Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2009, Article ID 246521, 9 pages
doi:10.1155/2009/246521

Research Article

Extended Cesàro Operators from Logarithmic-Type Spaces to Bloch-Type Spaces

Dinggui Gu

Department of Mathematics, JiaYing University, Meizhou, Guangdong 514015, China

Correspondence should be addressed to Dinggui Gu, gudinggui@163.com

Received 27 March 2009; Accepted 12 April 2009

Recommended by Stevo Stević

The boundedness and compactness of the extended Cesàro operator from logarithmic-type spaces to Bloch-type spaces on the unit ball are completely characterized in this paper.

1. Introduction

Let $\mathbb{B}_n$ be the unit ball of $\mathbb{C}^n$, $\partial\mathbb{B}_n$ the unit sphere of $\mathbb{C}^n$, $H(\mathbb{B}_n)$ the space of all holomorphic functions in $\mathbb{B}_n$, and H^∞ the space of all bounded holomorphic functions on $\mathbb{B}_n$. For $f \in H(\mathbb{B}_n)$, let

$$\Re f(z) = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z) \tag{1.1}$$

denote the radial derivative of $f \in H(\mathbb{B}_n)$.

A positive continuous function μ on the interval $[0,1)$ is called normal if there is $\delta \in [0,1)$ and s and t, $0 < s < t$ such that

$$\begin{aligned} &\frac{\mu(r)}{(1-r)^s} \text{ is decreasing on } [\delta,1), \quad \lim_{r\to 1}\frac{\mu(r)}{(1-r)^s} = 0, \\ &\frac{\mu(r)}{(1-r)^t} \text{ is increasing on } [\delta,1), \quad \lim_{r\to 1}\frac{\mu(r)}{(1-r)^t} = \infty. \end{aligned} \tag{1.2}$$

If we say that $\mu : \mathbb{B}_n \to [0,\infty)$ is normal we will also assume that $\mu(z) = \mu(|z|)$, $z \in \mathbb{B}_n$.

Let $\mu : \mathbb{B}_n \to [0, \infty)$ be normal. The Bloch-type space $\mathcal{B}_\mu = \mathcal{B}_\mu(\mathbb{B}_n)$ is the space of all functions $f \in H(\mathbb{B}_n)$ such that

$$b_\mu(f) = \sup_{z\in\mathbb{B}_n} \mu(z)\left|\Re f(z)\right| < \infty. \tag{1.3}$$

$\mathcal{B}_\mu$ is Banach space with the norm $\|f\|_{\mathcal{B}_\mu} = |f(0)| + b_\mu(f)$. The little Bloch-type space $\mathcal{B}_{\mu,0} = \mathcal{B}_{\mu,0}(\mathbb{B}_n)$ consists of all $f \in H(\mathbb{B}_n)$ such that

$$\lim_{|z|\to 1} \mu(z)\left|\Re f(z)\right| = 0. \tag{1.4}$$

It is easy to see that $\mathcal{B}_{\mu,0}$ is a closed subspace of $\mathcal{B}_\mu$. When $\mu(r) = (1-r^2)^\alpha$, $\alpha \in (0, \infty)$, we obtain so-called α-Bloch spaces and little α-Bloch spaces, respectively, which for $\alpha = 1$ are reduced to classical Bloch spaces (see, e.g., [1–4] and the references therein). When $\mu(r) = (1-r^2)\ln(e/(1-r^2))$, we obtain the logarithmic Bloch space $\mathcal{LB} = \mathcal{LB}(\mathbb{B}_n)$ and the little logarithmic Bloch space $\mathcal{LB}_0 = \mathcal{LB}_0(\mathbb{B}_n)$, respectively, (see [5]). It was shown that f is a multiplier of $\mathcal{B}$ if and only if $f \in H^\infty$ and $f \in \mathcal{LB}$ in [6].

An $f \in H(\mathbb{B}_n)$ is said to belong to the logarithmic-type space $H^\infty_{\log}$, if

$$\|f\|_{H^\infty_{\log}} = \sup_{z\in\mathbb{B}_n} \frac{|f(z)|}{\ln\left(e/\left(1-|z|^2\right)\right)} < \infty. \tag{1.5}$$

It is easy to see that $H^\infty_{\log}$ becomes a Banach space under the norm $\|\cdot\|_{H^\infty_{\log}}$, and that the inclusions $H^\infty \subset \mathcal{B} \subset H^\infty_{\log}$, hold. For some information of the space $H^\infty_{\log}$ see [7, 8].

