Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2008, Article ID 915292, 8 pages
doi:10.1155/2008/915292

Research Article

Approximation of Generalized Left Derivations

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Received 26 February 2008; Accepted 15 April 2008

Recommended by Paul Eloe

We need to take account of the superstability for generalized left derivations (resp., generalized derivations) associated with a Jensen-type functional equation, and we also deal with problems for the Jacobson radical ranges of left derivations (resp., derivations).

1. Introduction

Let $\mathcal{A}$ be an algebra over the real or complex field $\mathbb{F}$. An additive mapping $d : \mathcal{A} \to \mathcal{A}$ is said to be a *left derivation* (resp., *derivation*) if the functional equation $d(xy) = xd(y) + yd(x)$ (resp., $d(xy) = xd(y) + d(x)y$) holds for all $x, y \in \mathcal{A}$. Furthermore, if the functional equation $d(\lambda x) = \lambda d(x)$ is valid for all $\lambda \in \mathbb{F}$ and all $x \in \mathcal{A}$, then d is a *linear left derivation* (resp., *linear derivation*). An additive mapping $G : \mathcal{A} \to \mathcal{A}$ is called a *generalized left derivation* (resp., *generalized derivation*) if there exists a left derivation (resp., derivation) $\delta : \mathcal{A} \to \mathcal{A}$ such that the functional equation $G(xy) = xG(y) + y\delta(x)$ (resp., $G(xy) = xG(y) + \delta(x)y$) is fulfilled for all $x \in \mathcal{A}$. In addition, if the functional equations $G(\lambda x) = \lambda G(x)$ and $\delta(\lambda x) = \lambda\delta(x)$ hold for all $\lambda \in \mathbb{F}$ and all $x \in \mathcal{A}$, then G is a *linear generalized left derivation* (resp., *linear generalized derivation*).

It is of interest to consider the concept of stability for a functional equation arising when we replace the functional equation by an inequality which acts as a perturbation of the equation. The study of stability problems had been formulated by Ulam [1] during a talk in 1940: *"Under what condition does there exists a homomorphism near an approximate homomorphism?"* In the following year 1941, Hyers [2] was answered affirmatively the question of Ulam for Banach spaces, which states that *if* $\varepsilon > 0$ *and* $f : \mathcal{X} \to \mathcal{Y}$ *is a map with* $\mathcal{X}$*, a normed space,* $\mathcal{Y}$*, a Banach space, such that*

$$\left\| f(x+y) - f(x) - f(y) \right\| \le \varepsilon, \tag{1.1}$$

for all $x, y \in \mathcal{X}$, then there exists a unique additive map $T : \mathcal{X} \to \mathcal{Y}$ such that

$$\|f(x) - T(x)\| \le \varepsilon, \tag{1.2}$$

for all $x \in \mathcal{X}$. Moreover, if $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed x in $\mathcal{X}$, where $\mathbb{R}$ denotes the set of real numbers, then T is linear. This stability phenomenon is called the *Hyers-Ulam stability* of the additive functional equation $f(x + y) = f(x) + f(y)$. A generalized version of the theorem of Hyers for approximately additive mappings was given by Aoki [3] and for approximate linear mappings was presented by Rassias [4] in 1978 by considering the case when the inequality (1.1) is unbounded. Due to the fact that the additive functional equation $f(x + y) = f(x) + f(y)$ is said to have *the Hyers-Ulam-Rassias stability* property. The stability result concerning derivations between operator algebras was first obtained by Šemrl [5]. Recently, Badora [6] gave a generalization of the Bourgin's result [7]. He also dealt with the Hyers-Ulam stability and the Bourgin-type superstability of derivations in [8].

In 1955, Singer and Wermer [9] obtained a fundamental result which started investigation into the ranges of linear derivations on Banach algebras. The result, which is called the Singer-Wermer theorem, states that any continuous linear derivation on a commutative Banach algebra maps into the Jacobson radical. They also made a very insightful conjecture, namely, that the assumption of continuity is unnecessary. This was known as the Singer-Wermer conjecture and was proved in 1988 by Thomas [10]. The Singer-Wermer conjecture implies that any linear derivation on a commutative semisimple Banach algebra is identically zero which is the result of Johnson [11]. After then, Hatori and Wada [12] showed that a zero operator is the only derivation on a commutative semisimple Banach algebra with the maximal ideal space without isolated points. Note that this differs from the above result of B.E. Johnson. Based on these facts and a private communication with Watanabe [13], Miura et al. proved the Hyers-Ulam-Rassias stability and Bourgin-type superstability of derivations on Banach algebras in [13]. Various stability results are given by Moslehian and Park, see, for example, [14–18].

