<1.
\end{aligned} \tag{1.16}$$

We firstly investigate the global existence and the uniform boundedness of the solutions for (1.16), then prove the global asymptotic stability of the positive equilibrium (u^*, v^*, w^*) of (1.16) by an important lemma from [37]. The proof is complete and complement to the uniform convergence theorems in papers [38–40].

It is obvious that (u^*, v^*, w^*) is the unique positive equilibrium of the system (1.16) if (1.8) holds.

For simplicity, we denote $\|\cdot\|_{W_p^k(0,1)}$ by $|\cdot|_{k,p}$ and $\|\cdot\|_{L^p(0,1)}$ by $|\cdot|_p$. Our main results are as follows.

Theorem 1.1. *Let $u_0(x), v_0(x), w_0(x) \in W_2^2(0,1)$, (u,v,w) be the unique nonnegative solution of the system* (1.16) *in the maximal existence interval* $[0,T)$. *Assume that*

$$\begin{aligned} 8\alpha_{11}\alpha_{21}\alpha_{31} &> \alpha_{21}\alpha_{13}^2 + \alpha_{12}^2\alpha_{31}, \\ 8\alpha_{12}\alpha_{22}\alpha_{32} &> \alpha_{32}\alpha_{21}^2 + \alpha_{23}^2\alpha_{12}, \\ 8\alpha_{13}\alpha_{23}\alpha_{33} &> \alpha_{23}\alpha_{31}^2 + \alpha_{32}^2\alpha_{13}. \end{aligned} \tag{1.17}$$

Then there exist $t_0 > 0$ *and positive constants* M', M *which depend on* d_i, α_{ij} $(i,j = 1,2,3)$, a, b, e, l, A, B, D, E *and* L, *such that*

$$\sup\left\{|u(\cdot,t)|_{1,2}, |v(\cdot,t)|_{1,2}, |w(\cdot,t)|_{1,2} : t \in (t_0,T)\right\} \le M', \tag{1.18}$$

$$\max\{u(x,t), v(x,t), w(x,t) : (x,t) \in [0,1] \times (t_0,T)\} \le M, \tag{1.19}$$

and $T = +\infty$. *Moreover, in the case that* $d_1, d_2, d_3 \ge 1$, $d_2/d_1, d_3/d_1 \in [\underline{d}, \overline{d}]$, *where* $\underline{d}$ *and* $\overline{d}$ *are positive constants,* M', M *depend on* $\underline{d}, \overline{d}$, *but do not depend on* d_1, d_2, d_3.

Remark 1.2. Since the continuous embedding $H^1(\Omega) \hookrightarrow L^\infty(\Omega)$ holds only in one space dimension, we can only establish the uniform maximum-norn estimates about time for the solution in one space dimension.

Theorem 1.3. *Assume that all conditions in Theorem 1.1 are satisfied. Assume further that*

$$\begin{aligned} 4\alpha\beta u^* v^* w^* d_1 d_2 d_3 &> u^* M^2 \left(\alpha\alpha_{23} v^* + \beta\alpha_{32} w^*\right)^2 (d_1 + 2\alpha_{11} M + \alpha_{12} M + \alpha_{13} M) \\ &\quad + \alpha v^* M^2 \left(\alpha_{13} u^* + \beta\alpha_{31} w^*\right)^2 (d_2 + \alpha_{21} M + 2\alpha_{22} M + \alpha_{23} M) \\ &\quad + \beta w^* M^2 (\alpha_{12} u^* + \alpha\alpha_{21} v^*)^2 (d_3 + \alpha_{31} M + \alpha_{32} M + 2\alpha_{33} M), \end{aligned} \tag{1.20}$$

$$DE[(e-b) - b(e-b-1)] > A(e-b)[(E-B) - (e-b)](B-b), \tag{1.21}$$

and (1.8) *hold, where* $\alpha = a(1+bu^*)/el$, $\beta = A(1+Bu^*)/EL(1+Dw^*)$, M *is given by* (1.19). *Then the unique positive equilibrium* (u^*, v^*, w^*) *of* (1.16) *is globally asymptotically stable.*

Remark 1.4. The system (1.16) has no nonconstant positive steady-state solution if all conditions of Theorem 1.3 hold.

Examples. The following two examples satisfy all conditions of Theorem 1.3:

$$\begin{aligned}
&\alpha_{11} = \frac{1}{4}, \quad \alpha_{12} = 1, \quad \alpha_{13} = 1,\\
&\alpha_{21} = \frac{5}{2}, \quad \alpha_{22} = 1, \quad \alpha_{23} = \frac{1}{4},\\
&\alpha_{31} = 1, \quad \alpha_{32} = 1, \quad \alpha_{33} = 2,\\
&b = 1, \quad e = 3, \quad A = 1,\\
&B = \frac{1}{2}, \quad D = \frac{2}{3}, \quad E = 3,\\
&d_1, d_2, d_3 \gg 1;\\
&\alpha_{11} = 2, \quad \alpha_{12} = 1, \quad \alpha_{13} = 2,\\
&\alpha_{21} = \frac{3}{2}, \quad \alpha_{22} = \frac{1}{2}, \quad \alpha_{23} = 1,\\
&\alpha_{31} = 2, \quad \alpha_{32} = 2, \quad \alpha_{33} = 3,\\
&b = \frac{1}{2}, \quad e = \frac{5}{2}, \quad A = \frac{2}{3},\\
&B = \frac{1}{2}, \quad D = \frac{3}{4}, \quad E = 4,\\
&d_1, d_2, d_3 \gg 1.
\end{aligned} \tag{1.22}$$

2. Global Solutions

In order to establish the uniform W_2^1-estimates of the solutions for the system (1.16), the following Gagliardo-Nirenberg-type inequalities and the corresponding corollary play important roles (see [38, 41]).

Theorem 2.1. *Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with $\partial\Omega \in C^m$. For every function $u \in W_r^m(\Omega)$, $1 \le q, r \le \infty$, the derivative $D^j u$ $(0 \le j < m)$ satisfies the inequality*

$$\left|D^j u\right|_p \le C\left(|D^m u|_r^a |u|_q^{1-a} + |u|_q\right), \tag{2.1}$$

provided one of the following three conditions is satisfied: (1) $r \le q$, (2) $0 < n(r-q)/(mrq) < 1$, *or* (3) $n(r-q)/(mrq) = 1$ *and $m - n/q$ is not a nonnegative integer, where $1/p = j/n + a(1/r - m/n) + (1-a)/q$ for all $a \in [j/m, 1)$, and the positive constant C depends on n, m, j, q, r, a.*

Corollary 2.2. *There exists a universal constant C such that*

$$|u|_2 \le C\left(|u_x|_2^{1/3}|u|_1^{2/3} + |u|_1\right), \quad \forall u \in W_2^1(0,1), \tag{2.2}$$

$$|u|_4 \le C\left(|u_x|_2^{1/2}|u|_1^{1/2} + |u|_1\right), \quad \forall u \in W_2^1(0,1), \tag{2.3}$$

$$|u|_{5/2} \le C\left(|u_x|_2^{2/5}|u|_1^{3/5} + |u|_1\right), \quad \forall u \in W_2^1(0,1), \tag{2.4}$$

$$|u_x|_2 \le C\left(|u_{xx}|_2^{3/5}|u|_1^{2/5} + |u|_1\right), \quad \forall u \in W_2^2(0,1). \tag{2.5}$$

Throughout this paper, we always denote that C is a Sobolev embedding constant or other kind of universal constant, A_j, B_j, C_j are some positive constants which depend only on α_{ij} $(i,j = 1,2,3)$, a,b,e,l,A,B,D,E and L, K_j are positive constants depending on d_i, α_{ij} $(i,j = 1,2,3)$, a,b,e,l,A,B,D,E and L. When $d_1,d_2,d_3 \ge 1$, d_2/d_1, $d_3/d_1 \in [\underline{d}, \overline{d}]$, K_j depend on $\underline{d}, \overline{d}$, but do not depend on d_1,d_2,d_3.