Let $g \in H(\mathbb{B}_n)$. The extended Cesàro operator on $H(\mathbb{B}_n)$ is defined by

$$T_g f(z) = \int_0^1 f(tz)\Re g(tz)\frac{dt}{t}, \quad f \in H(\mathbb{B}_n),\ z \in \mathbb{B}_n. \tag{1.6}$$

This operator is a natural extension of a one-dimensional operator defined in [9]. Some other results on the one-dimensional operator can be found, for example, in [10, 11] (see also the references therein). For some extensions of operator (1.6) on the unit disk see [12–16]. On related operators on the unit polydisk see, for example, [17–21] and references therein. The boundedness and compactness of operator (1.6) between various spaces of holomorphic functions has been extensively studied recently, see, [17, 22–36]. For some integral operators on spaces of harmonic functions see, for example, [37] as well as the references therein. A new extension of operator (1.6) in the unit ball case have been recently introduced by Stević in [38] (see also [5, 39]).

In this paper, we study the extended Cesàro operator from $H^\infty_{\log}$ to Bloch-type spaces $\mathcal{B}_\mu$ and $\mathcal{B}_{\mu,0}$. Sufficient and necessary conditions for the extended Cesàro operator T_g to be bounded and compact are given.

Throughout the paper, constants are denoted by C, they are positive and may not be the same in every occurrence.

2. Main Results and Proofs

In this section, we give our main results and their proofs. Before stating these results, we need some auxiliary results, which are incorporated in the lemmas which follows.

Lemma 2.1. *Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \to [0,\infty)$ are normal. Then $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is compact if and only if $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in $H^\infty_{\log}$ which converges to zero uniformly on compact subsets of $\mathbb{B}_n$ as $k \to \infty$, one has $\|T_g f_k\|_{\mathcal{B}_\mu} \to 0$ as $k \to \infty$.*

The proof of Lemma 2.1 follows by standard arguments (see, e.g., Lemmas 3 in [20, 21, 29]). Hence, we omit the details.

Lemma 2.2. *Assume that $\mu : \mathbb{B}_n \to [0,\infty)$ is normal. A closed set K in $\mathcal{B}_{\mu,0}$ is compact if and only if it is bounded and satisfies*

$$\lim_{|z|\to 1} \sup_{f\in K} \mu(z)\left|\Re f(z)\right| = 0. \tag{2.1}$$

This lemma can be found in [5], and its proof is similar to the proof of Lemma 1 in [40]. Hence, it will be omitted.

The following result was proved in [8].

Lemma 2.3. *There exist two functions $f_1, f_2 \in H^\infty_{\log}(\mathbb{B}_1)$ such that*

$$\left|f_1(z)\right| + \left|f_2(z)\right| \geq \ln\frac{1}{1-|z|}, \quad z \in \mathbb{B}_1. \tag{2.2}$$

Now we are in a position to state and prove our main results.

Theorem 2.4. *Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \to [0,\infty)$ is normal. Then $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is bounded if and only if*

$$M = \sup_{z\in\mathbb{B}_n} \mu(z)\left|\Re g(z)\right| \ln\frac{e}{1-|z|^2} < \infty. \tag{2.3}$$

Moreover, if $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is bounded then the following asymptotic relation holds

$$\left\|T_g\right\|_{H^\infty_{\log}\to\mathcal{B}_\mu} \asymp \sup_{z\in\mathbb{B}_n} \mu(z)\left|\Re g(z)\right| \ln\frac{e}{1-|z|^2} < \infty. \tag{2.4}$$

Proof. Assume that (2.3) holds. Then, for any $f \in H^\infty_{\log}$, we have

$$\mu(z)\left|\Re(T_g f)(z)\right| = \mu(z)\left|\Re g(z)\right|\left|f(z)\right| \leq \mu(z)\left|\Re g(z)\right|\left(\ln\frac{e}{1-|z|^2}\right)\|f\|_{H^\infty_{\log}}. \tag{2.5}$$

In addition, it is easy to see that $T_g f(0) = 0$. Therefore we have

$$\|T_g f\|_{\mathcal{B}_\mu} = \sup_{z\in\mathbb{B}_n} \mu(z)\left|\Re(T_g f)(z)\right| \le M\|f\|_{H^\infty_{\log}} \tag{2.6}$$

as desired.