The main purpose of the present paper is to consider the superstability of generalized left derivations (resp., generalized derivations) on Banach algebras associated to the following Jensen type functional equation:

$$f\left(\frac{x+y}{k}\right) = \frac{f(x)}{k} + \frac{f(y)}{k}, \tag{1.3}$$

where $k > 1$ is an integer. This functional equation is introduced in [19]. Moreover, we will investigate the problems for the Jacobson radical ranges of left derivations (resp., derivations) on Banach algebras. We use the direct method and some ideas of Amyari et al. [19].

2. Main results

Throughout this paper, the element e of an algebra will denote a unit. We now establish the superstability of a generalized left derivation associated with the Jensen type functional equation as follows.

Theorem 2.1. *Let $\mathcal{A}$ be a Banach algebra with unit. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exists a mapping $g : \mathcal{A} \to \mathcal{A}$ such that the functional inequality:*

$$\left\| f\left(\frac{x+y}{k} + zw\right) - \frac{f(x)}{k} - \frac{f(y)}{k} - zf(w) - wg(z) \right\| \leq \varepsilon, \tag{2.1}$$

for all $x, y, z, w \in \mathcal{A}$. Then, f is a generalized left derivation, and g is a left derivation.

Proof. Substituting $w = 0$ in (2.1), we get

$$\left\| f\left(\frac{x+y}{k}\right) - \frac{f(x)}{k} - \frac{f(y)}{k} \right\| \leq \varepsilon, \tag{2.2}$$

for all $x, y \in \mathcal{A}$. Let us take $y = 0$ and replace x by kx in the above relation. Then, it becomes

$$\left\| f(x) - \frac{f(kx)}{k} \right\| \leq \varepsilon, \tag{2.3}$$

for all $x \in \mathcal{A}$. An induction implies that

$$\left\| \frac{f(k^n x)}{k^n} - f(x) \right\| \leq \frac{k}{k-1}\left(1 - \frac{1}{k^n}\right)\varepsilon, \tag{2.4}$$

for all $x \in \mathcal{A}$. By virtue of (2.4), one can easily check that for $n > m$,

$$\begin{aligned} \left\| \frac{f(k^n x)}{k^n} - \frac{f(k^m x)}{k^m} \right\| &= \frac{1}{k^m} \left\| \frac{f(k^{n-m} \cdot k^m x)}{k^{n-m}} - f(k^m x) \right\| \\ &\leq \frac{1}{k^{m-1}(k-1)}\left(1 - \frac{1}{k^{n-m}}\right)\varepsilon, \end{aligned} \tag{2.5}$$

for all $x \in \mathcal{A}$. So, the sequence $\{f(k^n x)/k^n\}$ is Cauchy. Since $\mathcal{A}$ is complete, $\{f(k^n x)/k^n\}$ converges. Let $d : \mathcal{A} \to \mathcal{A}$ be the mapping defined by $(x \in \mathcal{A})$

$$d(x) := \lim_{n \to \infty} \frac{f(k^n x)}{k^n}. \tag{2.6}$$

By letting $n \to \infty$ in (2.4), we get

$$\| f(x) - d(x) \| \leq \frac{k}{k-1}\varepsilon, \tag{2.7}$$

for all $x \in \mathcal{A}$.

Now, we assert that d is additive. Replacing x and y by $k^n x$ and $k^n y$ in (2.2), respectively, we have

$$\left\| \frac{1}{k^n} f\left(\frac{k^n x + k^n y}{k}\right) - \frac{1}{k}\frac{f(k^n x)}{k^n} - \frac{1}{k}\frac{f(k^n y)}{k^n} \right\| \leq \frac{1}{k^n}\varepsilon, \tag{2.8}$$

for all $x, y \in \mathcal{A}$, taking the limit as $n \to \infty$, we obtain

$$d\left(\frac{x+y}{k}\right) = \frac{d(x)}{k} + \frac{d(y)}{k}, \tag{2.9}$$

for all $x, y \in \mathcal{A}$. Letting $y = 0$ in the previous identity yields $d(x/k) = d(x)/k$ for all $x \in \mathcal{A}$. So, (2.9) becomes $d(x+y) = d(x) + d(y)$, for all $x, y \in \mathcal{A}$, namely, d is additive.

