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Research Article

Summability of Sequences and Selection Properties

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We prove that some classes of summable sequences of positive real numbers satisfy several selection principles related to a special kind of convergence.

1. Introduction

By $\mathbb{N}$, $\mathbb{R}$, and $\overline{\mathbb{R}}$ we denote the set of natural numbers, real numbers, and the extended real line $\mathbb{R} \cup \{-\infty, \infty\}$, respectively.

Let $\mathbb{S}$ denote the set of sequences $a = (a_n)_{n\in\mathbb{N}}$ of positive real numbers.

We begin with the following definitions of selection principles.

Let $\mathcal{A}$ and $\mathcal{B}$ be nonempty subsets of $\mathbb{S}$. Then the symbol $S_1(\mathcal{A}, \mathcal{B})$ denotes the selection principle.

For each sequence $(a_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $b = (b_n)_{n\in\mathbb{N}} \in \mathcal{B}$ such that $b_n \in a_n$ for each $n \in \mathbb{N}$.

The following infinitely long game $G_1(\mathcal{A}, \mathcal{B})$ is naturally associated to the previous selection principle.

> Two players, ONE and TWO, play a round for each positive integer. In the n-th round ONE chooses a sequence $a_n \in \mathcal{A}$, and TWO responds by choosing an element $b_n \in a_n$. TWO wins a play $(a_1, b_1; \ldots; a_n, b_n; \ldots)$ if $b = (b_n)_{n\in\mathbb{N}} \in \mathcal{B}$; otherwise, ONE wins.

Another selection principle $\mathsf{S}_{\text{fin}}(\mathcal{A}, \mathcal{B})$ is defined as follows.

> For each sequence $(a_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $b \in \mathcal{B}$ such that $b \cap a_n$ is finite for each $n \in \mathbb{N}$.

It is clear how the corresponding game $G_{\text{fin}}(\mathcal{A}, \mathcal{B})$ is defined.

A strategy of a player is a function σ from the set of all finite sequences of moves of the other player into the set of admissible moves of the strategy owner.

A strategy σ for the player TWO is a *coding strategy* if TWO remembers only the most recent move by ONE and by TWO before his next move. More precisely the moves of TWO are $b_1 = \sigma(a_1, \emptyset)$; $b_n = \sigma(a_n, b_{n-1})$, $n \geq 2$.

In this paper we introduce also the following game. Let $i \in \mathbb{N}$ be a fixed (but arbitrary) natural number. We define the game $\mathsf{G}_1^{(w=i)}(\mathcal{A}, \mathcal{B})$ for two players, ONE and TWO, who play a round for each $n \in \mathbb{N}$. In the i-th round ONE plays a sequence $a_i = (a_{i,m})_{m\in\mathbb{N}} \in \mathcal{A}$, and TWO responds by choosing a finite set $F_i = \{a_{i,m_{i_1}}, \ldots, a_{i,m_{i_k}}\}$. In the n-th round, $n \neq i$, ONE plays a sequence $a_n = (a_{n,m})_{m\in\mathbb{N}} \in \mathcal{A}$, and TWO responds by choosing an element $a_{n,m_n} \in a_n$. TWO wins a play if the sequence $b = (a_{1,m_1}, \ldots, a_{i-1,m_{i-1}}; a_{i,m_{i_1}}, \ldots, a_{i,m_{i_k}}; a_{i+1,m_{i+1}}, \ldots)$ belongs to $\mathcal{B}$; otherwise, ONE wins.

For more information on selection principles and games see the survey papers in [1, 2] and references therein.

In a number of papers by the authors published in the last few years it was demonstrated that some subclasses $\mathcal{A}$ and $\mathcal{B}$ of $\mathbb{S}$ satisfy certain selection principles and game theoretical statements (for $\mathcal{A}$ and $\mathcal{B}$ classes of divergent sequences related to celebrated Karamata's theory of regular variation [3–6] see [7–12], and for $\mathcal{A}$ and $\mathcal{B}$ classes of sequences converging to 0 see [13]). For other results concerning sequences and sequence spaces see [14–16].

In this paper our selections are related to special kinds of convergence of series. More precisely, we start by a sequence of summable sequences and during the selection process we control not only the convergence of series, but also the nature of that convergence.