Conversely, assume that $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is bounded. For $a \in \mathbb{B}_n$, set

$$f_a(z) = \ln\frac{e}{1-\langle z, a\rangle}. \tag{2.7}$$

It is easy to see that $f_a \in H^\infty_{\log}$ and $\sup_{a\in\mathbb{B}_n}\|f_a\|_{H^\infty_{\log}} < \infty$.

For any $b \in \mathbb{B}_n$, we have

$$\begin{aligned}
\infty &> \left\|T_g f_b\right\|_{\mathcal{B}_\mu} \\
&= \sup_{z\in\mathbb{B}_n} \mu(z)\left|\Re(T_g f_b)(z)\right| \\
&= \sup_{z\in\mathbb{B}_n} \mu(z)\left|\Re g(z)\right|\left|f_b(z)\right| \\
&\ge \mu(b)\left|\Re g(b)\right| \ln\frac{e}{1-|b|^2},
\end{aligned} \tag{2.8}$$

from which (2.3) follows, moreover

$$\sup_{z\in\mathbb{B}_n} \mu(z)\left|\Re g(z)\right| \ln\frac{e}{1-|z|^2} \le C\left\|T_g\right\|_{H^\infty_{\log}\to\mathcal{B}_\mu}. \tag{2.9}$$

From (2.6) and (2.9), we see that (2.4) holds. The proof is completed. □

Theorem 2.5. *Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \to [0,\infty)$ is normal. Then $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is compact if and only if*

$$\lim_{|z|\to 1} \mu(z)\left|\Re g(z)\right| \ln\frac{e}{1-|z|^2} = 0. \tag{2.10}$$

Proof. Suppose that $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is compact. Let $(z_k)_{k\in\mathbb{N}}$ be a sequence in $\mathbb{B}_n$ such that $\lim_{k\to\infty}|z_k| = 1$. Set

$$f_k(z) = \left(\ln\frac{e}{1-\langle z, z_k\rangle}\right)^2 \left(\ln\frac{e}{1-|z_k|^2}\right)^{-1}, \quad k \in \mathbb{N}. \tag{2.11}$$

It is easy to see that $\sup_{k\in\mathbb{N}}\|f_k\|_{H^\infty_{\log}} < \infty$. Moreover $f_k \to 0$ uniformly on compact subsets of $\mathbb{B}_n$ as $k \to \infty$. By Lemma 2.1,

$$\lim_{k\to\infty}\|T_g f_k\|_{\mathcal{B}_\mu} = 0. \tag{2.12}$$

In addition,

$$\|T_g f_k\|_{\mathcal{B}_\mu} = \sup_{z\in\mathbb{B}_n}\mu(z)\left|\Re g(z) f_k(z)\right| \geq \mu(z_k)\left|\Re g(z_k)\right|\ln\frac{e}{1-|z_k|^2}, \tag{2.13}$$

which together with (2.12) implies that

$$\lim_{k\to\infty}\mu(z_k)\left|\Re g(z_k)\right|\ln\frac{e}{1-|z_k|^2} = 0. \tag{2.14}$$

From the above inequality we see that (2.10) holds.

Conversely, assume that (2.10) holds. From Theorem 2.4 we see that $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is bounded. In order to prove that $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is compact, according to Lemma 2.1, it suffices to show that if $(f_k)_{k\in\mathbb{N}}$ is a bounded sequence in $H^\infty_{\log}$ converging to 0 uniformly on compact subsets of $\mathbb{B}_n$, then

$$\lim_{k\to\infty}\|T_g f_k\|_{\mathcal{B}_\mu} = 0. \tag{2.15}$$

Let $(f_k)_{k\in\mathbb{N}}$ be a bounded sequence in $H^\infty_{\log}$ such that $f_k \to 0$ uniformly on compact subsets of $\mathbb{B}_n$ as $k \to \infty$. By (2.10) we have that for any $\varepsilon > 0$, there is a constant $\delta \in (0,1)$, such that