To demonstrate the uniqueness of the additive mapping d subject to (2.7), we assume that there exists another additive mapping $D : \mathcal{A} \to \mathcal{A}$ satisfying the inequality (2.7), for all $x \in \mathcal{A}$. Since $D(k^n x) = k^n D(x)$ and $d(k^n x) = k^n d(x)$, we see that

$$\begin{aligned} \|D(x) - d(x)\| &= \frac{1}{k^n}\|D(k^n x) - d(k^n x)\| \\ &\le \frac{1}{k^n}\left[\|D(k^n x) - f(k^n x)\| + \|f(k^n x) - d(k^n x)\|\right] \\ &\le \frac{2}{k^{n-1}(k-1)}\varepsilon, \end{aligned} \tag{2.10}$$

for all $x \in \mathcal{A}$. By letting $n \to \infty$ in this inequality, we conclude that $D = d$, that is, d is unique.

Next, we are going to prove that f is a generalized left derivation. If we take $x = y = 0$ in (2.1), we also have

$$\|f(zw) - zf(w) - wg(z)\| \le \varepsilon, \tag{2.11}$$

for all $z, w \in \mathcal{A}$. Moreover, if we replace z and w with $k^n z$ and $k^n w$ in (2.11), respectively, and then divide both sides by k^{2n}, we get

$$\left\|\frac{f(k^{2n}zw)}{k^{2n}} - z\frac{f(k^n w)}{k^n} - w\frac{g(k^n z)}{k^n}\right\| \le \frac{1}{k^{2n}}\varepsilon, \tag{2.12}$$

for all $z, w \in \mathcal{A}$. Letting $n \to \infty$, we obtain

$$\lim_{n\to\infty} w\frac{g(k^n z)}{k^n} = d(zw) - zd(w), \tag{2.13}$$

for all $z, w \in \mathcal{A}$. Suppose that $w = e$ in the above equation. Then, it follows

$$\lim_{n\to\infty} \frac{g(k^n z)}{k^n} = d(z) - zd(e), \tag{2.14}$$

for all $z \in \mathcal{A}$. Thus, if $\delta(z) := d(z) - zd(e)$, then by the additivity of d, we get

$$\begin{aligned} \delta(x+y) &= d(x) + d(y) - xd(e) - yd(e) \\ &= (d(x) - xd(e)) + (d(y) - yd(e)) = \delta(x) + \delta(y), \end{aligned} \tag{2.15}$$

for all $x \in \mathcal{A}$. Hence, δ is additive.

Let $\Delta(z,w) = f(zw) - zf(w) - wg(z)$, for all $z,w \in \mathcal{A}$. Since, f and g satisfy the inequality given in (2.11), then

$$\lim_{n\to\infty} \frac{\Delta(k^n z, w)}{k^n} = 0, \tag{2.16}$$

for all $z,w \in \mathcal{A}$. We note that

$$\begin{aligned} d(zw) &= \lim_{n\to\infty} \frac{f(k^n zw)}{k^n} = \lim_{n\to\infty} \frac{f(k^n z \cdot w)}{k^n} \\ &= \lim_{n\to\infty} \frac{k^n z f(w) + w g(k^n z) + \Delta(k^n z, w)}{k^n} \\ &= \lim_{n\to\infty} \left\{ z f(w) + w \frac{g(k^n z)}{k^n} + \frac{\Delta(k^n z, w)}{k^n} \right\} \\ &= z f(w) + w\delta(z), \end{aligned} \tag{2.17}$$

for all $z,w \in \mathcal{A}$. Since δ is additive, we can rewrite (2.17) as

$$k^n z f(w) + k^n w \delta(z) = d(k^n z \cdot w) = d(z \cdot k^n w) = z f(k^n w) + k^n w \delta(z), \tag{2.18}$$

for all $z,w \in \mathcal{A}$. Based on the above relation, one has $zf(w) = z(f(k^n w)/k^n)$, for all $z,w \in \mathcal{A}$. Moreover, we can obtain $zf(w) = zd(w)$, for all $z,w \in \mathcal{A}$ as $n \to \infty$. If $z = e$, we also have that $f = d$. Therefore, we get

$$f(zw) = z f(w) + w\delta(z), \tag{2.19}$$

for all $z,w \in \mathcal{A}$.

We now want to verify that δ is a left derivation using the equations developed in the previous part. Indeed, using the facts that f satisfies (2.19), we have

$$\begin{aligned} \delta(xy) &= f(xy) - xyf(e) = xf(y) + y\delta(x) - xyf(e) \\ &= x\big(f(y) - yf(e)\big) + y\delta(x) = x\delta(y) + y\delta(x), \end{aligned} \tag{2.20}$$

for all $x,y \in \mathcal{A}$, which means that f is a generalized left derivation.