Proof of Theorem 1.1. Taking integration of the three equations in (1.16) over $(0,1)$, respectively, and combining the three integration equalities linearly, we have

$$\frac{d}{dt}\int_0^1 [(el+EL)u + av + Aw]dx \le \int_0^1 [(el+EL)u(1-u) - alv - ALw]dx. \tag{2.6}$$

It follows from the Young inequality and the Hölder inequality that

$$\frac{d}{dt}\int_0^1 [(el+EL)u + av + Aw]dx \le C_1 - k\int_0^1 [(el+EL)u + av + Aw]dx, \tag{2.7}$$

where $k = \min\{l, L\}$, $C_1 = (k+1)^2(el+EL)/4$. So there exist positive constants M_0 and τ_0 depending on a,b,e,l,A,B,D,E, and L, such that

$$\int_0^1 u\,dx, \int_0^1 v\,dx, \int_0^1 w\,dx \le M_0, \quad t \ge \tau_0. \tag{2.8}$$

Moreover, there exists a positive constant M_0' which depends on a,b,e,l,A,B,D,E,L and L^1-norm of u_0, v_0, w_0, such that

$$\int_0^1 u\,dx, \int_0^1 v\,dx, \int_0^1 w\,dx \le M_0', \quad t \ge 0. \tag{2.8}'$$

Multiplying the first three equations in the system (1.16) by u, v, w, respectively, and integrating over $(0,1)$, we have

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\int_0^1 u^2dx &\le -d_1\int_0^1 u_x^2dx - \int_0^1\Big[(2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w)u_x^2+(\alpha_{12}v_x+\alpha_{13}w_x)uu_x\Big]dx \\
&\quad + \int_0^1 u^2dx, \\
\frac{1}{2}\frac{d}{dt}\int_0^1 v^2dx &\le -d_2\int_0^1 v_x^2dx - \int_0^1\Big[(\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)v_x^2+\alpha_{21}(u_x+\alpha_{23}w_x)vv_x\Big]dx \\
&\quad + \frac{el}{b}\int_0^1 v^2dx, \\
\frac{1}{2}\frac{d}{dt}\int_0^1 w^2dx &\le -d_3\int_0^1 w_x^2dx - \int_0^1\Big[(\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w)w_x^2+(\alpha_{31}u_x+\alpha_{32}v_x)ww_x\Big]dx \\
&\quad + \frac{EL}{B}\int_0^1 w^2dx,
\end{aligned} \tag{2.9}$$

from which it follows that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_0^1\left(u^2+v^2+w^2\right)dx \\
&\quad\le -d\int_0^1\left(u_x^2+v_x^2+w_x^2\right)dx+\int_0^1 u^2dx+\frac{el}{b}\int_0^1 v^2dx+\frac{EL}{B}\int_0^1 w^2dx-\int_0^1 q(u_x,v_x,w_x)dx \\
&\quad\le -d\int_0^1\left(u_x^2+v_x^2+w_x^2\right)dx+\left(1+\frac{el}{b}+\frac{EL}{B}\right)\int_0^1\left(u^2+v^2+w^2\right)dx-\int_0^1 q(u_x,v_x,w_x)dx,
\end{aligned} \tag{2.10}$$

where $d=\min\{d_1,d_2,d_3\}$,

$$\begin{aligned}
q(u_x,v_x,w_x) &= (2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w)u_x^2+(\alpha_{12}u+\alpha_{21}v)u_xv_x+(\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)v_x^2 \\
&\quad +(\alpha_{13}u+\alpha_{31}w)u_xw_x+(\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w)w_x^2+(\alpha_{23}v+\alpha_{32}w)v_xw_x.
\end{aligned} \tag{2.11}$$

It is obvious that $q(u_x,v_x,w_x)$ is a positive definite quadratic form of u_x,v_x,w_x if (1.17) holds. So (1.17) implies that

$$\frac{1}{2}\frac{d}{dt}\int_0^1\left(u^2+v^2+w^2\right)dx\le -d\int_0^1\left(u_x^2+v_x^2+w_x^2\right)dx+\left(1+\frac{el}{b}+\frac{EL}{B}\right)\int_0^1\left(u^2+v^2+w^2\right)dx. \tag{2.12}$$

Now, we proceed in the following two cases.

(i) One has $t \geq \tau_0$. The inequality (2.2) implies that $|u|_2^6 \leq C(|u_x|_2^2|u|_1^4 + |u|_1^6) \leq CM_0^4(|u_x|_2^2 + M_0^2)$. So we have $\int_0^1 u_x^2 dx \geq (1/CM_0^4)(\int_0^1 u^2 dx)^3 - M_0^2$, and

$$-\int_0^1 \left(u_x^2 + v_x^2 + w_x^2\right) dx \leq -\frac{1}{9CM_0^4}\left[\int_0^1 \left(u^2 + v^2 + w^2\right) dx\right]^3 + 3M_0^2. \tag{2.13}$$

It follows from (2.12) and (2.13) that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_0^1 \left(u^2 + v^2 + w^2\right) dx \\ &\quad \leq d\left\{-C_2\left[\int_0^1 \left(u^2 + v^2 + w^2\right) dx\right]^3 + 3M_0^2 + \frac{1}{d}\left(1 + \frac{el}{b} + \frac{EL}{B}\right)\int_0^1 \left(u^2 + v^2 + w^2\right) dx\right\}.\end{aligned} \tag{2.14}$$

This means that there exist positive constants τ_1 and M_1 depending on $d_i (i = 1,2,3)$, a, b, e, l, A, B, D, E, and L, such that

$$\int_0^1 u^2 dx, \int_0^1 v^2 dx, \int_0^1 w^2 dx \leq M_1, \quad t \geq \tau_1. \tag{2.15}$$

When $d \geq 1$, M_1 is independent of d because the zero point of the right-hand side in (2.14) can be estimated by positive constants independent of d.

(ii) One has $t \geq 0$. Repeating estimates in (i) by $(2.8)'$, we can obtain that there exists a positive constant M_1' depending on d_i $(i = 1,2,3)$, $a, b, e, l, A, B, D, E, L$ and the L^1, L^2-norm of u_0, v_0, w_0, such that

$$\int_0^1 u^2 dx, \int_0^1 v^2 dx, \int_0^1 w^2 dx \leq M_1', \quad t \geq 0. \tag{2.15}'$$

When $d \geq 1, M_1'$ is independent of d.