$$\mu(z)\left|\Re g(z)\right|\ln\frac{e}{1-|z_k|^2} < \varepsilon \tag{2.16}$$

whenever $\delta < |z| < 1$. Let $K = \{z \in \mathbb{B}_n : |z| \leq \delta\}$. From (2.10) we see that $g \in \mathcal{B}_\mu$. Equality (2.16) along with the fact that $g \in \mathcal{B}_\mu$ implies

$$\begin{aligned}
\|T_g f_k\|_{\mathcal{B}_\mu} &= \sup_{z\in\mathbb{B}_n}\mu(z)\left|\Re(T_g f_k)(z)\right| \\
&= \sup_{z\in\mathbb{B}_n}\mu(z)\left|\Re g(z) f_k(z)\right| \\
&\leq \left(\sup_{\{z\in\mathbb{B}_n:|z|\leq\delta\}} + \sup_{\{z\in\mathbb{B}_n:\delta\leq|z|<1\}}\right)\mu(z)\left|\Re g(z)\right|\left|f_k(z)\right| \\
&\leq \|g\|_{\mathcal{B}_\mu}\sup_{z\in K}\left|f_k(z)\right| + \sup_{\{z\in\mathbb{B}_n:\delta\leq|z|<1\}}\mu(z)\left|g(z)\right|\left(\ln\frac{e}{1-|z|^2}\right)\|f_k\|_{H^\infty_{\log}} \\
&\leq \|g\|_{\mathcal{B}_\mu}\sup_{z\in K}\left|f_k(z)\right| + C\varepsilon.
\end{aligned} \tag{2.17}$$

Observe that K is a compact subset of $\mathbb{B}_n$, so that

$$\lim_{k\to\infty}\sup_{z\in K}\left|f_k(z)\right| = 0. \tag{2.18}$$

Therefore

$$\limsup_{k\to\infty}\left\|T_g f_k\right\|_{\mathcal{B}_\mu} \le C\varepsilon. \tag{2.19}$$

Since $\varepsilon > 0$ is an arbitrary positive number it follows that the last limit is equal to zero. Therefore, $T_g : H^\infty_{\log} \to \mathcal{B}_\mu$ is compact. The proof is completed. □

Remark 2.6. From [24] and Theorems 2.4 and 2.5, we see that $T_g : H^\infty_{\log} \to \mathcal{B}$ is bounded if and only if $T_g : \mathcal{B} \to \mathcal{B}$ is bounded; $T_g : H^\infty_{\log} \to \mathcal{B}$ is compact if and only if $T_g : \mathcal{B} \to \mathcal{B}$ is compact.

Theorem 2.7. *Assume that $g \in H(\mathbb{B}_n)$ and $\mu : \mathbb{B}_n \to [0,\infty)$ is normal. Then the following statements are equivalent:*

(a) $T_g : H^\infty_{\log} \to \mathcal{B}_{\mu,0}$ *is bounded;*

(b) $T_g : H^\infty_{\log} \to \mathcal{B}_{\mu,0}$ *is compact;*

(c)

$$\lim_{|z|\to 1}\mu(z)\left|\Re g(z)\right|\ln\frac{e}{1-|z|^2} = 0. \tag{2.20}$$

Proof. (b)⇒(a). This implication is obvious.

(a)⇒(c). Assume that $T_g : H^\infty_{\log} \to \mathcal{B}_{\mu,0}$ is bounded. Now we prove that (2.20) holds. Note that (2.20) is equivalent with

$$\lim_{|z|\to 1}\mu(z)\left|\Re g(z)\right|\ln\frac{1}{1-|z|} = 0. \tag{2.21}$$

Hence we only need to show that (2.21) holds. This can be done by contradiction. Now assume that the condition (2.21) does not hold. If it was, then it would exist $\varepsilon_0 > 0$ and a sequence $(z^{(j)})_{j\in\mathbb{N}} \in \mathbb{B}_n$, such that $z^{(j)} \to \partial\mathbb{B}_n$, and

$$\mu\left(z^{(j)}\right)\left|\Re g\left(z^{(j)}\right)\right|\ln\frac{1}{1-\left|z^{(j)}\right|} \ge \varepsilon_0 > 0 \tag{2.22}$$

for sufficiently large j. We may assume that $\lim_{j\to\infty}z^{(j)} = (1,0,\ldots,0)$ and also

$$1-\left|z^{(j)}\right| > \frac{1-\left|z_1^{(j)}\right|}{2}, \quad j\in\mathbb{N}. \tag{2.23}$$