We finally need to show that g is a left derivation. Let us replace w by $k^n w$ in (2.11). Then,

$$\left\| \frac{f(k^n zw)}{k^n} - z\frac{f(k^n w)}{k^n} - wg(z) \right\| \le \frac{1}{k^n}\varepsilon, \tag{2.21}$$

for all $z,w \in \mathcal{A}$. Passing the limit as $n \to \infty$, we get

$$d(zw) - zd(w) - wg(z) = 0, \tag{2.22}$$

for all $z,w \in \mathcal{A}$. This implies that $d(zw) = zd(w) + wg(z)$, for all $z,w \in \mathcal{A}$, and thus if $w = e$, we deduce that $g(z) + zd(e) = d(z)$, for all $z \in \mathcal{A}$. Hence, we get $g(z) = d(z) - zd(e) = \delta(z)$, for all $z \in \mathcal{A}$. Since, δ is a left derivation, we can conclude that g is a left derivation as well. This completes the proof of the theorem. □

Employing the similar way as in the proof of Theorem 2.1, we get the following result for a generalized derivation.

Theorem 2.2. *Let $\mathcal{A}$ be a Banach algebra with unit. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exists a mapping $g : \mathcal{A} \to \mathcal{A}$ such that*

$$\left\| f\left(\frac{x+y}{k} + zw\right) - \frac{f(x)}{k} - \frac{f(y)}{k} - zf(w) - g(z)w \right\| \leq \varepsilon, \tag{2.23}$$

for all $x, y, z, w \in \mathcal{A}$. Then, f is a generalized derivation, and g is a derivation.

In view of the Thomas' result [10], derivations on Banach algebras now belong to the noncommutative setting. Among various noncommutative version of the Singer-Wermer theorem, Brešar and Vukman [20] proved the following. *Any continuous linear left derivation on a Banach algebra maps into its Jacobson radical and also any left derivation on a semiprime ring is a derivation which maps into its center.*

The following is the functional inequality with the problem as in the above Brešar and Vukman's result.

Theorem 2.3. *Let $\mathcal{A}$ be a semiprime Banach algebra with unit. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exists a mapping $g : \mathcal{A} \to \mathcal{A}$ such that the functional inequality:*

$$\left\| f\left(\frac{\alpha x+\beta y}{k} + zw\right) - \alpha\frac{f(x)}{k} - \beta\frac{f(y)}{k} - zf(w) - wg(z) \right\| \leq \varepsilon, \tag{2.24}$$

for all $x, y, z, w \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U} = \{z \in \mathbb{C} : |z| = 1\}$. Then, f is a linear generalized left derivation. In this case, g is a linear derivation which maps $\mathcal{A}$ into the intersection of its center $Z(\mathcal{A})$ and its Jacobson radical $\operatorname{rad}(\mathcal{A})$.

Proof. We consider $\alpha = \beta = 1 \in \mathbb{U}$ in (2.24) and then f satisfies the inequality (2.1). It follows from Theorem 2.1 that f is a generalized left derivation, and g is a left derivation, where

$$f(x) := \lim_{n\to\infty} \frac{f(k^n x)}{k^n}, \qquad g(x) := f(x) - xf(e), \tag{2.25}$$

for all $x \in \mathcal{A}$. Letting $w = 0$ in (2.24), we have

$$\left\| f\left(\frac{\alpha x+\beta y}{k}\right) - \alpha\frac{f(x)}{k} - \beta\frac{f(y)}{k} \right\| \leq \varepsilon, \tag{2.26}$$

for all $x, y \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$. If we also replace x and y with $k^n x$ and $k^n y$ in (2.26), respectively, and then divide both sides by k^n, we see that

$$\left\| \frac{1}{k^n} f\left(\frac{\alpha k^n x+\beta k^n y}{k}\right) - \alpha\frac{1}{k}\frac{f(k^n x)}{k^n} - \beta\frac{1}{k}\frac{f(k^n y)}{k^n} \right\| \leq \frac{1}{k^n}\varepsilon \longrightarrow 0, \tag{2.27}$$

for all $x, y \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$, as $n \to \infty$. So, we get

$$f\left(\frac{\alpha x+\beta y}{k}\right) = \alpha\frac{f(x)}{k} + \beta\frac{f(y)}{k}, \tag{2.28}$$

for all $x, y \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$. From the additivity of f, we find that