To estimate $|u_x|_2, |v_x|_2, |w_x|_2$, we introduce the following scaling:

$$\widehat{u} = \frac{u}{d_1}, \qquad \widehat{v} = \frac{v}{d_1}, \qquad \widehat{w} = \frac{w}{d_1}, \qquad \widehat{t} = d_1 t. \tag{2.16}$$

Denoting $\xi = d_2/d_1, \eta = d_3/d_1$, and using u, v, w, t instead of $\widehat{u}, \widehat{v}, \widehat{w}, \widehat{t}$, respectively, then the system (1.16) reduces to

$$\begin{aligned}
&u_t = P_{xx} + f(u,v,w), \quad 0 < x < 1,\ t > 0,\\
&v_t = Q_{xx} + g(u,v,w), \quad 0 < x < 1,\ t > 0,\\
&w_t = R_{xx} + h(u,v,w), \quad 0 < x < 1,\ t > 0,\\
&u_x(x,t) = v_x(x,t) = w_x(x,t) = 0, \quad x = 0,1,\ t > 0,\\
&u(x,0) = \widehat{u}_0(x), \qquad v(x,0) = \widehat{v}_0(x), \qquad w(x,0) = \widehat{w}_0(x), \quad 0 < x < 1,
\end{aligned} \tag{2.17}$$

where $P = u+\alpha_{11}u^2+\alpha_{12}uv+\alpha_{13}uw,\ Q = \xi v+\alpha_{21}uv+\alpha_{22}v^2+\alpha_{23}vw,\ R = \eta w+\alpha_{31}uw+\alpha_{32}vw+\alpha_{33}w^2, f(u,v,w) = d_1^{-1}u - u^2 - (auv/(1+bd_1u)) - (Auw/(1+Bd_1u+Dd_1w)),\ g(u,v,w) = lv(-d_1^{-1} + (eu/(1+bd_1u))), h(u,v,w) = Lw(-d_1^{-1} + (Eu/(1+Bd_1u+Dd_1w)))$.

We still proceed in the following two cases.

(i) One has $t \geq \tau_1^* = d_1\tau_1$. It is not hard to verify that

$$\begin{aligned}
&\int_0^1 u\,dx, \int_0^1 v\,dx, \int_0^1 w\,dx \leq M_0 d_1^{-1},\\
&\int_0^1 u^2 dx, \int_0^1 v^2 dx, \int_0^1 w^2 dx \leq M_1 d_1^{-2},\\
&|P|_1, |Q|_1, |R|_1 \leq A_1 K_1 d_1^{-1},
\end{aligned} \tag{2.18}$$

where $K_1 = (1+\xi+\eta) + M_1 d_1^{-1},\ A_1 = \max\{M_0, \alpha_{11}+\alpha_{12}+\alpha_{13}, \alpha_{21}+\alpha_{22}+\alpha_{23}, \alpha_{31}+\alpha_{32}+\alpha_{33}\}$.

Multiplying the first three equations in (2.17) by P_t, Q_t, R_t, integrating them over the domain $(0,1)$, respectively, and then adding up the three integration equalities, we have

$$\begin{aligned}
\frac{1}{2}\overline{y}'(t) = &-\int_0^1 u_t^2 dx - \xi\int_0^1 v_t^2 dx - \eta\int_0^1 w_t^2 dx - \int_0^1 q(u_t, v_t, w_t)dx\\
&+\int_0^1 \left[(1+2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w)u_t f + \alpha_{12}uv_t f + \alpha_{13}uw_t f\right]dx\\
&+\int_0^1 \left[(\xi+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)v_t g + \alpha_{21}vu_t g + \alpha_{23}vw_t g\right]dx\\
&+\int_0^1 \left[(\eta+\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w)w_t h + \alpha_{31}wu_t h + \alpha_{32}wv_t h\right]dx,
\end{aligned} \tag{2.19}$$

where $\overline{y}(t) = \int_0^1 (P_x^2 + Q_x^2 + R_x^2)dx$. Notice by (1.17) that there exists a positive constant C_3 depending only on α_{ij} $(i,j = 1,2,3)$, such that

$$q(u_t, v_t, w_t) \geq C_3(u+v+w)\left(u_t^2 + v_t^2 + w_t^2\right). \tag{2.20}$$

Thus,

$$\begin{aligned}\frac{1}{2}\overline{y}'(t) \le & -\int_0^1 u_t^2 dx - \xi\int_0^1 v_t^2 dx - \eta\int_0^1 w_t^2 dx - C_3\int_0^1 (u+v+w)\left(u_t^2+v_t^2+w_t^2\right)dx \\ & + \int_0^1 \left[(1+2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w)u_t f + \alpha_{12}uv_t f + \alpha_{13}uw_t f\right]dx \\ & + \int_0^1 \left[(\xi+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)v_t g + \alpha_{21}vu_t g + \alpha_{23}vw_t g\right]dx \\ & + \int_0^1 \left[(\eta+\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w)w_t h + \alpha_{31}wu_t h + \alpha_{32}wv_t h\right]dx.\end{aligned} \tag{2.21}$$

Using the Young inequality, Hölder inequality, and (2.18), we can obtain the following estimates:

$$\begin{aligned}&\int_0^1 u^4 dx \le \left(\int_0^1 u^2 dx\right)^{1/3}\left(\int_0^1 u^5 dx\right)^{2/3} \le M_1^{1/3} d_1^{-2/3}\left(\int_0^1 u^5 dx\right)^{2/3}, \\ &\int_0^1 uv^2 dx \le \left(\int_0^1 u^2 dx\right)^{1/2}\left(\int_0^1 v^2 dx\right)^{1/6}\left(\int_0^1 v^5 dx\right)^{1/3} \le M_1^{2/3} d_1^{-4/3}\left(\int_0^1 v^5 dx\right)^{1/3}, \\ &\int_0^1 u^3 dx \le \left(\int_0^1 u^2 dx\right)^{2/3}\left(\int_0^1 u^5 dx\right)^{1/3} \le M_1^{2/3} d_1^{-4/3}\left(\int_0^1 u^5 dx\right)^{1/3}, \\ &\int_0^1 u^4 v\, dx \le \frac{4}{5}\int_0^1 u^5 dx + \frac{1}{5}\int_0^1 v^5 dx \le \frac{4}{5}\int_0^1 \left(u^5+v^5\right)dx.\end{aligned} \tag{2.22}$$