According to Lemma 2.3 we know that there exist two functions $h_1, h_2 \in H^\infty_{\log}(\mathbb{B}_1)$ such that

$$|h_1(z_1)| + |h_2(z_1)| \geq \ln\frac{1}{1-|z_1|}, \quad z_1 \in \mathbb{B}_1. \tag{2.24}$$

Let

$$F_1(z) = h_1(z_1), \quad F_2(z) = h_2(z_1), \quad z = (z_1, \ldots, z_n) \in \mathbb{B}_n. \tag{2.25}$$

Then clearly $F_1, F_2 \in H^\infty_{\log}$. By the boundedness of $T_g : H^\infty_{\log} \to \mathcal{B}_{\mu,0}$ we have

$$T_gF_1, T_gF_2 \in \mathcal{B}_{\mu,0}. \tag{2.26}$$

On the other hand,

$$\begin{aligned}
&\mu\left(z^{(j)}\right)\left(\left|\Re(T_gF_1)\left(z^{(j)}\right)\right| + \left|\Re(T_gF_2)\left(z^{(j)}\right)\right|\right)\\
&\quad= \mu\left(z^{(j)}\right)\left|\Re g\left(z^{(j)}\right)\right|\left(\left|F_1\left(z^{(j)}\right)\right| + \left|F_2\left(z^{(j)}\right)\right|\right)\\
&\quad= \mu\left(z^{(j)}\right)\left|\Re g\left(z^{(j)}\right)\right|\left(\left|h_1\left(z_1^{(j)}\right)\right| + \left|h_2\left(z_1^{(j)}\right)\right|\right)\\
&\quad\geq \mu\left(z^{(j)}\right)\left|\Re g\left(z^{(j)}\right)\right|\ln\frac{1}{1-\left|z_1^{(j)}\right|}\\
&\quad\geq C\mu\left(z^{(j)}\right)\left|\Re g\left(z^{(j)}\right)\right|\ln\frac{1}{1-\left|z^{(j)}\right|}\\
&\quad\geq C\varepsilon_0 > 0
\end{aligned} \tag{2.27}$$

for sufficiently large j. Since $z^{(j)} \to \partial\mathbb{B}_n$, from the above inequality we obtain that $T_gF_1, T_gF_2 \notin \mathcal{B}_{\mu,0}$, which is a contradiction.

(c)$\Rightarrow$(b). From (1.5) we have that

$$\mu(z)\left|\Re(T_gf)(z)\right| \leq \mu(z)\left|\Re g(z)\right|\left(\ln\frac{e}{1-|z|^2}\right)\|f\|_{H^\infty_{\log}}. \tag{2.28}$$

Taking the supremum in the above inequality over all $f \in H^\infty_{\log}$ such that $\|f\|_{H^\infty_{\log}} \leq 1$, then letting $|z| \to 1$, by (2.20) we arrive at

$$\lim_{|z|\to 1}\sup_{\|f\|_{H^\infty_{\log}}\leq 1}\mu(z)\left|\Re(T_g(f))(z)\right| = 0. \tag{2.29}$$

From this and by employing Lemma 2.2, we see that $T_g : H^\infty_{\log} \to \mathcal{B}_{\mu,0}$ is compact. The proof is completed. □

From Theorems 2.4, 2.5, and 2.7, we have the following corollary.

Corollary 2.8. *Let $g \in H(\mathbb{B}_n)$. Then*

(1) $T_g : H^\infty_{\log} \to \mathcal{LB}$ *is bounded if and only if*

$$\sup_{z\in\mathbb{B}_n}\left(1-|z|^2\right)\left|\Re g(z)\right|\left(\ln\frac{e}{1-|z|^2}\right)^2<\infty. \tag{2.30}$$

(2) $T_g : H^\infty_{\log} \to \mathcal{LB}$ *is compact if and only if* $T_g : H^\infty_{\log} \to \mathcal{LB}_0$ *is bounded if and only if* $T_g : H^\infty_{\log} \to \mathcal{LB}_0$ *is compact if and only if*

$$\lim_{|z|\to 1}\left(1-|z|^2\right)\left|\Re g(z)\right|\left(\ln\frac{e}{1-|z|^2}\right)^2=0. \tag{2.31}$$

Acknowledgment

The author is supported partly by the Natural Science Foundation of Guangdong Province (no. 07006700).