2. Results

We use the following notations for the classes of sequences we deal with:

$$\begin{aligned} \ell^1 &= \left\{ a \in \mathbb{S} : \sum_{n=1}^{\infty} a_n < \infty \right\}, \\ \ell^{1,S} &= \left\{ a \in \mathbb{S} : \sum_{n=1}^{\infty} a_n = S \right\}, \quad \text{for } S \in (0,\infty], \\ \ell^{1,(\alpha,\beta)} &= \left\{ a \in \ell^{1,S} : S \in (\alpha,\beta) \right\}, \quad \text{for } \alpha,\beta \in (0,\infty), \\ \ell^{1,(\alpha,\beta]} &= \ell^{1,(\alpha,\beta)} \bigcup \ell^{1,\beta}, \quad \text{for } \alpha,\beta \in (0,\infty). \end{aligned} \tag{2.1}$$

Notice that the sequence $x = (x_n)_{n\in\mathbb{N}}$, $x_n = S/2^n$, belongs to the class $\ell^{1,S}$, so that all the classes above are nonempty.

Theorem 2.1. *For each $S \in (0,\infty)$ and each $\varepsilon = \varepsilon(S) \in (0,S)$ TWO has a winning coding strategy in the game $\mathsf{G}_1^{(w=1)}(\ell^{1,S}, \ell^{1,(S-\varepsilon,S]})$.*

Proof. Let σ denote a strategy of TWO, and let $S > 0$ and $\varepsilon = \varepsilon(S) \in (0,S)$ be fixed. Suppose that in the first round ONE chooses a sequence $x_1 = (x_{1,m})_{m\in\mathbb{N}}$ from $\ell^{1,S}$. There is $k \in \mathbb{N}$

such that $\sum_{m=k+1}^{\infty} x_{1,m} < \varepsilon/2$, and thus $M = S - \sum_{m=1}^{k} x_{1,m} \in (0, \varepsilon/2)$. Player TWO plays $\sigma(x_1) = \{x_{1,1}, \ldots, x_{1,k}\}$—a finite subset of x_1.

In the second round ONE chooses a sequence $x_2 = (x_{2,m})_{m\in\mathbb{N}} \in \ell^{1,S}$, and then TWO responds by choosing $\sigma(x_2, \sigma(x_1)) = x_{2,m_2}$ such that $x_{2,m_2} < M/2$ (which is possible because $\lim_{m\to\infty} x_{2,m} = 0$).

In the n-th round, $n \geq 3$, ONE chooses $x_n = (x_{n,m})_{m\in\mathbb{N}} \in \ell^{1,S}$, and TWO's response is $\sigma(x_n, x_{n-1,m_{n-1}}) = x_{n,m_n}$ such that $x_{n,m_n} < x_{n-1,m_{n-1}}/2^{n-1} < M/2^{n-1}$, and so on.

Set $y_n = x_{1,n}$ for $n \leq k$ and $y_n = x_{n-k+1,m_{n-k+1}}$ for $n > k$. Let us prove $y = (y_n)_{n\in\mathbb{N}} \in \ell^{1,(S-\varepsilon,S]}$. We have

$$\begin{aligned}\sum_{n=1}^{\infty} y_n &= \sum_{n=1}^{k} y_n + \sum_{n=k+1}^{\infty} y_n = \sum_{m=1}^{k} x_{1,m} + \sum_{n=k+1}^{\infty} y_n \\ &= S - M + \sum_{n=k+1}^{\infty} y_n < S - M + M\left(\sum_{i=1}^{\infty} \frac{1}{2^i}\right) = S.\end{aligned} \tag{2.2}$$

On the other hand,

$$\sum_{n=1}^{\infty} y_n > \sum_{m=1}^{k} x_{1,m} = S - M > S - \frac{\varepsilon}{2}. \tag{2.3}$$

That is, $y \in \ell^{1,(S-\varepsilon,S]}$. □

Corollary 2.2. *For each $S \in (0, \infty)$ and each $\varepsilon = \varepsilon(S) \in (0, S)$ the selection principle $\mathsf{S}_{\text{fin}}(\ell^{1,S}, \ell^{1,(S-\varepsilon,S]})$ is true.*

Notice that one can prove a refinement of Theorem 2.1 (and Corollary 2.2) in the sense that it is possible to have additional control of selections giving the sequence y. For this we need the following definitions and notation.