Applying the above estimates and Gagliardo-Nirenberg-type inequalities to the terms on the right-hand side of (2.21), we have

$$\begin{aligned}&-\int_0^1 u_t^2 dx \le -\frac{1}{2}\int_0^1 P_{xx}^2 dx + \int_0^1 f^2 dx, \\ &-\xi\int_0^1 v_t^2 dx \le -\frac{\xi}{2}\int_0^1 Q_{xx}^2 dx + \xi\int_0^1 g^2 dx, \\ &-\eta\int_0^1 w_t^2 dx \le -\frac{\eta}{2}\int_0^1 R_{xx}^2 dx + \eta\int_0^1 h^2 dx,\end{aligned}$$

$$\begin{aligned}
\int_0^1 f^2dx &\le \int_0^1\left(d_1^{-2}u^2+u^4+\frac{a^2v^2}{(bd_1)^2}+\frac{A^2w^2}{(Bd_1)^2}+2\frac{au^2v}{bd_1}+2\frac{Au^2w}{Bd_1}+2\frac{aAvw}{bBd_1^2}\right)dx\\
&\le \left[1+2\left(\frac{a}{b}\right)^2+2\left(\frac{A}{B}\right)^2\right]M_1d_1^{-4}+M_1^{1/3}d_1^{-2/3}\left(\int_0^1 u^5dx\right)^{2/3}\\
&\quad+2\left(\frac{a}{b}+\frac{A}{B}\right)M_1^{2/3}d_1^{-7/3}\left(\int_0^1 u^5dx\right)^{1/3},\\
\xi\int_0^1 g^2dx &\le \xi\int_0^1\left(l^2d_1^{-2}v^2+\frac{e^2l^2v^2}{(bd_1)^2}\right)dx\le \xi l^2\left[1+\left(\frac{e}{b}\right)^2\right]M_1d_1^{-4},\\
\eta\int_0^1 h^2dx &\le \eta\int_0^1\left(L^2d_1^{-2}w^2+\frac{E^2L^2w^2}{(Bd_1)^2}\right)dx\le \eta L^2\left[1+\left(\frac{E}{B}\right)^2\right]M_1d_1^{-4},
\end{aligned}\tag{2.23}$$

$$\begin{aligned}
&-\int_0^1 u_t^2dx-\xi\int_0^1 v_t^2dx-\eta\int_0^1 w_t^2dx\\
&\quad\le -\frac{1}{2}\int_0^1 P_{xx}^2dx-\frac{\xi}{2}\int_0^1 Q_{xx}^2dx-\frac{\eta}{2}\int_0^1 R_{xx}^2dx\\
&\qquad+\left\{1+2\left[\left(\frac{a}{b}\right)^2+\left(\frac{A}{B}\right)^2\right]+\xi l^2\left[1+\left(\frac{e}{b}\right)^2\right]+\eta L^2\left[1+\left(\frac{E}{B}\right)^2\right]\right\}M_1d_1^{-4}\\
&\qquad+M_1^{1/3}d_1^{-2/3}\left(\int_0^1 u^5dx\right)^{2/3}+2\left(\frac{a}{b}+\frac{A}{B}\right)M_1^{2/3}d_1^{-7/3}\left(\int_0^1 u^5dx\right)^{1/3}.
\end{aligned}\tag{2.24}$$

Similarly, we can obtain

$$\begin{aligned}
\int_0^1 u_tf\,dx &\le \int_0^1 u_t\left(d_1^{-1}u+u^2+\frac{a}{bd_1}v+\frac{A}{Bd_1}w\right)dx\\
&\le \frac{d_1^{-2}}{2\varepsilon}\int_0^1 u\,dx+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx+\frac{1}{2\varepsilon}\int_0^1 u^3dx+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx\\
&\quad+\frac{(a/bd_1)^2}{2\varepsilon}\int_0^1 v\,dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx+\frac{(A/Bd_1)^2}{2\varepsilon}\int_0^1 w\,dx+\frac{\varepsilon}{2}\int_0^1 wu_t^2dx\\
&\le \frac{1}{2\varepsilon}\left[1+\left(\frac{a}{b}\right)^2+\left(\frac{A}{B}\right)^2\right]M_0d_1^{-3}+\frac{1}{2\varepsilon}M_1^{2/3}d_1^{-4/3}\left(\int_0^1 u^5dx\right)^{1/3}\\
&\quad+\varepsilon\int_0^1 uu_t^2dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx+\frac{\varepsilon}{2}\int_0^1 wu_t^2dx,
\end{aligned}$$

$$
\begin{aligned}
2\alpha_{11}\int_0^1 uu_t f\,dx &\le 2\alpha_{11}\int_0^1 uu_t\left(d_1^{-1}u+u^2+\frac{a}{bd_1}v+\frac{A}{Bd_1}w\right)dx\\
&\le \frac{\alpha_{11}^2 d_1^{-2}}{\varepsilon}\int_0^1 u^3dx+\varepsilon\int_0^1 uu_t^2dx+\frac{\alpha_{11}^2}{\varepsilon}\int_0^1 u^5dx+\varepsilon\int_0^1 uu_t^2dx\\
&\quad+\frac{\alpha_{11}^2a^2}{\varepsilon b^2d_1^2}\int_0^1 u^2v\,dx+\varepsilon\int_0^1 vu_t^2dx+\frac{\alpha_{11}^2A^2}{\varepsilon B^2d_1^2}\int_0^1 u^2w\,dx+\varepsilon\int_0^1 wu_t^2dx\\
&\le \frac{\alpha_{11}^2}{\varepsilon}\left[1+\left(\frac{a}{b}\right)^2+\left(\frac{A}{B}\right)^2\right]M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\alpha_{11}^2}{\varepsilon}\int_0^1 u^5dx\\
&\quad+2\varepsilon\int_0^1 uu_t^2dx+\varepsilon\int_0^1 vu_t^2dx+\varepsilon\int_0^1 wu_t^2dx,
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{12}\int_0^1 vu_t f\,dx &\le \alpha_{12}\int_0^1 vu_t\left(d_1^{-1}u+u^2+\frac{a}{bd_1}v+\frac{A}{Bd_1}w\right)dx\\
&\le \frac{\alpha_{12}^2 d_1^{-2}}{2\varepsilon}\int_0^1 uv^2dx+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx+\frac{\alpha_{12}^2}{2\varepsilon}\int_0^1 u^4v\,dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx\\
&\quad+\frac{\alpha_{12}^2a^2}{2\varepsilon b^2d_1^2}\int_0^1 v^3dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx+\frac{\alpha_{12}^2A^2}{2\varepsilon B^2d_1^2}\int_0^1 v^2w\,dx+\frac{\varepsilon}{2}\int_0^1 wu_t^2dx\\
&\le \frac{\alpha_{12}^2}{2\varepsilon}\left[1+\left(\frac{a}{b}\right)^2+\left(\frac{A}{B}\right)^2\right]M_1^{2/3}d_1^{-10/3}\left(\int_0^1 v^5dx\right)^{1/3}+\frac{2\alpha_{12}^2}{5\varepsilon}\int_0^1\left(u^5+v^5\right)dx\\
&\quad+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx+\varepsilon\int_0^1 vu_t^2dx+\frac{\varepsilon}{2}\int_0^1 wu_t^2dx,
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{13}\int_0^1 wu_t f\,dx &\le \alpha_{13}\int_0^1 wu_t\left(d_1^{-1}u+u^2+\frac{a}{bd_1}v+\frac{A}{Bd_1}w\right)dx\\
&\le \frac{\alpha_{13}^2 d_1^{-2}}{2\varepsilon}\int_0^1 uw^2dx+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx+\frac{\alpha_{13}^2}{2\varepsilon}\int_0^1 u^4w\,dx+\frac{\varepsilon}{2}\int_0^1 wu_t^2dx\\
&\quad+\frac{\alpha_{13}^2a^2}{2\varepsilon b^2d_1^2}\int_0^1 w^2v\,dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx+\frac{\alpha_{13}^2A^2}{2\varepsilon B^2d_1^2}\int_0^1 w^3dx+\frac{\varepsilon}{2}\int_0^1 wu_t^2dx\\
&\le \frac{\alpha_{13}^2}{2\varepsilon}\left[1+\left(\frac{a}{b}\right)^2+\left(\frac{A}{B}\right)^2\right]M_1^{2/3}d_1^{-10/3}\left(\int_0^1 w^5dx\right)^{1/3}+\frac{2\alpha_{13}^2}{5\varepsilon}\int_0^1\left(u^5+w^5\right)dx\\
&\quad+\frac{\varepsilon}{2}\int_0^1 uu_t^2dx+\varepsilon\int_0^1 wu_t^2dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2dx,
\end{aligned}
$$