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y), \tag{2.29}$$

for all $x, y \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U}$. Let us now assume that λ is a nonzero complex number and that L a positive integer greater than $|\lambda|$. Then by applying a geometric argument, there exist $\lambda_1, \lambda_2 \in \mathbb{U}$ such that $2(\lambda/L) = \lambda_1 + \lambda_2$. In particular, due to the additivity of f, we obtain $f((1/2)x) = (1/2)f(x)$ for all $x \in \mathcal{A}$. Thus, we have

$$\begin{aligned} f(\lambda x) &= f\left(\frac{L}{2} \cdot 2 \cdot \frac{\lambda}{L} x\right) = Lf\left(\frac{1}{2} \cdot 2 \cdot \frac{\lambda}{L} x\right) = \frac{L}{2} f((\lambda_1 + \lambda_2)x) \\ &= \frac{L}{2}(\lambda_1 + \lambda_2) f(x) = \frac{L}{2} \cdot 2 \cdot \frac{\lambda}{L} f(x) = \lambda f(x), \end{aligned} \tag{2.30}$$

for all $x \in \mathcal{A}$. Also, it is obvious that $f(0x) = 0 = 0f(x)$, for all $x \in \mathcal{A}$, that is, f is $\mathbb{C}$-linear. Therefore, f is a linear generalized left derivation, and so g is also a linear left derivation. According to the Brešar and Vukman's result which tells us that g is a linear derivation which maps $\mathcal{A}$ into its center $Z(\mathcal{A})$. Since $Z(\mathcal{A})$ is a commutative Banach algebra, the Singer-Wermer conjecture tells us that $g_{|Z(\mathcal{A})}$ maps $Z(\mathcal{A})$ into $\operatorname{rad}(Z(\mathcal{A})) = Z(\mathcal{A}) \cap \operatorname{rad}(\mathcal{A})$ and thus $g^2(\mathcal{A}) \subseteq \operatorname{rad}(\mathcal{A})$. Using the semiprimeness of $\operatorname{rad}(\mathcal{A})$ as well as the identity, we have

$$2g(x)yg(x) = g^2(xyx) - xg^2(yx) - g^2(xy)x + xg^2(y)x, \tag{2.31}$$

for all $x, y \in \mathcal{A}$, we have $g(\mathcal{A}) \subseteq \operatorname{rad}(\mathcal{A})$, that is, g is a linear derivation which maps $\mathcal{A}$ into the intersection of its center $Z(\mathcal{A})$ and its Jacobson radical $\operatorname{rad}(\mathcal{A})$. The proof of the theorem is ended. □

The next corollary is the Brešar and Vukman's result.

Corollary 2.4. *Let $\mathcal{A}$ be a Banach algebra with unit. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a continuous mapping with $f(0) = 0$ for which there exists a mapping $g : \mathcal{A} \to \mathcal{A}$ such that the functional inequality* (2.26)*. Then, $f : \mathcal{A} \to \mathcal{A}$ is a linear generalized left derivation. In this case, g maps $\mathcal{A}$ into its Jacobson radical* $\operatorname{rad}(\mathcal{A})$.

Proof. On account of Theorem 2.3, g is a linear left derivation on $\mathcal{A}$. Hence, g maps $\mathcal{A}$ into its Jacobson radical $\operatorname{rad}(\mathcal{A})$ by the Brešar and Vukman's result, which completes the proof. □

With the help of Theorem 2.2, the following property can be derived along the same argument in the proof of Theorem 2.3.

Theorem 2.5. *Let $\mathcal{A}$ be a commutative Banach algebra with unit. Suppose that $f : \mathcal{A} \to \mathcal{A}$ is a mapping with $f(0) = 0$ for which there exists a mapping $g : \mathcal{A} \to \mathcal{A}$ such that the functional inequality:*

$$\left\| f\left(\frac{\alpha x + \beta y}{k} + zw\right) - \alpha \frac{f(x)}{k} - \beta \frac{f(y)}{k} - zf(w) - g(z)w \right\| \leq \varepsilon, \tag{2.32}$$

for all $x, y, z, w \in \mathcal{A}$ and all $\alpha, \beta \in \mathbb{U} = \{z \in \mathbb{C} : |z| = 1\}$. Then, f is a linear generalized derivation. In this case, g maps $\mathcal{A}$ into its Jacobson radical $\operatorname{rad}(\mathcal{A})$.

Remark 2.6. We can generalize our results by substituting another functions or another forms satisfying suitable conditions (see, e.g., [19, 21]) for the bound ε of the functional inequalities connected to the Jensen type functional equation.

Acknowledgments

The authors would like to thank referees for their valuable comments regarding a previous version of this paper. The corresponding author dedicates this paper to his late father.

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