Definition 2.3 (see [13]). A sequence $(x_n)_{n\in\mathbb{N}} \in \mathbb{S}$ is said to belong to the class $\mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})$ if for each $\lambda \geq 1$ it satisfies

$$\lim_{n\to\infty} \frac{x_{[n+\lambda]}}{x_n} = 0, \tag{2.4}$$

where $[r]$ denotes the integer part of $r \in \mathbb{R}$.

Definition 2.4 (see [9]). For a sequence $x = (x_n)_{n\in\mathbb{N}} \in \mathbb{S}$, the *Landau-Hurwicz sequence* $w(x) = (w_n(x))_{n\in\mathbb{N}}$ of x is defined by

$$w_n(x) := \sup\{|x_m - x_k| : m \geq n, k \geq n\}, \qquad n \in \mathbb{N}. \tag{2.5}$$

Given a sequence $x = (x_n)_{n\in\mathbb{N}} \in \mathbb{S}$ we denote by $S_x = (S_n(x))_{n\in\mathbb{N}}$ the sequence defined by

$$S_n(x) = \sum_{i=1}^{n} x_i, \quad n \in \mathbb{N}. \tag{2.6}$$

Let $\ell^{1,(\alpha,\beta]}_{\mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})}$ be the set of all sequences $a = (a_n)_{n\in\mathbb{N}} \in \ell^{1,(\alpha,\beta]}$ such that $w(S_a) \in \mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})$.

Theorem 2.5. *For each $S \in (0,\infty)$ and each $\varepsilon = \varepsilon(S) \in (0,S)$ TWO has a winning coding strategy in the game $\mathtt{G}_1^{(w=1)}(\ell^{1,S}, \ell^{1,(S-\varepsilon,S]}_{\mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})})$.*

Proof. The strategy σ of player TWO and the sequence $y = (y_n)_{n\in\mathbb{N}}$ are actually from the proof of Theorem 2.1. Therefore, $y \in \ell^{1,(S-\varepsilon,S]}$. Besides, since, by construction, the series

$$\sum_{n=1}^{\infty} \frac{y_{n+1}}{y_n} \tag{2.7}$$

is convergent, we have

$$\lim_{n\to\infty}\left(\sum_{k=n}^{\infty} \frac{y_{k+1}}{y_k}\right) = 0. \tag{2.8}$$

Consider now the sequence $S_y = (S_n(y))_{n\in\mathbb{N}}$. This sequence is convergent (by the d'Alembert criterion), and let $S(y)$ be its limit. It remains to prove $w(S_y) = (w_n(S_y))_{n\in\mathbb{N}} \in \mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})$. It is enough to prove

$$\lim_{n\to\infty} \frac{w_{n+1}(S_y)}{w_n(S_y)} = 0. \tag{2.9}$$

First, notice that

$$w_n(S_y) = S(y) - S_n(y), \quad n \in \mathbb{N}. \tag{2.10}$$

Thus we get

$$\lim_{n\to\infty} \frac{w_{n+1}(S_y)}{w_n(S_y)} = \lim_{n\to\infty} \frac{S(y) - S_{n+1}(y)}{S(y) - S_n(y)} = 1 - \lim_{n\to\infty} \frac{y_{n+1}}{y_{n+1} + y_{n+2} + \cdots} = 0. \tag{2.11}$$

That is (2.9), since by (2.8) and the fact that for n sufficiently large it holds

$$\frac{y_{n+2}}{y_{n+1}} + \frac{y_{n+3}}{y_{n+1}} + \cdots = \frac{y_{n+2}}{y_{n+1}} + \frac{y_{n+3}}{y_{n+2}} \cdot \frac{y_{n+2}}{y_{n+1}} + \cdots \le \frac{y_{n+2}}{y_{n+1}} + \frac{y_{n+3}}{y_{n+2}} + \cdots, \tag{2.12}$$

we have

$$\lim_{n\to\infty}\frac{y_{n+1}}{y_{n+1}+y_{n+2}+\cdots}=\lim_{n\to\infty}\frac{1}{1+\left(y_{n+2}/y_{n+1}\right)+\left(y_{n+3}/y_{n+1}\right)+\cdots}=1. \tag{2.13}$$

The theorem is proved. □

Corollary 2.6. *The selection principle* $\mathsf{S}_{\text{fin}}(\ell^{1,S},\ell^{1,(S-\varepsilon,S]}_{\text{Tr}(\mathbb{R}_{-\infty,s})})$ *is true.*

The following two theorems give other selection results for defined classes of sequences: one of the S_{fin}-type and the other of the S_1-type.