$$\begin{aligned}
\alpha_{12}\int_0^1 uv_t f\,dx &\le \alpha_{12}\int_0^1 uv_t\left(d_1^{-1}u+u^2+\frac{a}{bd_1}v+\frac{A}{Bd_1}w\right)dx\\
&\le \frac{\alpha_{12}^2 d_1^{-2}}{2\varepsilon}\int_0^1 u^3dx+\frac{\varepsilon}{2}\int_0^1 uv_t^2dx+\frac{\alpha_{12}^2}{2\varepsilon}\int_0^1 u^5dx+\frac{\varepsilon}{2}\int_0^1 uv_t^2dx\\
&\quad+\frac{\alpha_{12}^2a^2}{2\varepsilon b^2d_1^2}\int_0^1 u^2v\,dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx+\frac{\alpha_{12}^2A^2}{2\varepsilon B^2d_1^2}\int_0^1 u^2w\,dx+\frac{\varepsilon}{2}\int_0^1 wv_t^2dx\\
&\le \frac{\alpha_{12}^2}{2\varepsilon}\left[1+\left(\frac{a}{b}\right)^2+\left(\frac{A}{B}\right)^2\right]M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\alpha_{12}^2}{2\varepsilon}\int_0^1 u^5dx\\
&\quad+\varepsilon\int_0^1 uv_t^2dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx+\frac{\varepsilon}{2}\int_0^1 wv_t^2dx,
\end{aligned}$$

$$\begin{aligned}
\alpha_{13}\int_0^1 uw_t f\,dx &\le \alpha_{13}\int_0^1 uw_t\left(d_1^{-1}u+u^2+\frac{a}{bd_1}v+\frac{A}{Bd_1}w\right)dx\\
&\le \frac{\alpha_{13}^2 d_1^{-2}}{2\varepsilon}\int_0^1 u^3dx+\frac{\varepsilon}{2}\int_0^1 uw_t^2dx+\frac{\alpha_{13}^2}{2\varepsilon}\int_0^1 u^5dx+\frac{\varepsilon}{2}\int_0^1 uw_t^2dx\\
&\quad+\frac{\alpha_{13}^2a^2}{2\varepsilon b^2d_1^2}\int_0^1 u^2v\,dx+\frac{\varepsilon}{2}\int_0^1 vw_t^2dx+\frac{\alpha_{13}^2A^2}{2\varepsilon B^2d_1^2}\int_0^1 u^2w\,dx+\frac{\varepsilon}{2}\int_0^1 ww_t^2dx,\\
&\le \frac{\alpha_{13}^2}{2\varepsilon}\left[1+\left(\frac{a}{b}\right)^2+\left(\frac{A}{B}\right)^2\right]M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\alpha_{13}^2}{2\varepsilon}\int_0^1 u^5dx\\
&\quad+\varepsilon\int_0^1 uw_t^2dx+\frac{\varepsilon}{2}\int_0^1 vw_t^2dx+\frac{\varepsilon}{2}\int_0^1 ww_t^2dx,
\end{aligned}$$

$$\begin{aligned}
\xi\int_0^1 v_t g\,dx &\le \xi\int_0^1 v_t lv\left(d_1^{-1}+\frac{e}{bd_1}\right)dx\le\frac{\xi^2l^2d_1^{-2}(1+(e/b))^2}{2\varepsilon}\int_0^1 v\,dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx\\
&\le \frac{\xi^2l^2(1+(e/b))^2}{2\varepsilon}M_0d_1^{-3}+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx,
\end{aligned}$$

$$\begin{aligned}
\alpha_{21}\int_0^1 uv_t g\,dx &\le \alpha_{21}\int_0^1 uv_t lv\left(d_1^{-1}+\frac{e}{bd_1}\right)dx\\
&\le \frac{\alpha_{21}^2l^2d_1^{-2}(1+(e/b))^2}{2\varepsilon}\int_0^1 u^2v\,dx+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx\\
&\le \frac{\alpha_{21}^2l^2(1+(e/b))^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 u^5dx\right)^{1/3}+\frac{\varepsilon}{2}\int_0^1 vv_t^2dx,
\end{aligned}$$

$$2\alpha_{22}\int_0^1 vv_t g\,dx \le 2\alpha_{22}\int_0^1 v^2 v_t l\left(d_1^{-1}+\frac{e}{bd_1}\right)dx$$
$$\le \frac{\alpha_{22}^2 l^2 d_1^{-2}(1+(e/b))^2}{\varepsilon}\int_0^1 v^3\,dx+\varepsilon\int_0^1 vv_t^2\,dx$$
$$\le \frac{\alpha_{21}^2 l^2(1+(e/b))^2}{\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 v^5\,dx\right)^{1/3}+\varepsilon\int_0^1 vv_t^2\,dx,$$

$$\alpha_{23}\int_0^1 wv_t g\,dx \le \alpha_{23}\int_0^1 wv_t lv\left(d_1^{-1}+\frac{e}{bd_1}\right)dx$$
$$\le \frac{\alpha_{23}^2 l^2 d_1^{-2}(1+(e/b))^2}{2\varepsilon}\int_0^1 v^2 w\,dx+\frac{\varepsilon}{2}\int_0^1 wv_t^2\,dx$$
$$\le \frac{\alpha_{23}^2 l^2(1+(e/b))^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 v^5\,dx\right)^{1/3}+\frac{\varepsilon}{2}\int_0^1 wv_t^2\,dx,$$

$$\alpha_{21}\int_0^1 vu_t g\,dx \le \alpha_{21}\int_0^1 v^2 v_t l\left(d_1^{-1}+\frac{e}{bd_1}\right)dx$$
$$\le \frac{\alpha_{21}^2 l^2 d_1^{-2}(1+(e/b))^2}{2\varepsilon}\int_0^1 v^3\,dx+\frac{\varepsilon}{2}\int_0^1 vu_t^2\,dx$$
$$\le \frac{\alpha_{21}^2 l^2(1+(e/b))^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 v^5\,dx\right)^{1/3}+\frac{\varepsilon}{2}\int_0^1 vu_t^2\,dx,$$

$$\alpha_{23}\int_0^1 vw_t g\,dx \le \alpha_{23}\int_0^1 v^2 w_t l\left(d_1^{-1}+\frac{e}{bd_1}\right)dx$$
$$\le \frac{\alpha_{23}^2 l^2 d_1^{-2}(1+(e/b))^2}{2\varepsilon}\int_0^1 v^3\,dx+\frac{\varepsilon}{2}\int_0^1 vw_t^2\,dx$$
$$\le \frac{\alpha_{23}^2 l^2(1+(e/b))^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 v^5\,dx\right)^{1/3}+\frac{\varepsilon}{2}\int_0^1 vw_t^2\,dx,$$