References

[1] D. D. Clahane and S. Stević, "Norm equivalence and composition operators between Bloch/Lipschitz spaces of the ball," *Journal of Inequalities and Applications*, vol. 2006, Article ID 61018, 11 pages, 2006.
[2] S. Li and S. Stević, "Some characterizations of the Besov space and the α-Bloch space," *Journal of Mathematical Analysis and Applications*, vol. 346, no. 1, pp. 262–273, 2008.
[3] S. Li and H. Wulan, "Characterizations of α-Bloch spaces on the unit ball," *Journal of Mathematical Analysis and Applications*, vol. 343, no. 1, pp. 58–63, 2008.
[4] K. Zhu, *Spaces of Holomorphic Functions in the Unit Ball*, vol. 226 of *Graduate Texts in Mathematics*, Springer, New York, NY, USA, 2005.
[5] S. Stević, "On a new operator from the logarithmic Bloch space to the Bloch-type space on the unit ball," *Applied Mathematics and Computation*, vol. 206, no. 1, pp. 313–320, 2008.
[6] K. H. Zhu, "Multipliers of BMO in the Bergman metric with applications to Toeplitz operators," *Journal of Functional Analysis*, vol. 87, no. 1, pp. 31–50, 1989.
[7] X. Fu and X. Zhu, "Weighted composition operators on some weighted spaces in the unit ball," *Abstract and Applied Analysis*, vol. 2008, Article ID 605807, 8 pages, 2008.
[8] D. Girela, J. Á. Peláez, F. Pérez-González, and J. Rättyä, "Carleson measures for the Bloch space," *Integral Equations and Operator Theory*, vol. 61, no. 4, pp. 511–547, 2008.
[9] Ch. Pommerenke, "Schlichte Funktionen und analytische Funktionen von beschränkter mittlerer Oszillation," *Commentarii Mathematici Helvetici*, vol. 52, no. 4, pp. 591–602, 1977.
[10] A. Aleman and J. A. Cima, "An integral operator on H^p and Hardy's inequality," *Journal d'Analyse Mathématique*, vol. 85, pp. 157–176, 2001.
[11] A. G. Siskakis and R. Zhao, "A Volterra type operator on spaces of analytic functions," in *Function Spaces*, vol. 232 of *Contemporary Mathematics*, pp. 299–311, American Mathematical Society, Providence, RI, USA, 1999.
[12] S. Li and S. Stević, "Generalized composition operators on Zygmund spaces and Bloch type spaces," *Journal of Mathematical Analysis and Applications*, vol. 338, no. 2, pp. 1282–1295, 2008.
[13] S. Li and S. Stević, "Products of composition and integral type operators from H^∞ to the Bloch space," *Complex Variables and Elliptic Equations*, vol. 53, no. 5, pp. 463–474, 2008.

[14] S. Li and S. Stević, “Products of Volterra type operator and composition operator from H^∞ and Bloch spaces to Zygmund spaces,” *Journal of Mathematical Analysis and Applications*, vol. 345, no. 1, pp. 40–52, 2008.

[15] S. Li and S. Stević, “Products of integral-type operators and composition operators between Bloch-type spaces,” *Journal of Mathematical Analysis and Applications*, vol. 349, no. 2, pp. 596–610, 2009.

[16] S. Stević, “Norms of some operators from Bergman spaces to weighted and Bloch-type spaces,” *Utilitas Mathematica*, vol. 76, pp. 59–64, 2008.

[17] D.-C. Chang, S. Li, and S. Stević, “On some integral operators on the unit polydisk and the unit ball,” *Taiwanese Journal of Mathematics*, vol. 11, no. 5, pp. 1251–1285, 2007.

[18] D.-C. Chang and S. Stević, “Estimates of an integral operator on function spaces,” *Taiwanese Journal of Mathematics*, vol. 7, no. 3, pp. 423–432, 2003.

[19] S. Stević, “Cesàro averaging operators,” *Mathematische Nachrichten*, vol. 248-249, no. 1, pp. 185–189, 2003.

[20] S. Stević, “Boundedness and compactness of an integral operator on a weighted space on the polydisc,” *Indian Journal of Pure and Applied Mathematics*, vol. 37, no. 6, pp. 343–355, 2006.