Theorem 2.7. *For each* $S\in(0,\infty]$ *the selection principle* $\mathsf{S}_{\text{fin}}(\ell^{1,S},\ell^{1,\infty})$ *is satisfied.*

Proof. Consider first the case $S\in(0,\infty)$. Let $(x_n : n\in\mathbb{N})$, $x_n=(x_{n,m})_{m\in\mathbb{N}}$, be a sequence of elements of $\ell^{1,S}$. For each $n\in\mathbb{N}$ let $z_{n_i}=x_{n,i}$, $i\le k=k(n)$, be a finite subset of x_n such that $S/2<\sum_{i=1}^{k}z_{n_i}<S$. Arrange now z_{n_p}, $n\in\mathbb{N}$, $p\in\{1,2,\ldots,k(n)\}$, in the sequence $y=(y_j)_{j\in\mathbb{N}}$ in which the position of an element is determined first by n and then by p, that is,

$$y=\left(z_{1_1},\ldots,z_{1_{k(1)}};\ldots;z_{n_1},\ldots,z_{n_{k(n)}};\ldots\right). \tag{2.14}$$

We have

$$n\cdot\frac{S}{2}<\sum_{m=1}^{n}\sum_{i=1}^{k(m)}z_{m_i}=\sum_{j=1}^{k(n)}y_j, \tag{2.15}$$

where $y_{k(n)}$ is the last element of x_n belonging to the sequence y. Therefore,

$$\sum_{j=1}^{\infty}y_j=\lim_{n\to\infty}\sum_{j=1}^{k(n)}y_j>\lim_{n\to\infty}\left(n\cdot\frac{S}{2}\right)=\infty. \tag{2.16}$$

That is, $y\in\ell^{1,\infty}$.

Suppose now that $S=\infty$. This case is treated similarly to the previous case, but here we require $\sum_{i=1}^{k}z_{n_i}>1$ for each $n\in\mathbb{N}$; the sequence $y=(y_j)_{j\in\mathbb{N}}$ is formed in a similar way as in the first case. So we have $n\cdot 1<\sum_{j=1}^{k(n)}y_j$ for each $n\in\mathbb{N}$, hence

$$\sum_{j=1}^{\infty}y_j=\lim_{n\to\infty}\sum_{j=1}^{k(n)}y_j>\lim_{n\to\infty}(n\cdot 1)=\infty. \tag{2.17}$$

That is, $y\in\ell^{1,\infty}$. The theorem is proved. □

Theorem 2.8. *For each* $S\in(0,\infty)$ *and each* $\alpha>0$ *the selection principle* $\mathsf{S}_1(\ell^{1,S},\ell^{1,(0,\alpha)})$ *is true.*

Proof. Let $(x_n : n\in\mathbb{N})$, $x_n=(x_{n,m})_{m\in\mathbb{N}}$, be a sequence of elements in $\ell^{1,S}$. For each $n\in\mathbb{N}$ take $y_n=x_{n,m_n}\in x_n$ so that $y_1\in(0,\alpha)$ (which is possible since $x_{1,m}\to 0$ as $m\to\infty$) and

$y_n < \alpha - y_1/2^{n-1}$ for $n \geq 2$. Then the sequence $y = (y_n)_{n\in\mathbb{N}}$ witnesses that the statement is true, because

$$\sum_{n=1}^{\infty} y_n = y_1 + \sum_{n=2}^{\infty} y_n < y_1 + (\alpha - y_1) = \alpha. \tag{2.18}$$

That is, $y \in \ell^{1,(0,\alpha)}$. □

For the next result we have to define the following selection principles [2, 17]. Notice that in [18] we developed an interesting technique for proving results concerning these selection principles and certain classes of sequences from $\mathbb{S}$. In [19] we proposed the use of this technique (and these selection principles) in other fields of mathematics and its applications.