$$\eta\int_0^1 w_t h\,dx \le \eta\int_0^1 w_t Lw\left(d_1^{-1}+\frac{E}{Bd_1}\right)dx$$
$$\le \frac{\eta^2 L^2 d_1^{-2}(1+(E/B))^2}{2\varepsilon}\int_0^1 w\,dx+\frac{\varepsilon}{2}\int_0^1 ww_t^2\,dx$$

$$\leq \frac{\eta^2 L^2 (1+(E/B))^2}{2\varepsilon} M_0 d_1^{-3} + \frac{\varepsilon}{2}\int_0^1 w w_t^2 dx,$$

$$\begin{aligned}
\alpha_{31}\int_0^1 u w_t h\,dx &\leq \alpha_{31}\int_0^1 u w_t L w\left(d_1^{-1}+\frac{E}{Bd_1}\right)dx \\
&\leq \frac{\alpha_{31}^2 L^2 d_1^{-2}(1+(E/B))^2}{2\varepsilon}\int_0^1 w^2 u\,dx + \frac{\varepsilon}{2}\int_0^1 u w_t^2 dx \\
&\leq \frac{\alpha_{31}^2 L^2 (1+(E/B))^2}{2\varepsilon} M_1^{2/3} d_1^{-10/3}\left(\int_0^1 w^5 dx\right)^{1/3} + \frac{\varepsilon}{2}\int_0^1 u w_t^2 dx,
\end{aligned}$$

$$\begin{aligned}
\alpha_{32}\int_0^1 v w_t h\,dx &\leq \alpha_{32}\int_0^1 v w_t L w\left(d_1^{-1}+\frac{E}{Bd_1}\right)dx \\
&\leq \frac{\alpha_{32}^2 L^2 d_1^{-2}(1+(E/B))^2}{2\varepsilon}\int_0^1 w^2 v\,dx + \frac{\varepsilon}{2}\int_0^1 v w_t^2 dx \\
&\leq \frac{\alpha_{32}^2 L^2 (1+(E/B))^2}{2\varepsilon} M_1^{2/3} d_1^{-10/3}\left(\int_0^1 w^5 dx\right)^{1/3} + \frac{\varepsilon}{2}\int_0^1 v w_t^2 dx,
\end{aligned}$$

$$\begin{aligned}
2\alpha_{33}\int_0^1 w w_t h\,dx &\leq 2\alpha_{33}\int_0^1 w^2 w_t L\left(d_1^{-1}+\frac{E}{Bd_1}\right)dx \\
&\leq \frac{\alpha_{33}^2 L^2 d_1^{-2}(1+(E/B))^2}{\varepsilon}\int_0^1 w^3 dx + \varepsilon\int_0^1 w w_t^2 dx \\
&\leq \frac{\alpha_{33}^2 L^2 (1+(E/B))^2}{\varepsilon} M_1^{2/3} d_1^{-10/3}\left(\int_0^1 w^5 dx\right)^{1/3} + \varepsilon\int_0^1 w w_t^2 dx,
\end{aligned}$$

$$\begin{aligned}
\alpha_{31}\int_0^1 w u_t h\,dx &\leq \alpha_{31}\int_0^1 w^2 u_t L\left(d_1^{-1}+\frac{E}{Bd_1}\right)dx \\
&\leq \frac{\alpha_{31}^2 L^2 d_1^{-2}(1+(E/B))^2}{2\varepsilon}\int_0^1 w^3 dx + \frac{\varepsilon}{2}\int_0^1 w u_t^2 dx \\
&\leq \frac{\alpha_{31}^2 L^2 (1+(E/B))^2}{2\varepsilon} M_1^{2/3} d_1^{-10/3}\left(\int_0^1 v^5 dx\right)^{1/3} + \frac{\varepsilon}{2}\int_0^1 w u_t^2 dx,
\end{aligned}$$

$$\begin{aligned}\alpha_{32}\int_0^1 wv_t h\,dx &\le \alpha_{32}\int_0^1 w^2 v_t L\left(d_1^{-1}+\frac{E}{Bd_1}\right)dx\\ &\le \frac{\alpha_{32}^2L^2d_1^{-2}(1+(E/B))^2}{2\varepsilon}\int_0^1 w^3dx+\frac{\varepsilon}{2}\int_0^1 wv_t^2dx\\ &\le \frac{\alpha_{32}^2L^2(1+(E/B))^2}{2\varepsilon}M_1^{2/3}d_1^{-10/3}\left(\int_0^1 w^5dx\right)^{1/3}+\frac{\varepsilon}{2}\int_0^1 wv_t^2dx.\end{aligned} \tag{2.25}$$

By the above inequalities and the condition (1.17), we have

$$\begin{aligned}&\int_0^1\left[(1+2\alpha_{11}u+\alpha_{12}v+\alpha_{13}w)u_tf+\alpha_{12}uv_tf+\alpha_{13}uw_tf\right]dx\\ &\quad+\int_0^1\left[(\xi+\alpha_{21}u+2\alpha_{22}v+\alpha_{23}w)v_tg+\alpha_{21}vu_tg+\alpha_{23}vw_tg\right]dx\\ &\quad+\int_0^1\left[(\eta+\alpha_{31}u+\alpha_{32}v+2\alpha_{33}w)w_th+\alpha_{31}wu_th+\alpha_{32}wv_th\right]dx\\ &\le\overline{\lambda}\varepsilon\int_0^1(u+v+w)\left(u_t^2+v_t^2+w_t^2\right)dx+\frac{C_4}{\varepsilon}\left(1+\xi^2+\eta^2\right)M_0d_1^{-3}\\ &\quad+\frac{C_5}{\varepsilon}\left(1+d_1^{-2}\right)M_1^{2/3}d_1^{-4/3}\left[\int_0^1\left(u^5+v^5+w^5\right)dx\right]^{1/3}+\frac{C_6}{\varepsilon}\int_0^1\left(u^5+v^5+w^5\right)dx,\end{aligned} \tag{2.26}$$

where $\overline{\lambda}$ is a constant depending only on $\varepsilon(\alpha_{ij})$ $(i,j=1,2,3)$. Choose a small enough positive number ε which depends on α_{ij} $(i,j=1,2,3)$, a,b,e,l,A,B,D,E, and L, such that $\overline{\lambda}\varepsilon<C_3$. Substituting inequalities (2.24) and (2.26) into (2.21), one can obtain

$$\begin{aligned}\frac{1}{2}\overline{y}'(t)\le&-\frac{1}{2}\int_0^1P_{xx}^2dx-\frac{\xi}{2}\int_0^1Q_{xx}^2dx-\frac{\eta}{2}\int_0^1R_{xx}^2dx\\ &+B_1K_2d_1^{-3}+B_2\Upsilon+B_3K_3d_1^{-2/3}\Upsilon^{2/3}+B_4K_4d_1^{-4/3}\Upsilon^{1/3},\end{aligned} \tag{2.27}$$

where $\Upsilon=\int_0^1(u^5+v^5+w^5)dx$, $K_2=(1+\xi^2+\eta^2)M_0+(1+\xi+\eta)M_1d_1^{-1}$, $K_3=M_1^{1/3}$, $K_4=M_1^{2/3}(1+d_1^{-2})$.