[21] S. Stević, “Boundedness and compactness of an integral operator in a mixed norm space on the polydisk,” *Sibirskiĭ Matematicheskiĭ Zhurnal*, vol. 48, no. 3, pp. 694–706, 2007.

[22] K. Avetisyan and S. Stević, “Extended Cesàro operators between different Hardy spaces,” *Applied Mathematics and Computation*, vol. 207, no. 2, pp. 346–350, 2009.

[23] Z. Hu, “Extended Cesàro operators on mixed norm spaces,” *Proceedings of the American Mathematical Society*, vol. 131, no. 7, pp. 2171–2179, 2003.

[24] Z. Hu, “Extended Cesáro operators on the Bloch space in the unit ball of $\mathbb{C}^n$,” *Acta Mathematica Scientia. Series B*, vol. 23, no. 4, pp. 561–566, 2003.

[25] Z. Hu, “Extended Cesàro operators on Bergman spaces,” *Journal of Mathematical Analysis and Applications*, vol. 296, no. 2, pp. 435–454, 2004.

[26] S. Li, “Riemann-Stieltjes operators from $F(p,q,s)$ spaces to α-Bloch spaces on the unit ball,” *Journal of Inequalities and Applications*, vol. 2006, Article ID 27874, 14 pages, 2006.

[27] S. Li and S. Stević, “Integral type operators from mixed-norm spaces to α-Bloch spaces,” *Integral Transforms and Special Functions*, vol. 18, no. 7-8, pp. 485–493, 2007.

[28] S. Li and S. Stević, “Riemann-Stieltjes operators on Hardy spaces in the unit ball of $\mathbb{C}^n$,” *Bulletin of the Belgian Mathematical Society. Simon Stevin*, vol. 14, no. 4, pp. 621–628, 2007.

[29] S. Li and S. Stević, “Riemann-Stieltjes-type integral operators on the unit ball in $\mathbb{C}^n$,” *Complex Variables and Elliptic Equations*, vol. 52, no. 6, pp. 495–517, 2007.

[30] S. Li and S. Stević, “Compactness of Riemann-Stieltjes operators between $F(p,q,s)$ spaces and α-Bloch spaces,” *Publicationes Mathematicae Debrecen*, vol. 72, no. 1-2, pp. 111–128, 2008.

[31] S. Li and S. Stević, “Riemann-Stieltjes operators between different weighted Bergman spaces,” *Bulletin of the Belgian Mathematical Society. Simon Stevin*, vol. 15, no. 4, pp. 677–686, 2008.

[32] S. Li and S. Stević, “Riemann-Stieltjes operators between mixed norm spaces,” *Indian Journal of Mathematics*, vol. 50, no. 1, pp. 177–188, 2008.

[33] S. Li and S. Stević, “Cesàro-type operators on some spaces of analytic functions on the unit ball,” *Applied Mathematics and Computation*, vol. 208, no. 2, pp. 378–388, 2009.

[34] S. Stević, “On an integral operator on the unit ball in $\mathbb{C}^n$,” *Journal of Inequalities and Applications*, vol. 2005, no. 1, pp. 81–88, 2005.

[35] X. Tang, “Extended Cesàro operators between Bloch-type spaces in the unit ball of $\mathbb{C}^n$,” *Journal of Mathematical Analysis and Applications*, vol. 326, no. 2, pp. 1199–1211, 2007.

[36] J. Xiao, “Riemann-Stieltjes operators on weighted Bloch and Bergman spaces of the unit ball,” *Journal of the London Mathematical Society. Second Series*, vol. 70, no. 1, pp. 199–214, 2004.

[37] S. Stević, “Compactness of the Hardy-Littlewood operator on some harmonic function spaces,” *Siberian Mathematical Journal*, vol. 50, no. 1, pp. 167–180, 2009.

[38] S. Stević, “On a new operator from H^∞ to the Bloch-type space on the unit ball,” *Utilitas Mathematica*, vol. 77, pp. 257–263, 2008.

[39] S. Stević, “On a new integral-type operator from the weighted Bergman space to the Bloch-type space on the unit ball,” *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 154263, 14 pages, 2008.

[40] K. Madigan and A. Matheson, “Compact composition operators on the Bloch space,” *Transactions of the American Mathematical Society*, vol. 347, no. 7, pp. 2679–2687, 1995.