Let, as before, $\mathcal{A}$ and $\mathcal{B}$ be certain nonempty subfamilies of $\mathbb{S}$. Then the symbol $\alpha_i(\mathcal{A}, \mathcal{B})$, $i = 2, 3, 4$, denotes the selection hypothesis that for each sequence $(a_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is an element $b \in \mathcal{B}$ such that:

α_2(A,B): for each $n \in \mathbb{N}$ the set $a_n \cap b$ is infinite;

$\alpha_3(\mathcal{A}, \mathcal{B})$: for infinitely many $n \in \mathbb{N}$ the set $a_n \cap b$ is infinite;

$\alpha_4(\mathcal{A}, \mathcal{B})$: for infinitely many $n \in \mathbb{N}$ the set $a_n \cap b$ is nonempty.

Theorem 2.9. *For each $S \in (0, \infty)$ and each $\alpha > 0$ the selection principles $\alpha_i(\ell^{1,S}, \ell^{1,(0,\alpha)}_{\mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})})$, $i = 2, 3, 4$, are satisfied.*

Proof. We prove that the principle α_2 is true (hence also α_3 and α_4). Let $(x_n : n \in \mathbb{N})$, $x_n = (x_{n,m})_{m\in\mathbb{N}}$, be a sequence of sequences from $\ell^{1,S}$. Let $m_1 \in \mathbb{N}$ be such that $\sum_{m=m_1+1}^{\infty} x_{1,m} < \alpha/2$. For $k \leq 2$ let m_k be a natural number such that $\sum_{m=m_k+1}^{\infty} x_{k,m} < \alpha/2^k$. Consider the sequence $y = (y_j)_{j\in\mathbb{N}}$ defined in this way:

$$y = (x_{1,m_1+1}, x_{1,m_2+2}, \ldots; x_{2,m_2+1}, x_{2,m_2+2}, \ldots; x_{k,m_k+1}, x_{k,m_k+2}, \ldots). \tag{2.19}$$

Then $y \cap x_n$ is infinite for each $n \in \mathbb{N}$. Further, $y \in \ell^{1,(0,\alpha)}$ because

$$0 < \sum_{j=1}^{\infty} y_j = \sum_{k=1}^{\infty} \sum_{m=m_k+1}^{\infty} x_{k,m} < \sum_{k=1}^{\infty} \frac{\alpha}{2^k} = \alpha. \tag{2.20}$$

We construct now a new sequence $z = (z_i)$ in the way described in Table 1 .

Evidently, $z \cap x_n$ is infinite for each $n \in \mathbb{N}$. Also, $0 < \sum_{i=1}^{\infty} z_i \leq \sum_{j=1}^{\infty} y_j < \alpha$, that is, $z \in \ell^{1,(0,\alpha)}$. By a minor modification of the proof of Theorem 2.5 we obtain $w(S_z) \in \mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})$. This means $z \in \ell^{1,(0,\alpha)}_{\mathrm{Tr}(\mathtt{R}_{-\infty,\mathtt{s}})}$. □

Table 1

	x_1	x_2	x_3	x_4	How
z_1	$z_1 \in y \cap x_1$	—	—	—	any
z_2	—	$z_2 \in y \cap x_2$	—	—	$z_2/z_1 < 1/2$
z_3	$z_3 \in y \cap x_1$	—	—	—	$z_3/z_2 < 1/2^2$
z_4	—	—	$z_4 \in y \cap x_3$	—	$z_4/z_3 < 1/2^3$
z_5	—	$z_5 \in y \cap x_2$	—	—	$z_5/z_4 < 1/2^4$
z_6	$z_6 \in y \cap x_1$	—	—	—	$z_6/z_5 < 1/2^5$
z_7	—	—	—	$z_7 \in y \cap x_4$	$z_7/z_6 < 1/2^6$
z_8	—	—	$z_8 \in y \cap x_3$	—	$z_8/z_7 < 1/2^7$
z_9	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

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