Note that

$$P\ge\alpha_{11}u^2,\qquad Q\ge\alpha_{22}v^2,\qquad R\ge\alpha_{33}w^2. \tag{2.28}$$

It follows from (2.18) and (2.4) to functions $P,\ Q,\ R$ that

$$\begin{aligned} &Y \le B_5 \int_0^1 \left(P^{5/2} + Q^{5/2} + R^{5/2}\right) dx \le B_6 K_1^{3/2} d_1^{-3/2} \overline{y}^{1/2} + B_6 K_1^{5/2} d_1^{-5/2}, \\ &Y^{1/3} \le B_7 K_1^{1/2} d_1^{-1/2} \overline{y}^{1/6} + B_7 K_1^{5/6} d_1^{-5/6}, \\ &Y^{2/3} \le B_8 K_1 d_1^{-1} \overline{y}^{1/3} + B_8 K_1^{5/3} d_1^{-5/3}. \end{aligned} \tag{2.29}$$

Moreover, one can obtain by (2.5) and (2.18) that

$$\begin{aligned} &-\frac{1}{2}\int_0^1 P_{xx}^2 dx - \frac{\xi}{2}\int_0^1 Q_{xx}^2 dx - \frac{\eta}{2}\int_0^1 R_{xx}^2 dx \\ &\qquad \le -B_9 \min\{1,\xi,\eta\} K_1^{-4/3} d_1^{4/3} \overline{y}^{5/3} + (1+\xi+\eta) K_1^2 d_1^{-2}. \end{aligned} \tag{2.30}$$

Combining (2.27),(2.29), and (2.30), we have

$$\begin{aligned} \frac{1}{2}\overline{y}'(t) \le &- A_1 \min\{1,\xi,\eta\} K_1^{-4/3} d_1^{4/3} \overline{y}^{5/3} \\ &+ A_2\left[(1+\xi+\eta) K_1^2 d_1^{-2} + K_2 d_1^{-3} + K_1^{5/2} d_1^{-5/2} + K_1^{5/3} K_3 d_1^{-7/3} + K_1^{5/6} K_4 d_1^{-13/6}\right] \\ &+ A_3 K_1^{3/2} d_1^{-3/2} \overline{y}^{1/2} + A_4 K_1 K_3 d_1^{-5/3} \overline{y}^{1/3} + A_5 K_1^{1/2} K_4 d_1^{-11/6} \overline{y}^{1/6}. \end{aligned} \tag{2.31}$$

Multiplying inequality (2.31) by d_1^2, we have

$$\begin{aligned} \frac{1}{2}y'(t) \le &- A_1 \min\{1,\xi,\eta\} K_1^{-4/3} y^{5/3} \\ &+ A_2\left[(1+\xi+\eta) K_1^2 + K_2 d_1^{-1} + K_1^{5/2} d_1^{-1/2} + K_1^{5/3} K_3 d_1^{-1/3} + K_1^{5/6} K_4 d_1^{-1/6}\right] \\ &+ A_3 K_1^{3/2} d_1^{-1/2} y^{1/2} + A_4 K_1 K_3 d_1^{-1/3} y^{1/3} + A_5 K_1^{1/2} K_4 d_1^{-1/6} y^{1/6}, \end{aligned} \tag{2.32}$$

where $y = \int_0^1 [(d_1 P_x)^2 + (d_1 Q_x)^2 + (d_1 R_x)^2] dx$. The inequality (2.32) implies that there exist $\tilde{\tau}_2 > 0$ and positive constant $\widetilde{M}_2$ depending on d_i, α_{ij} $(i,j = 1,2,3)$, a, b, e, l, A, B, D, E, and L, such that

$$\int_0^1 (d_1 P_x)^2 dx, \int_0^1 (d_1 Q_x)^2 dx, \int_0^1 (d_1 R_x)^2 dx \le \widetilde{M}_2, \quad t \ge \tilde{\tau}_2. \tag{2.33}$$

In the case that $d_1, d_2, d_3 \ge 1$, $\xi, \eta \in [\underline{d}, \overline{d}]$, the coefficients of inequality (2.31) can be estimated by some constants which depend on $\underline{d}, \overline{d}$, but do not depend on d_1, d_2, d_3. So $\widetilde{M}_2$ depends

on α_{ij} $(i,j = 1,2,3)$, $a,b,e,l,A,B,D,E,L,\underline{d}$, and $\overline{d}$, but it is irrelevant to d_1,d_2,d_3, when $d_1,d_2,d_3 \geq 1$ and $\xi,\eta \in [\underline{d},\overline{d}]$. Since

$$\begin{pmatrix} P_x \\ Q_x \\ R_x \end{pmatrix} = \begin{pmatrix} P_u & P_v & P_w \\ Q_u & Q_v & Q_w \\ R_u & R_v & R_w \end{pmatrix} \begin{pmatrix} u_x \\ v_x \\ w_x \end{pmatrix}, \tag{2.34}$$

we can transform the formulations of u_x, v_x, w_x into fraction representations, then distribute the denominators of the absolute value of the fractions to the numerators item and enlarge the term concerning with u_x, v_x or w_x to obtain

$$|d_1 u_x| + |d_1 v_x| + |d_1 w_x| \leq C'(|d_1 P_x| + |d_1 Q_x| + |d_1 R_x|), \quad 0 < x < 1,\ t > 0, \tag{2.35}$$

where C' is a constant depending only on ξ,η,α_{ij} $(i,j = 1,2,3)$. After scaling back and contacting estimates (2.33) and (2.35), there exist positive constant M_2 depending on d_i,α_{ij} $(i,j = 1,2,3)$, a,b,e,l,A,B,D,E,L, and $\tau_2 > 0$, such that

$$\int_0^1 u_x^2 dx, \int_0^1 v_x^2 dx, \int_0^1 w_x^2 dx \leq M_2, \quad t \geq \tau_2. \tag{2.36}$$

When $d_1,d_2,d_3 \geq 1$ and $\xi,\eta \in [\underline{d},\overline{d}]$, M_2 is independent of d_1,d_2,d_3.

(ii)One has $t \geq 0$. Modifying the dependency of the coefficients in inequalities (2.17)–(2.22), namely, replacing M_0, M_1 with M_0', M_1', there exists a positive constant M_2' depending on $d_i,\alpha_{ij}(i,j = 1,2,3)$, $a,b,e,l,A,B,$ D, E,L, and the W_2^1-norm of u_0,v_0,w_0, such that

$$\int_0^1 u_x^2 dx, \int_0^1 v_x^2 dx, \int_0^1 w_x^2 dx \leq M_2', \quad t \geq 0. \tag{2.36}'$$

Furthermore, in the case that $d_1,d_2,d_3 \geq 1$, $\xi,\eta \in [\underline{d},\overline{d}]$, M_2' depends on $\underline{d},\overline{d}$, but does not depend on d_1,d_2,d_3.

Summarizing estimates (2.8), (2.15), (2.36) and Sobolev embedding theorem, there exist positive constants M, M' depending only on d_i,α_{ij} $(i,j = 1,2,3)$, $a,b,e,l,A,B,D,E,$ and L, such that (1.18) and (1.19) hold. In particular, M, M' depend only on α_{ij} $(i,j = 1,2,3)$, $a,b,e,l,A,B,D,E,L,\underline{d}$, and $\overline{d}$, but do not depend on d_1,d_2,d_3, when $d_1,d_2,d_3 \geq 1$ and $\xi,\eta \in [\underline{d},\overline{d}]$.

Similarly, according to $(2.8)'$, $(2.15)'$, $(2.36)'$, there exists a positive constant M'' depending on d_i,α_{ij} $(i,j = 1,2,3)$,a,b,e,l,A,B,D,E,L and the initial functions u_0,v_0,w_0, such that

$$|u(\cdot,t)|_{1,2}, |v(\cdot,t)|_{1,2}, |w(\cdot,t)|_{1,2} \leq M'', \quad t \geq 0. \tag{2.37}$$

Further, in the case that $d_1,d_2,d_3 \geq 1$, $\xi,\eta \in [\underline{d},\overline{d}]$, M'' depends only on $\underline{d},\overline{d}$, but do not depend on d_1,d_2,d_3. Thus, $T = +\infty$. This completes the proof of Theorem 1.1. □

3. Global Stability

In order to obtain the uniform convergence of the solution for the system (1.16), we recall the following result which can be found in [37, 42].

Lemma 3.1. *Let a and b be positive constants. Assume that $\varphi, \psi \in C^1([a,+\infty)), \psi(t) \geq 0$ and φ is bounded from below. If $\varphi'(t) \leq -b\psi(t)$ and $\psi'(t)$ is bounded from above in $[a,+\infty)$, then* $\lim_{t\to\infty}\psi(t) = 0$.

Proof of Theorem 1.3. Let (u,v,w) be a solution for the system (1.16) with initial functions $u_0(x), v_0(x), w_0(x) \geq (\not\equiv)0$. From the strong maximum principle for parabolic equations, it is not hard to verify that $u, v, w > 0$ for $t > 0$. Define the function

$$\begin{aligned} H(u,v,w) = &\int_0^1 \left(u - u^* - u^* \ln \frac{u}{u^*}\right)dx + \alpha \int_0^1 \left(v - v^* - v^* \ln \frac{v}{v^*}\right)dx \\ &+ \beta \int_0^1 \left(w - w^* - w^* \ln \frac{w}{w^*}\right)dx. \end{aligned} \tag{3.1}$$

Then the time derivative of $H(u,v,w)$ for the system (1.16) satisfies

$$\begin{aligned} \frac{dH}{dt} = &\int_0^1 \frac{u-u^*}{u}u_t dx + \alpha \int_0^1 \frac{v-v^*}{v}v_t dx + \beta \int_0^1 \frac{w-w^*}{w}w_t dx \\ = &-\int_0^1 \left[\frac{u^*}{u^2}(d_1 + 2\alpha_{11}u + \alpha_{12}v + \alpha_{13}w)u_x^2 + \frac{u^*}{u}(\alpha_{12}v_x + \alpha_{13}w_x)u_x \right. \\ &+ \frac{\alpha v^*}{v^2}(d_2 + \alpha_{21}u + 2\alpha_{22}v + \alpha_{23}w)v_x^2 + \frac{\alpha v^*}{v}(\alpha_{21}u_x + \alpha_{23}w_x)v_x \\ &\left. + \frac{\beta w^*}{w^2}(d_3 + \alpha_{31}u + \alpha_{32}v + 2\alpha_{33}w)w_x^2 + \frac{\beta w^*}{w}(\alpha_{31}u_x + \alpha_{13}v_x)w_x\right]dx \\ &+ \int_0^1 (u-u^*)\left(1 - u - \frac{av}{1+bu} - \frac{Aw}{1+Bu+Dw}\right)dx + \alpha \int_0^1 (v-v^*)m\left(-1 + \frac{eu}{1+bu}\right)dx \\ &+ \beta \int_0^1 (w-w^*)M\left(-1 + \frac{Eu}{1+Bu+Dw}\right)dx. \end{aligned} \tag{3.2}$$

The first integrand in the right hand of (3.2) is positive definite if

$$\begin{aligned} &4\alpha\beta u^* v^* w^* (d_1 + 2\alpha_{11}u + \alpha_{12}v + \alpha_{13}w)(d_2 + \alpha_{21}u + 2\alpha_{22}v + \alpha_{23}w)(d_3 + \alpha_{31}u + \alpha_{32}v + 2\alpha_{33}w) \\ &> u^*\left(\alpha\alpha_{23}v^*w + \beta\alpha_{32}w^*v\right)^2(d_1 + 2\alpha_{11}u + \alpha_{12}v + \alpha_{13}w) \\ &\quad + \alpha v^*\left(\alpha_{13}u^*w + \beta\alpha_{31}w^*u\right)^2(d_2 + \alpha_{21}u + 2\alpha_{22}v + \alpha_{23}w) \\ &\quad + \beta w^*(\alpha_{12}u^*v + \alpha\alpha_{21}v^*u)^2(d_3 + \alpha_{31}u + \alpha_{32}v + 2\alpha_{33}w). \end{aligned} \tag{3.3}$$

By the maximum-norm estimate in Theorem 1.1, the condition (1.20) implies (3.3). Therefore, we have

$$\begin{aligned}\frac{dH}{dt} &\le -\int_0^1 \left[1 - \frac{abv^*}{(1+bu^*)(1+bu)} - \frac{ABw^*}{(1+Bu^*+Dw^*)(1+Bu+Dw)}\right](u-u^*)^2dx \\ &\quad -\beta\int_0^1 \frac{MDEu^*}{(1+Bu^*+Dw^*)(1+Bu+Dw)}(w-w^*)^2dx \\ &\le -\int_0^1 \left[1 - \frac{abv^*}{1+bu^*} - \frac{ABw^*}{1+Bu^*+Dw^*}\right](u-u^*)^2dx - \beta\int_0^1 \frac{MDEu^*}{1+Bu^*+Dw^*}(w-w^*)^2dx \\ &\triangleq -l_1\int_0^1 (u-u^*)^2dx - \beta l_2\int_0^1 (w-w^*)^2dx,\end{aligned} \tag{3.4}$$

where

$$\begin{aligned} l_1 &= 1 - \frac{abv^*}{1+bu^*} - \frac{ABw^*}{1+Bu^*+Dw^*}, \\ l_2 &= \frac{MDEu^*}{1+Bu^*+Dw^*} > 0. \end{aligned} \tag{3.5}$$

The condition (1.21) implies $l_1 > 0$. Using the similar argument in the proof of Theorem 4.2 in [42], by the maximum-norm estimate in Theorem 1.1 and some tedious calculations, we can prove

$$\lim_{t\to\infty}\int_0^1 (u-u^*)^2dx = \lim_{t\to\infty}\int_0^1 (v-v^*)^2dx = \lim_{t\to\infty}\int_0^1 (w-w^*)^2dx = 0. \tag{3.6}$$

It follows from (3.6) and Gagliardo-Nirenberg-type inequality $|u|_\infty \le C|u|_{1,2}^{1/2}|u|_2^{1/2}$ that (u,v,w) converges uniformly on (u^*,v^*,w^*). By the fact that $H(u,v,w)$ is decreasing for $t \ge 0$, it is obvious that (u^*,v^*,w^*) is globally asymptotically stable. So the proof of Theorem 1.3 is completed. □

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